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MYERS ARITHMETIC

FOR

GRAMMAR SCHOOLS

BY

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PREFACE

This book continues the work of the Myers Elementary Arithmetic and covers comprehensively the work of the last four years of the elementary school. It is a topical treatment of the subject and is thus in itself a complete arithmetic.

The educational reason for arithmetic as a school subject is that it is a type of the organization of experience that the world deems useful in daily life. The experiences which arithmetic should organize are sensations of measuring and numbering. In order that this measuring and numbering experience may be organized into the most efficient practical knowledge, the work must be done in harmony with the mind's way of construing these sensations into relationships. Nothing is more fundamentally important than that the pupil actually have the experiences that are to be organized. The ready-made experiences of others cannot be substituted for those of the learner himself. Furthermore, the pupil should organize as well as experience these sensations.

The requirement that the sensations to be organized shall be the pupil's own can be met only by the plentiful use of concrete material. The objects, pictures, real problems, and other concrete materials must be not merely talked about by the teacher, the pupil passively listening and looking on. The pupil must handle the objects, draw the sketches, examine them and even make verbal problems. He must be allowed to make discoveries for himself. The concrete material used may be very simple indeed, as sticks, blocks, dots, marks, squares, oblongs, verbal problems, if the pupil himself do the handling,

counting, grouping, measuring, to the end that he *experience* the relations to be organized. A variety of simple objects will in fact serve the purposes of number sensations better than will a few elaborate and artificial objects, made for the pure purpose of exemplifying number relations. For example, so simple a group of number truths, as 2 dots and 3 dots are 5 dots, 2 marks and 3 marks are 5 marks, 2 inches and 3 inches are 5 inches, 2 children and 3 children are 5 children, 2 sticks and 3 sticks are 5 sticks, 2 things and 3 things are 5 things, given in quick succession, leading the child to say, or to feel, "Why, it's always that way; 2 and 3 are always 5," is for number purposes as good an exercise in child-like generalizing as can be devised. Then the correlative subtraction truths 3 from 5 and 2 from 5, and the supplementary addition truth 3 and 2 are 5 should be presented at the same time. All the simple addition combinations should first be arrived at by the pupil in this fashion. Such truths as $3+2=5$, $4+5=9$, would then stand for organized experience.

But important as is the concrete material as an aid in early work, this material is only an aid. The chief business is to organize the number truths exemplified by this material into definite arithmetical knowledge. It is important not only at the beginning but all along the way that the arithmetical truths be based on the concrete, but the weightier matter is to bring out clearly and to teach thoroughly the number truths.

The use of concrete material is to a large extent a function of the teaching. And yet a text-book, by its plan, by the nature of its problems and problem material, and by the mode and order of presentation, can be of real assistance to the busy teacher. Accordingly, the problems and modes of treatment of this book may be regarded as in the nature of types and suggestions. It may even be that teachers may here and there prefer problem material of a more distinctly local interest than are the problems

of the text. The treatments here given will suggest how such substituted material may best be handled to secure the desired arithmetical result.

Experimental pedagogy has been active in recent years in testing school work by objective standards. No subject has come in for a larger share of this testing than arithmetic. Some of the findings for arithmetic are:—

1. That what we speak of as arithmetical ability is in reality a complex of many separate abilities.

2. That the several abilities, such as those used in the process of addition and subtraction, and that for analysis, may or may not represent a single unified arithmetical ability. When the separate abilities do not combine to produce a unified ability, high power and skill in any one line may co-exist with very low power and skill in others. When they do combine, power and skill of one kind serve to heighten power and skill of related kinds. The more completely the several processes and kinds of work are kept together in teaching, and the more persistently they are returned to from time to time, and from year to year, the more completely are the several particular abilities fused into a composite single ability.

3. That methods of teaching can be weighed and tested with precise objective certainty.

4. That the most effective method is that in which arithmetical truths are grouped into small unities in the early grades, into ever larger unities as the grades are passed, and into topical units in the later grades.

5. That in the grammar school a forecast of the main theme of each year's work should be given at the close of the preceding year, while its basal notions are most vivid and that at the beginning of each year a summarizing review and synopsis of the work of the preceding year should be given. This brings each main topic repeatedly into the learner's consciousness in a fuller way as he proceeds.

There has been a very earnest attempt to make the plan and organization of this book conform to these practical findings. This will be seen from the following outline of the several parts.

Part I, for the fifth grade, teaches the Arabic and Roman notations, the fundamental processes, including the most useful forms of factoring and cancellation, and then reviews the processes thoroughly, both formally and through real problems. It then gives all the most useful aspects of common fractions, and makes the transition from common to decimal fractions, closing with a general review. The core of the year's work is the topical treatment of Notation and Numeration, the Fundamental Processes, and Common Fractions.

Part II, for the sixth grade, reviews all former processes rapidly through problem material of such character that many practical mensuration truths are learned, treats decimal fractions and compound denominate numbers topically, and passes from decimals to an easy first view of both percentage and interest. The year's work closes with a careful review. The main themes of this year are Decimal Fractions and Compound Denominate Numbers.

Part III, for the seventh grade, is organized on the plan of a general review through practical problems of preceding topics, including accounts and account-keeping incidentally, then ratio and proportion, percentage and interest, and a very vigorous review. In connection with the review of decimal fractions, the circumference and the area of the circle are taught. The main subjects of this year's work are Ratio and Proportion, Percentage, and Interest.

Part IV, for grade eight, opens with a review, passes on through the modeling of solids, and the treatment of angles and arcs, to mensuration proper. Here all preceding mensuration truths are summarized and the mensuration of the common solids is taught. Square root

is carefully treated and enough cube root for elementary school possibilities and needs. Then follow more difficult applications of percentage and interest, such as insurance, taxes, bank discount, stocks and bonds, etc. A little work on general number and a thorough review through formal and practical problems of all arithmetic close the year. The main themes of this year's work are Rationalized Mensuration, Longitude and Time, Square Root, and the difficult Applications of Percentage.

So much of the material of this book has been carefully tested from so many practical points of view, that it cannot be looked upon as new or experimental. Numerous suggestions from quarters so diverse as to make it impossible to accredit them properly have been incorporated to the very great improvement of the book. The writer is no less grateful for the suggestions because he finds it impossible to name their authors.

The author's acknowledgments are especially due to Mr. F. W. Buchholz, of The Chicago Teachers College, for painstaking criticisms and suggestions that have greatly improved the book.

Chicago, May, 1912. THE AUTHOR.

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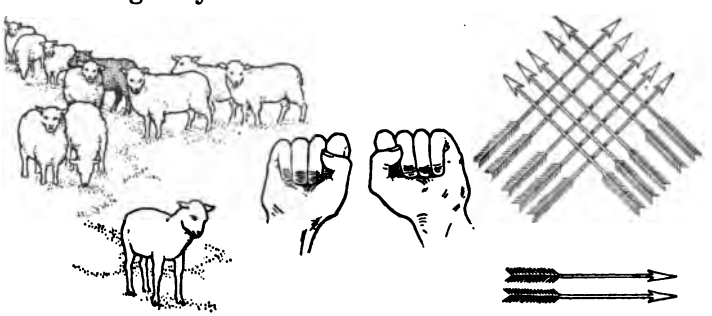
PART I

NOTATION AND NUMERATION

§1. Number Names. The pupil has already learned the common number names: one, two, three, four, five, six, seven, eight, nine, ten.

The story of *eleven* and *twelve* is interesting.

Early people did not count as we do. In reality instead of counting they **tallied.**



Instead of saying one sheep, two sheep, three sheep, and so on, they turned down one finger for each sheep. When all the fingers of one hand had been used as tallies, they said they had **a hand of sheep.** Then if there were more sheep, they bent down a finger on the other hand, then another finger, and so on, until all fingers and thumbs were used as tallies, and they then said **a man of sheep.***

They counted other objects, as warriors, tribesmen, arrows, axes, etc., in the same way. If after "a man of objects" had been tallied off, one object was left over, they said **ain-lif,** meaning *one left.* From this word *ain-lif* we

*Some tribes used the toes also in tallying. With these tribes "a man" of objects meant 20 objects.

get our word "eleven." Our word eleven then means: *One left after 10 are tallied, or counted.*

If after ten objects had been tallied off, two were left untallied, they said **twa-lif,** meaning *two left.* From this *twa-lif* comes our word "twelve." It means: *Two left after 10 are tallied, or counted.*

From here on our number names follow a regular law of formation.

In number words the syllable **-teen** means *ten,* and **-ty** means *tens.*

Thirteen means 3 and 10, **fourteen** means 4 and 10, and so on.

1. Give the meanings of fifteen; sixteen; seventeen; eighteen; nineteen.

Twenty means two tens; **thirty** means three tens, and so on.

2. Give the meanings of forty; fifty; sixty; seventy; eighty; ninety.

We have also the number words: **hundred,** standing for 10 tens; **thousand,** for 10 hundreds; **million,** for 1000 thousands; **billion,** for 1000 millions; **trillion,** for 1000 billions, and so on for **quadrillion, quintillion, sextillion, septillion, octillion, nonillion,** and **decillion.**

3. With the last four names compare the names of the months of the year.

To count through millions we have to learn the meanings and the use of only 15 number names and two number syllables, or suffixes.

All number names are made up of these 15 names and two suffixes.

It is also because men used their 10 fingers, or *digits,* so generally in tallying and counting that they came to use the word **digits** for the number symbols that are commonly called figures, and that they chose 10 as the **base** of our system of numbers. We shall soon see that 10 lies at the *basis* of place value of the digits.

§2. Digits. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

1. How many characters are given above? By what names do you know the first character? It is called *zero*, or *naught*.

These ten characters are called DIGITS or figures.

By means of these ten digits all numbers may be expressed.

§3. Place and Name Values of Digits.

1. Read the following numbers:

$30	3	3000
300 mi.	30	30,000
3 yd.	300	300,000

What is the same in all these numbers? What is different in the numbers?

2. Read:

7 feet 7 days 7 hundreds 7 millions 7 tenths

In all the numbers above, notice that the name of the *digit* is the same; but that the name of the unit of the *number* is different.

3. In the number 666, what does the first 6 on the right mean? The second 6? The third? *1st Ans.* 6 ones, *or* 6 units.

4. The value which depends on the name of a digit is called its *name value*. The value of the place of a digit is called its *place value*.

5. How many values has a digit? What gives the first value? The second? Which is the permanent (unchangeable) value? Which is the changeable value?

§4. Reading Numbers.

For convenience in reading, numbers are separated into groups of 3 digits, or figures, beginning at ones. These groups are often separated by commas. The 3-figure groups are called **periods,** thus:

2d Period			1st Period		
Thousands			Units		
Hundreds	Tens	Ones	Hundreds	Tens	Ones
	6	123			
	49	852			
795		648			

In reading these numbers the figures of each period are read as though they stood alone, then the name of the *period* is added, thus:

6 thousand, 123;
49 thousand, 852;
795 thousand, 648.

After the last period, the units' period, the name of the period that is the word "units," is understood and is therefore omitted.

1. Read the following numbers:

8000	64,387	910,643
4345	30,300	606,606
5060	82,082	450,450

The number 2,461,375 shows the beginning of a new period, called *millions.* This number is read:

2 million, 461 thousand, 375.

2. Read the following numbers:

7,000,000	5,406,406
82,000,000	202,202,202

3. Write the following, using digits:

Nine thousand, ten; ten thousand, nine; six hundred thousand, six hundred; five million, fifty thousand, three; eighty thousand, eighty.

The first three periods with their names and the names of the places in the periods are shown here:

Millions			Thousands			Units		
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
3	2	1	5	8	6	4	9	7

321,586,497

The number, 321,586,497, is read 321 million, 586 thousand, 497. The three-figure groups are read just as any three-figure number, then the name of the period is pronounced.

The next period above millions is called **billions.**

4. The following facts about our nation show the need for large numbers. Read some of them:

The public debt in 1910.....$2,652,665,838
Value of farm live stock, 1900.....$3,978,050,000
Value of farm land and buildings, 1900.....$16,674,690,247
Value of all farm property, 1900.....$20,514,001,838
Value of farm products, 1900.....$4,739,118,752
Corn crop, 1910, 2,772,376,000 bu.; its value. $1,652,822,000
Corn crop, 1909, 2,668,651,000 bu.; its value. $1,616,145,000

Value of farms and estimated wealth of the United States for three years.

	Value Farms	Est'd Wealth
1850	$3,967,343,580	$7,135,780,000
1880	$12,180,501,533	$42,642,000,000
1900	$13,039,279,566	$94,300,000,000

§5. Writing Numbers.

Write in figures the following:

1. Two hundred thirteen million, one hundred twenty-two thousand, six hundred seven; four hundred fifty million, three hundred twenty-seven thousand, one.

2. Two hundred million; one hundred ninety thousand, four hundred two; three million, one hundred twenty-four; fifteen thousand, nine; ninety; one hundred eighty-six million, ninety; one hundred forty-seven million, one hundred thousand, one hundred.

3. The value of the farm horses of the United States in 1910 was two billion, seven hundred seventy-six million, three hundred sixty-three thousand dollars.

4. The public debt of the nation in 1910 was two billion, six hundred fifty-two million, six hundred sixty-five thousand, eight hundred thirty-eight dollars.

The digits were first used by the Hindus. They were introduced into Europe by the Arabs, and we call the way of writing numbers with digits the **Arabic notation.**

Numbers written with these digits are called Arabic numerals.

§6. Roman Notation.

The Romans used letters instead of digits to represent numbers. This notation is still used in numbering chapters and volumes of books and upon monuments and buildings. It is called the **Roman notation.**

1. Write the Arabic numerals for each of the following:

I,	II,	III,	IV,	V,	VI,
VII,	VIII,	IX,	X,	XI,	XX.

In the Roman notation,

L, stands for 50; C, for 100; D, for 500; and M, for 1000.

The laws for using the Roman notation are:

I. *When a letter, or symbol of less value, as I, precedes one of greater value, as V, the value of the less is to be taken from that of the greater; thus, IV means 4, and XC means 90.*

II. *When a letter, or symbol of less value follows one of greater value its value is added to that of the greater; as, VI for 6, XV for 15, and CX for 110.*

III. *Repeating a letter repeats its value; as, XX for 20, CC for 200.*

IV. *A horizontal line over a letter increases its value a thousandfold; as, $\overline{C}$ meaning 100,000, $\overline{D}$ meaning 500,000.*

To change to Arabic notation numbers written in the Roman notation, *always begin on the right.*

2. Change MCDXIX to Arabic numerals:

X means 10
IX means $10-1=9$, by law I
XIX means $9+10=19$, by law II
DXIX means $19+500=519$, by law II
CDXIX means $519-100=419$, by law I
MCDXIX means $419+1000=1419$, by law II

3. Give the value in the Arabic notation of the following numbers:

XIV	DIX	XVI	XXVII	CCCXVI
XIX	LCI	XCXI	XXXIX	MCLXIX
LIV	XLIV	LXIX	CLXXI	MCLVI

Thus, besides the symbol MCDXIX for 1419, MCCCCXIX, or DDCDXIX, might be used; but both are longer than MCDXIX.

4. Write the following numbers in the Roman notation:

9, 4, 6, 14, 29, 13, 78, 44, 83,
59, 94, 96, 104, 199, 335, 549, 2000, 1908.

5. Change from Roman to Arabic numerals:

XV	MD	XIX	DDCV
XX	DM	XV	MDCCC
CCC	MCD	LIX	MCM
XC	MDC	LXI	$\overline{\text{MD}}$

6. Change from Arabic to Roman numerals:

20	69	800	505	1908
38	101	765	900	2000
60	305	648	1900	200,000

Write answers to the following in Roman numerals:

7. Columbus discovered America in MCDXCII. 20 years later Florida was explored. In what year was Florida explored?

8. The first battle of the Revolution was fought in MDCCLXXV. The last battle was fought 6 years later. What was the year of the last battle?

9. Washington was elected President in MDCCLXXXIX. Roosevelt was elected 115 years afterward. In what year was Roosevelt elected?

ADDITION

§7. Definitions.

Answer all you can orally

1. A bicyclist rides 19 miles Monday and 10 miles Tuesday. How far does he ride in both days?

2. I paid \$18 for a suit of clothes and \$30 for a bicycle. How much did I pay for both?

The answer to problem 1 is the same as the answer to the question: "What single distance is just as long as the distances 19 mi. and 10 mi. combined?" Answering problem 2 answers the question: "What single amount equals \$18 and \$30 combined?"

ADDITION is combining numbers into a single number.
THE SUM is the result of addition.
THE ADDENDS are the numbers to be combined or added.
The sign of addition is +. It is read "plus."
The sign, =, when written between two numbers means that they are equal. It should be read "equal," or "equals."

3. Read and answer these questions:

(1) $3+8+4=?$ (3) $12+7+8=?$ (5) $3+15+4+9=?$
(2) $7+6+8=?$ (4) $6+13+7=?$ (6) $6+16+8+9=?$

4. In these questions, what numbers should stand in place of the question mark?

(1)* 2 8's+5 8's=? 8's (3) 7 x's+5 x's=? x's
(2) 12 13's+7 13's=? 13's (4) 9 y's+8 y's=? y's

§8. Adding and Checking.

1. In five trips a street-car carried the following numbers of passengers: 30, 42, 45, 60, 65. Find the whole number of passengers carried.

* Read (1) thus, "2 eights plus 5 eights equal how many eights?"

2. On these trips the conductor collected \$1.50, \$2.10, \$2.25, \$3.00, and \$3.25. The fares for the five trips amounted to how much?

3. In one week a man's street-car fare was 35¢ 25¢, 40¢, 25¢, 30¢, and 30¢. Find the whole amount.

4. A train moves the following distances in five successive hours: 40 mi., 45 mi., 50 mi., 48 mi., 48 mi. What distance is traveled in the given time?

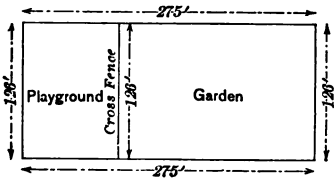


FIGURE 1

5. An eighth grade class staked out a rectangular (square cornered) plot of ground to be used for a playground and school garden. They fenced in the plot and ran a cross fence separating the parts. The entire plot was 126 ft. wide by 275 ft. long. How many feet of fence were needed?

CONVENIENT FORM

275 ft.
126 ft.
275 ft. } Addends
126 ft.
126 ft.
928 ft. Sum

Arrange the addends in convenient form. Beginning at the bottom of units' column, think the sums thus, 6, 12, 17, 23, 28 equals 2 tens and 8 units. Write the 8 in units' column. Add the 2 tens to the tens' digits, thinking (not speaking) 4, 6, 13, 15, 22 tens, equal 2 hundreds and 2 tens, and so on. Since we are adding feet, the sum is 928 ft.

Test the correctness of the sum by beginning at the top of each column and adding downward. If the sum is the same the work is probably correct. This is called **checking** the work.

§9. Exercises.

1. Add:

(1) 187, 892, and 478.
(2) 324, 407, and 282.
(3) 648, 376, and 197.
(4) 97, 526, and 712.
(5) 841, 107, and 358.
(6) 263, 509, and 118.

In adding long columns the number to be carried may be written underneath the column as shown in exercise (1) below.

2. Add:

(1)	(2)	(3)	(4)	(5)	(6)
7843	32765	25987	96875	81818	247
8789	89247	6586	40984	92193	91838
9576	76348	78379	50839	87689	9705
2589	20873	96468	9787	76434	87278
8956	94608	980	67898	68979	3849
3210	13495	20876	76580	37590	89878
7029	68950	67099	54777	89763	79929
47992					
334					

Check the work of problem 2 by adding from the top, downward.

3. Add the following problems in the usual way:

(1)	(2)	(3)	(4)
68	238	237	24567
97	472	48984	89012
89	836	3789	4567
43	297	980 ____ ?	54976 ____ ?	89876 ____ ?
30		722	500	54378
78		146	3897	98989
83		348	57878	24864
29	220	765 ____ ?	36498 ____ ?	3099 ____ ?
48		897	9889	87655
67		305	65847	98788
98		969	678	4890
88	301	878 ____ ?	88769 ____ ?	86767 ____ ?
818	818			

Another way to check is to add the columns as parts, and then add the partial sums as shown in the exercises above.

§10. Populations of Four Divisions of the United States.

Pupils may select such problems from this list as have a special geographical interest to their school.

Check the correctness of your work in the following problems.

1. From the table given below, find the total population of the Middle Atlantic division for 1900; for 1910.

2. Find the total population of the New England division for 1900; for 1910.

3. Find the totals for the Southern division for 1900; for 1910.

4. Find the totals for the North Central division for both dates.

State	1910	1900	State	1910	1900
N. Y.	9,113,614	7,268,894	Me.	742,371	694,466
N. J.	2,537,167	1,883,669	N. H.	430,572	411,588
Penn.	7,665,111	6,302,115	Vt.	355,956	343,641
			Mass.	3,366,416	2,805,346
			R. I.	542,674	428,556
			Conn.	1,114,756	908,420
Mid. Atlantic Division			New Eng. Division		

Ala.	2,138,093	1,828,697	Ohio	4,767,121	4,157,545
Ark.	1,574,449	1,311,564	Ind.	2,700,876	2,516,462
Fla.	752,615	528,542	Ill.	5,638,591	4,821,550
Ga.	2,609,121	2,216,331	Mich.	2,810,173	2,420,982
Ky.	2,289,905	2,147,174	Wis.	2,333,860	2,069,042
La.	1,656,388	1,381,625	Minn.	2,075,708	1,751,394
Miss.	1,797,114	1,551,270	Ia.	2,224,771	2,231,853
No. Carolina.	2,206,287	1,893,810	Mo.	3,293,335	3,106,665
Okla.	1,657,155	1,414,177	Kan.	1,690,949	1,470,495
So. Carolina.	1,515,400	1,340,316	Neb.	1,192,214	1,066,300
Tenn.	2,184,789	2,020,616	N. D.	577,056	319,146
Texas	3,896,542	3,048,710	S. D.	583,888	401,570
Va.	2,061,612	1,854,184			
W. Va.	1,221,119	958,800			
Southern Division			North Central Div.		

§11. Adding Products of the Same Number.

Answer all you can orally

1. How many twos are 3 twos and 5 twos? 8 2's and 6 2's? 6 2's+6 2's? $4\times2+5\times2$?

2. How many 7's are 4 7's and 5 7's? 9 7's+6 7's? $6\times7+5\times7$?

3. How many 10's are 3 10's and 7 10's? 6 10's+7 10's? 9 10's+7 10's? $8\times10+5\times10$?

4. How many 25's are 6 25's and 4 25's? 7 25's and 8 25's? $9\times25+4\times25$?

5. How many a's are $5a$'s and $8a$'s? $4\times a$ and $6\times a$? $7\times a+3\times a$?

6. How many 9's are 8 9's and 7 9's? How many 12's are 8 12's and 7 12's? How many x's are $8x$ and $7x$? $8z + 7z = ?$

7. Add these numbers:

(1)	(2)	(3)	(4)	(5)	(6)
$9a$	$8a$	$26x$	$48z$	$76y$	$69b$
$8a$	$6a$	$47x$	$98z$	$49y$	$79b$

(7)	(8)	(9)	(10)	(11)	(12)
$6m$	$68n$	$84c$	$73d$	$786x$	$960x$
$8m$	$75n$	$76c$	$39d$	$319x$	$869x$
$9m$	$98n$	$37c$	$79d$	$876x$	$48x$

SUBTRACTION

§12. Definitions.

Answer all you can orally

1. In a month a man earns \$180 and spends \$140. What is the difference between his earnings and expenditures?

2. One farmer owns 320 acres of land; another 80 acres. Find the difference in size of the two farms.

3. From a cheese weighing 43 lb., 23 lb. were sold. How many pounds remained unsold?

4. From a bin containing 72 bu. of oats, I use 32 bu. How many bushels remain unused?

I. SUBTRACTION is the way of finding:

(1) The difference of two numbers.

(2) The remainder after taking part of a number away from the number.

THE MINUEND is the number from which we subtract.
THE SUBTRAHEND is the number to be subtracted.
THE DIFFERENCE, or REMAINDER, is the result.

II. SUBTRACTION is also the way of finding either one of two addends when their sum and the other addend are known.

The known sum is the minuend. The known addend is the subtrahend. The unknown addend is the difference, or remainder.

The sign of subtraction, −, is read "minus."

The minuend is always written before the sign.

5. Tell what number these letters stands for:

(1) $8 - 5 = x$ (4) $25 - y = 8$
(2) $17 - 9 = x$ (5) $15 - 8 = z$
(3) $16 - 7 = y$ (6) $16 - z = 8$

§13. Subtracting and Checking.

Use pencil and paper only when needed

1. A baseball team won 58 games during the season, and lost 32. How many more games were won than lost?

2. A boy is 54 inches tall and his father is 72 inches tall. How much taller is the father than the boy?

3. A man has a journey of 437 mi. to make and he has already traveled 199 mi. of it. How much farther must he go?

Convenient Form	
437 mi.	Minuend
199 mi.	Subtrahend
238 mi.	Remainder

Beginning on the right, 9 units can not be taken from 7 units. But 37 is 3 tens and 7 units, or 2 tens and 17 units. Then 17 units − 9 units = 8 units. Write 8 in units' column. Passing to tens' column, we cannot take 9 tens from the 2 tens remaining, but 42 tens are 3 hundreds and 12 tens. Then 12 tens − 9 tens = 3 tens. Write 3 in tens' column. Finally, 3 hundreds − 1 hundred = 2 hundreds. Write the 2 in the hundreds' column. The remainder is 238 miles.

4. Prove 238 mi. to be the correct remainder, or difference, in problem 3 by adding it to the subtrahend, 199 miles.

This is a way of checking subtraction.

5. The minuend is 1284, and the subtrahend is 876. What is the difference, or remainder? Prove.

6. A subtrahend is 469, and the remainder is 299. What is the minuend?

7. From a wire 1 yard long a 16-inch piece is cut. How long is the remainder?

8. From 72 hr. subtract 1 da. and 4 hours.

A correct answer to this problem would be 72 hr. − 1 da. and 4 hr. But to express the answer in a single unit, or name, first change 1 da. and 4 hr. to hours. 1 da. = 24 hr.; 24 hr. + 4 hr. = 28 hr.; 72 hr. − 28 hr. = 44 hours.

9. From 36 ft. take 5 yd. What must first be done to obtain the result in a single unit?

10. 20 qt. − 2 pk. = x qt. What is the first thing to be done? Find the value of x.

Only numbers expressed in the same unit can be added or subtracted if the result is to be expressed in a single unit.

§14. Exercises and Problems.

1. Subtract the following and prove by adding:

(1)	(2)	(3)	(4)	(5)	(6)
8346	24890	36745	48234	57855	72180
5838	17901	17829	29018	29666	23092
(7)	(8)	(9)	(10)	(11)	(12)
62387	83475	90281	38297	27666	87726
34299	56077	37345	19088	18785	68640
(13)	(14)	(15)	(16)	(17)	(18)
418967	687240	723485	868240	927200	707241
229875	478119	438907	372906	418117	354438
(19)	(20)	(21)	(22)	(23)	(24)
381487	592173	600840	821380	727248	917288
191598	394205	236450	291653	570649	129399

2. A man began business with \$5275.75; in five years he had \$22,794.50. How much had he gained?

3. One road is 20 mi. 160 rods long, another is 14 mi. 80 rd. long; how much longer is the first than the second?

4. A cotton dealer bought 328,900 lb. of cotton one year, and 715,600 lb. the next. How many pounds more did he buy the second year than the first?

5. One vessel is valued at \$1,250,000; another at \$975,800. What is the difference in value?

6. During one week a merchant's transactions (business dealings) at his bank were as follows: deposits (money put in), \$217.40, \$343.27, \$290.00, \$365.18, \$380.24, \$415.17; withdrawals (money taken out), \$200, \$320.25, \$195.75, \$286, \$240.15, \$395.25. Which item was the greater at the close of the week, and how much?

7. During one week of November, 1901, 63,188 cattle were received at the Chicago stock yards; during the corresponding week of 1899, 32,722 were received. What was the increase?

8. A man had \$95 and bought a suit with part of this money. He had \$71 left. How much did the suit cost him?

§15. Applications to Geography.

1. Great Salt Lake is 4200 ft. above sea level, and Lake Superior 602 ft. The difference in elevation is x feet. Find the number x stands for.

2. Cuba contains 45,884 sq. mi., and the Philippines, 114,410 sq. mi. Find the difference in size.

3. Mt. Everest is 29,002 ft. above sea level, and Mt. Blanc 15,744 ft. What is the difference in altitude?

4. The Mississippi basin has an area of 1,350,500 sq. mi.; the Amazon, of 2,500,000 sq. mi. How much greater is the Amazon basin than the Mississippi basin?

5. The area of the Philippines is 114,410 sq. mi.; that of California, 158,360 sq. mi. How much larger is the state than the islands?

6. Compare the same islands with Texas, area 265,780 sq. mi.; with Illinois, area 56,650 sq. mi.; with Rhode Island, area 1250 square miles.

7. In 1900, of a population of 6,302,115 in Pennsylvania, 1,151,880 were pupils in various schools. How many persons were not attending school?

8. In a certain year the Hawaiian Islands produced 221,694 tons of sugar, while Cuba produced 880,372 tons. How much more did Cuba produce than the Hawaiian Islands?

9. In 1890, 25,403 manufacturing companies in New York City paid for wages $230,102,167; for materials, $366,422,722; for miscellaneous expenses, $59,991,710. They sold the products for $777,222,721. What was the total profit for all the companies?

For the numbers for problems 10-14, see the table on page 24.

10. Find by comparing the population for 1910 with that for 1900, the increase of population for the ten years, in each state of the Middle Atlantic division; in the entire division.

Check the results in the following problems

11. Which had the larger population in 1900, and by how much, the Middle Atlantic division or the New England division? In 1910? The New England or the Southern division in 1900? In 1910? The North Central or the Middle Atlantic division in 1900? In 1910?

12. Compare the populations of separate states near you.

13. Find, by comparing the population for 1910 with that for 1900, the increase of population for the ten years in each state of the North Central division; in the whole division.

14. Find the increase of population in your own state from 1900 to 1910. What is the average yearly increase?

§16. Subtracting Dates.

Answer all you can orally

	Born	Died
Paul Revere	1735	1818
Benjamin Franklin	1706	1790
Abraham Lincoln	1809	1865
George Washington	1732	1799
James R. Lowell	1819	1891
H. W. Longfellow	1807	1882
Andrew Jackson	1767	1845
Washington Irving	1783	1859
Patrick Henry	1736	1799

1. In what year were you born?

2. Give your age in years.

3. Find how many years each person in the table lived.

4. Which was the older when he died and how much older, James R. Lowell or Henry W. Longfellow? Abraham Lincoln or Patrick Henry?

5. How many years ago (now) was each person in the above list born?

	Dates of Admission		Dates of Admission
Ohio	Nov. 29, 1802	Iowa	Mar. 3, 1845
Ind	Dec. 11, 1816	Mo	Mar. 2, 1821
Ill	Dec. 3, 1818	Neb	Mar. 1, 1867
Mich	Jan. 26, 1837	Kan	Jan. 29, 1861
Wis	May 29, 1848	N. D	Nov. 2, 1889
Minn	May 11, 1858	S. D	Nov. 2, 1889

6. How long was it July 4, 1910, since Ohio was admitted as a state?

Call:

Jan. (1)	April (4)	July (7)	Oct. (10)
Feb. (2)	May (5)	Aug. (8)	Nov. (11)
Mar. (3)	June (6)	Sept. (9)	Dec. (12)

Show then that the date of admission of Ohio may be written thus: 1802-11-29, and that July 4, 1910 may be written, 1910-7-4.

Convenient Form

yr.	mo.	da.
1910—	7—	4
1802—	11—	29
107—	7—	5

29 da. cannot be taken from 4 da. But 7 mo. and 4 da. is the same as 6 mo. and 34 da., as a month in all such problems as this is called 30 da. 29 da. from 34 da. leaves 5 da. Write 5 da. in the result.

11 mo. cannot be taken from 6 mo., but 10 yr. and 6 mo. is 9 yr. and 18 mo.; 11 mo. from 18 mo. is 7 mo. Write 7 mo. in result. In the year column 2 from 9 leaves 7, etc. The answer is, 107 yr., 7 mo., 5 da.

7. How many years, months, and days since Illinois was admitted as a state? Since your native state was admitted into the Union?

8. Make similar problems from the table, solve them, and check your work.

§17. Subtraction of Literal Numbers.

1. How many five-cent pieces are 7 five-cent pieces − 3 five-cent pieces?

2. How many dimes are 12 dimes − 5 dimes?

3. How many 9's are 16 9's − 9 9's?

4. How many c's are $13c - 7c$?

5. Write the differences:

$18x$	$26y$	$43z$	$68z$	$683a$	$1021z$
$9x$	$8y$	$37z$	$39z$	$97a$	$879z$

6. A lot contains $160x$ sq. ft., and the house on it covers $40x$ sq. ft. How many square feet of the lot are not covered by the house?

7. A boy earned $15x$ ¢ on Friday and spent $10x$ ¢ on Saturday. How many cents were not spent?

8. Tell what number x stands for in these problems:

(1) $12-x=7$	(5) $3x-2x=8$	(9) $7x=63$
(2) $65-x=20$	(6) $7x-5x=10$	(10) $9x=108$
(3) $x-35=60$	(7) $16x-13x=15$	(11) $50x=100$
(4) $x-19=7$	(8) $9x-5x=20$	(12) $48x-36x=24$

Numbers denoted by letters are called literal numbers.

MULTIPLICATION

§18. Definitions.

Answer all you can orally

1. At 75¢ a day, how many dollars does a boy earn in 6 days?

This problem may be solved in two ways. We may say he earned the sum of 75¢+75¢+75¢+75¢+75¢+75¢, which is \$4.50.

Or, we may say he earned 6 times 75¢ (6×75¢), or \$4.50.

In either case, his earnings are found to be \$4.50.

MULTIPLICATION is a short way of finding the sum of equal addends when the number of addends and one of them are given.

The MULTIPLICAND is the given addend.
The MULTIPLIER is the number of equal addends.
The PRODUCT is the result, or sum.
The multiplicand and multiplier are factors of the product.
The sign of multiplication, ×, is read "multiplied by" when it is written after the multiplicand and "times" when it is written before the multiplicand.

Thus, 22 mi.×6 = 132 mi., is read "22 mi. multiplied by 6 equals 132 mi.," and 6×22 mi. = 132 mi., is read "6 times 22 mi. equals 132 miles."

An expression like 6×22 mi. = 132 mi., is called an EQUATION.

2. Read and solve the following:

(1) 8×\$75	= ?	(5) 15 bbl.×18	= ?
(2) 80 rd.×7	= ?	(6) 24 doz.×10	= ?
(3) 11×64 ft.	= ?	(7) 45 bu. × 9	= ?
(4) 12×36 gal.	= ?	(8) 68×10 qt.	= ?

When a problem is expressed in an equation, the equation is called the *statement* of the problem.

§19. Multiplying and Checking.

Use pencil and paper only when needed

Make statements and solve:

1. There are 128 cubic feet in 1 cord of wood. How many cubic feet in 15 cords?

STATEMENT. 15×128 cu. ft. $= ?$
Or, 15×128 cu. ft. $= x$ cu. ft.

Find the number which should stand in place of x.

The number which should stand in place of x in an equation is called the *value* of x.

2. There are 5280 ft. in a mile. How many ft. in 24 miles?

3. Find the cost of 275 fence posts at 65¢ each.

\$.65 = cost of one post;
275 = number of posts;
\$.65×275 = \$178.75 (cost of 275 posts) is the equation of the problem.

\$.65	In this problem, which number is the multiplicand? Which the multiplier?
275	
325	This problem may be solved also thus: If each post cost 1¢, 275 posts would cost \$2.75; but as each post costs 65¢, the cost of all will be 65×\$2.75, and the work may be arranged as shown. Or, we may say 275 times \$.65 is the same as 65 times \$2.75, and multiply as above.
455	
130	
\$178.75	

\$2.75
65
1375
1650
\$178.75

Since the numerical product of factors is the same, whichever is used as the multiplier, the smaller number is generally taken for the multiplier *for convenience.*

Either of these ways of multiplying furnishes a means of checking the other.

4. Multiply 892. The Multiplicand.
by 235. The Multiplier.

Full Form		Convenient Form
Multiply 892		892
by 235		235
4460	= 5×892	4460
26760	= 30×892	2676
178400	= 200×892	1784
209620	= 235×892	209620

5. Multiply the following:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
563	589	789	582	678	765	899
365	589	579	376	497	689	655

6. Multiply (1) 729 by 460; (2) 476 by 308.

(1)	(2)
729	476
460	308
43740	3808
2916	1428

7. In 64 pk. there are how many quarts?

8. How many pounds in 494 bu. of corn, if in 1 bu. there are 56 lb.? How many pounds in 25 bushels?

9. There are 231 cu. inches in 1 gallon. How many cu. inches in 587 gallons? In y gallons?

§20. Exercises and Problems.

1. Find the products of the following, and prove results:

(1)	(2)	(3)	(4)
573×248	384×537	735×376	4005×129
858×305	508×607	506×429	2060×456
968×675	651×234	309×150	3070×250
809×584	475×406	847×907	1022×284

2. What will a peck of peanuts cost at 5¢ a pint?

3. Milk costs 6¢ a qt. and a family buys 2 qt. a day. What is the milk bill for April?

4. The average milk yield of a cow was 8 qt. a da. for 120 days. If this milk was sold at 6¢ a qt., how much did the owner receive for it?

5. The cow's feed (problem 4) cost the owner $2.50 per mo. of 30 days. How much profit did the owner receive from the cow's milk during the 120 days?

6. 68 qt. of a certain Jersey cow's milk yield 7 lb. of butter. If milk is selling at 6¢ per qt. and butter at 35¢ per lb., is it more profitable to sell 68 qt. of the milk of this cow, or to make butter of it and sell the butter? How much more profitable?

7. The pulse of the average healthy man beats 72 times per minute. How many times does it beat in 1 hour? In 1 da. at the same rate?

8. Count your own pulse beats for 1 min. and find how many times it beats in a day at this rate.

9. An elevator in a tall building goes up 165 ft. and down the same distance each trip. How many feet does it go in 152 trips?

10. It costs 56¢ a word for a cablegram from Japan to Chicago. How much will a message of 1213 words cost?

11. There are 40 street lamps on 1 mile of street, each burning 18 cu. ft. of gas every night. How much gas will they burn in the month of April?

12. There are 897 hills of corn in a single row and 364 such rows in a field. How many hills of corn in the field?

13. There are 43,560 square feet in an acre. If 189 pounds of water fall on one square foot in a year, how many pounds will fall on an acre in a year?

14. There are 12 elevators in a building. Each makes 94 round trips in a day and carries, on an average, 4 passengers each way. How many passengers make single trips on all of them in a day?

15. In a single layer in a wall there are 964 bricks. There are 197 layers in the wall. How many bricks does it contain?

SPECIAL FORMS OF MULTIPLIER

For checking and for other practical advantages, special forms of multiplying should be learned.

§21. Multiplying by Factors.

1. $7\times8=56$ $28\times2=56$ $14\times4=56$

7, 8, 28, 2, 14, and 4 are all *factors* of 56.

2. Write the factors of 44, 48, 96, 35, 84, 81, 49, 108.

3. The two equal factors of 144 are 12 and 12. Give the two equal factors of 4, 9, 25, 49, 36, 121.

4.	$8\times2=?$	$8\times2\times5=?$	$8\times10=?$
5.	$12\times2=?$	$12\times2\times3=?$	$12\times\ 6=?$
6.	$20\times4=?$	$20\times4\times3=?$	$20\times12=?$
7.	$25\times4=?$	$25\times4\times6=?$	$25\times24=?$
8.	$64\times9=?$	$64\times9\times7=?$	$64\times63=?$

9. Instead of multiplying a number by 15, by what two numbers in succession may I multiply and get the same product?

10. Multiplying any number by 21 gives the same product as multiplying by what two numbers in succession?

11. $56\times2\times4$ gives the same product as 56 multiplied by what single number?

12. Multiplying a number by several factors one after another gives the same product as multiplying it by what single number?

A number which has factors other than itself and 1 is called a COMPOSITE NUMBER.

A number such as 3, 5, 17, etc., which has no factors other than 1 and itself, is called a PRIME NUMBER.

13. Instead of multiplying a number by a composite number, in what other way may I multiply it to obtain the same product?

14. Which of these numbers are prime, and which are composite:

24, 25, 19, 6, 2, 21, 23, 84, 45, 43, 27, 28?

§22. Multiplying by 10, 100, 1000, 10,000.

1. Multiply 84 by 10; by 100; by 1000; by 10,000.

2. Multiply 327 by 10; by 100; by 1000; by 10,000.

3. How may any whole number be quickly multiplied by 10, 100, 1000, or 10,000?

§23. Multiplying by a Number Near 10, 100, 1000, 10,000.

1. $746\times9=?$

This is once 746 less than 10×746, or $7460-746=6714$.

2. $965\times 99=?$ *Ans.* $96{,}500-965=95{,}535$.

3. $965\times 98=?$

4. $965\times 101=?$ *Ans.* $96{,}500+965=97{,}465$.

5. $965\times 102=?$

6. $873\times 11=?$

7. $8473\times1001=?$

8. $8473\times 999=?$

§24. Multiplying by 25, 50, $12\frac{1}{2}$, 75, 500, or 250.

1. $65\times25=?$ $25=\frac{1}{4}$ of 100; $\begin{cases}(\frac{1}{4}\text{ of }100)\times 65=\\ \frac{1}{4}\text{ of }65\times100=1625\end{cases}$

2. $83\times50=?$

3. $638\times12\frac{1}{2}=?$

4. $132\times75=?$ $75=\frac{3}{4}$ of 100; $\frac{3}{4}$ of $132\times100=9900$.

5. $640\times500=?$ $500=\frac{1}{2}$ of 1000.

6. $740\times250=?$ $250=\frac{1}{4}$ of 1000.

We see from problem 1 that to multiply by 25, we may multiply by 100 and take $\frac{1}{4}$ of the product.

7. Make a rule, different from the ordinary rule, for multiplying quickly by 50; by $12\frac{1}{2}$; by 250; by 500.

8. Make a rule for multiplying quickly by $33\frac{1}{3}$; for multiplying quickly by $333\frac{1}{3}$.

This work shows the importance of remembering that:

$\frac{1}{2}$ of $100=50$	$\frac{1}{16}$ of $100=6\frac{1}{4}$	$\frac{3}{8}$ of $1000=375$
$\frac{1}{4}$ of $100=25$	$\frac{1}{3}$ of $100=33\frac{1}{3}$	$\frac{5}{8}$ of $1000=625$
$\frac{3}{4}$ of $100=75$	$\frac{2}{3}$ of $100=66\frac{2}{3}$	$\frac{7}{8}$ of $1000=875$
$\frac{1}{8}$ of $100=12\frac{1}{2}$	$\frac{1}{2}$ of $1000=500$	$\frac{1}{16}$ of $1000=62\frac{1}{2}$
$\frac{3}{8}$ of $100=37\frac{1}{2}$	$\frac{1}{4}$ of $1000=250$	$\frac{1}{3}$ of $1000=333\frac{1}{3}$
$\frac{5}{8}$ of $100=62\frac{1}{2}$	$\frac{3}{4}$ of $1000=750$	$\frac{2}{3}$ of $1000=666\frac{2}{3}$
$\frac{7}{8}$ of $100=87\frac{1}{2}$	$\frac{1}{8}$ of $1000=125$	

9. Make rules for multiplying quickly by the numbers on the right-hand side of the above equations.

10. Find mentally the cost of 160 acres of land at \$50 per acre; at \$75 per acre; at $\$12\frac{1}{2}$ per acre; at \$25; at $\$37\frac{1}{2}$; at $\$87\frac{1}{2}$; at $\$62\frac{1}{2}$.

11. Multiply mentally 48 by $33\frac{1}{3}$; 125; $66\frac{2}{3}$; 250; 500; 750; $6\frac{1}{4}$.

12. Multiply quickly 96 by $62\frac{1}{2}$; $333\frac{1}{3}$; 375; 625; $666\frac{2}{3}$.

APPLICATIONS OF MULTIPLICATION

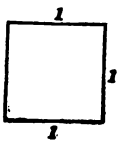


FIGURE 1

§25. Measurements and Area.

Answer all you can orally

A square unit, or a unit square, is a square each side of which is 1 unit long.

The area of a surface is the number of square units needed to cover it.

1. What is a square inch? A square foot? A square yard? A square mile? A square rod?

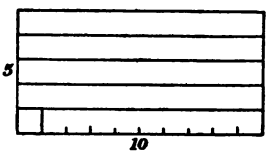


FIGURE 2

2. If a rectangle is made up of 5 rows of 10 square units each, what is its area? How many tens of square units are in the rectangle?

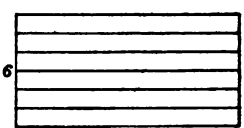


FIGURE 3

3. If a rectangle is 12 units long and 6 units wide, how many twelves of unit squares are there in the area?

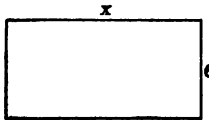


FIGURE 4

4. If a rectangle is x units long and 9 units wide, how many x's of unit squares are in the area?

Seven x's is written $7x$ and is read "seven x." It means 7 times x.

5. How wide must a rectangle z inches long be to contain $6z$ (6 times z) sq. in.? $15z$ square inches?

6. How long must a rectangle 15 in. wide be to contain $15a$ (15 times a) square inches?

7. Draw diagrams on a scale of 1 inch to 3 feet for rooms of the following dimensions. Give their areas.

12 feet long and 9 feet wide
24 feet long and 18 feet wide
27 feet long and 15 feet wide
24 feet long and w feet wide

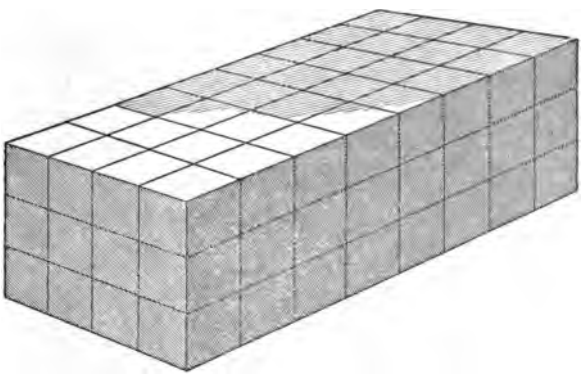
Give perimeters (distance around) in feet. In yards.

8. Draw diagrams on a scale of 1 inch to 3 yards for lots of the following dimensions and give areas:

15 yards long and 12 yards wide
18 yards long and 15 yards wide
12 yards long and 6 yards wide
33 yards long and 15 yards wide

Give the perimeters in yards. In feet.

§26. Measurement and Volume.



Answer all you can orally

1. In the lower layer of cubic blocks in this solid, how many rows of 4 blocks each? How many blocks in the layer? How is this number found? In all 3 layers, or the entire solid, how many blocks are there? How is this number found? How then is the number of cubic units in any square-cornered solid found?

2. How many inch cubes are there in a block 2 inches long, 2 inches wide, and 1 inch high? In a block 4 inches long, 2 inches wide, and 1 inch high?

3. How many inch cubes are there in a block 6 inches long, 4 inches wide, and 1 inch high? How many inch cubes in one row? How many of these rows in the block?

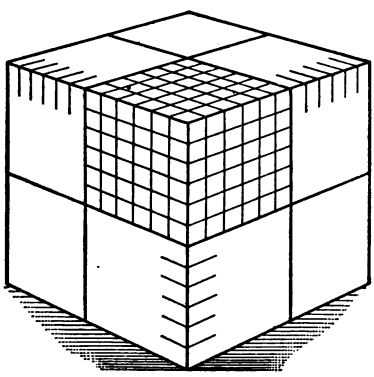
4. How many cubic inches are there in a box that is 4 inches long, 3 inches wide, and 1 inch high? 2 inches high? 3 inches high?

5. How many cubic inches are there in a box 4 inches long, 2 inches wide, and 2 inches high?

6. A block is 5 inches long, 2 inches wide, and 3 inches high. How many cubic inches are there in it?

§27. The Cube.

Answer all you can orally



1. How many edges do you see on this block? How many edges has it?

2. If the edges were all 1 inch long, what would such a block be called?

3. If 12 such cubes are placed in a row, how long is the row?

4. If 12 of these rows are placed side by side, how wide is the whole?

5. How many square inches would there be in the top surface? How many inch-cubes would you use in forming a layer over this surface?

6. If you should place another layer of cubes on those already used, how many cubic inches would you have? How many, if you used 3 layers? 4 layers? 5 layers? 6 layers? 7 layers?

7. The picture represents 12 layers of inch-cubes. Each layer contains 12 rows of 12 cubes each. How long is each edge? What would you call such a cube? How many cubic inches does it contain?

8. If you should divide one-half of this cube into 4 equal cubes, what would each cube measure?

9. What part of the whole cube would each of these 4 equal cubes be?

10. How many cubic inches are there in each of these equal cubes?

11. How many cubic inches in three-eighths of a cubic foot? In seven-eighths? In one-fourth?

12. A gallon contains 231 cubic inches. How many cubic inches are there in 15 gallons?

13. How many boxes 12 inches long, 6 inches wide, and 3 inches high can be packed in a case 6 feet long, 4 feet wide, and 4 feet high?

14. How many cubic feet of water will a glass tank hold, that is 24 inches long, 18 inches wide, and 12 inches high?

15. From a vessel holding 2 cubic feet of water 864 cubic inches were taken. How many cubic inches remained?

§28. Problems with Literal Numbers.

$4 \times x$ is written $4x$. If $4x = 24$, what is the value of x?

Notice that the equation $4x = 24$ is the statement of this problem. A boy reads 24 pages of a book in 4 days. If he reads the same number of pages each day how many pages does he read a day? What does x stand for in this problem?

1. What number does x stand for in these equations?

(1) $3x = 18$	(5) $x - 18 = 4$	(9) $x + 16 = 25$
(2) $6x = 48$	(6) $x - 9 = 8$	(10) $5x + 2x = 70$
(3) $8x = 72$	(7) $x + 9 = 17$	(11) $9x - 3x = 36$
(4) $x - 8 = 15$	(8) $x + 6 = 22$	(12) $8x + 5x = 26$

§29. Mental Drills.

A multiplication table for numbers larger than 12 is not needed. This will be seen by the following:

$2\times13=2\times10$ and $2\times3=20+6=26$
$4\times14=4\times10$ and $4\times4=40+16=56$
$3\times15=30+15=45$

This work should be done mentally and rapidly, the pencil being used only to write the final product.

1. Solve mentally:

$3\times13=?$	$6\times13=?$	$9\times13=?$
$4\times13=?$	$7\times13=?$	$10\times13=?$
$5\times13=?$	$8\times13=?$	$11\times13=?$

2. Solve mentally:

$2\times14=?$	$5\times14=?$	$8\times14=?$
$3\times14=?$	$6\times14=?$	$9\times14=?$
$4\times14=?$	$7\times14=?$	$11\times14=?$

3. How many are 26 and 10? 26 and 20? 26 and 30? 36 and 10? 39 and 30? 49 and 30? 49 and 20? 49 and 40? 42 and 20? 42 and 30? 42 and 50? 42 and 40?

4. Count by 10's from 13 to 103. From 27 to 127. From 46 to 116. From 87 to 137.

5. Count by 20's from 14 to 134. From 4 to 144. From 33 to 153. By 30's from 16 to 166. From 9 to 189.

To add 38 and 23, first add to 38 the 10's in 23, then the 1's, using pencil only to write the last result. Thus, 38 and 20 are 58 and 3 are 61.

6. Solve mentally:

$26+13=?$	$28+14=?$	$43+15=?$	$75+15=?$
$39+13=?$	$48+16=?$	$42+14=?$	$32+16=?$
$56+14=?$	$64+16=?$	$65+13=?$	$43+16=?$

If products like 13×2, and 14×4, are wanted, turn them mentally into 2×13, and 4×14.

7. Solve mentally:

$13\times2=?$	$14\times5=?$	$13\times4=?$	$13\times\ 9=?$
$14\times2=?$	$14\times6=?$	$14\times4=?$	$15\times\ 4=?$
$14\times3=?$	$15\times3=?$	$13\times5=?$	$14\times\ 8=?$
$13\times3=?$	$13\times6=?$	$14\times7=?$	$14\times10=?$

8. Complete the table of 13's from 1×13 to 12×13 as suggested in problem 1, page 44.

9. Complete the table of 14's as suggested in problem 2, page 44.

10. Make a table of 15's similarly.

11. Make a table of 16's the same way; of 17's; of 20's.

12. Solve mentally:

$23+10=?$	$54-10=?$	$54+20=?$	$83-30=?$
$47+20=?$	$65+20=?$	$65-20=?$	$63+30=?$

13. Solve mentally, by adding, or subtracting the 10's, then the 1's:

$36+12=?$	$57+32=?$	$67-34=?$	$55+34=?$
$46+23=?$	$74-22=?$	$55+24=?$	$65-34=?$

14. Solve mentally, as rapidly as you can give results correctly; first adding, or subtracting the 10's, then the 1's:

$38+22=?$	$38+43=?$	$38-22=?$	$45+26=?$
$38+32=?$	$38+54=?$	$45+16=?$	$45+37=?$

15. How many 2's in 4? How many 2's in 2 fours? In 3 fours? In 4 fours? In 5 fours? In 6 fours?

16. How many 4's in 8? In 2 8's? In 3 8's? In 4 8's? In 6 8's? In 7 8's? In 10 8's?

17. How many 3's in 6? In 2 6's? In 3 6's? In 5 6's? In 7 6's? In 2 12's? In 3 12's?

18. How many 5's in 10? In 2 10's? In 3 10's? In 4 10's? In 15? In 2 15's? In 5 15's?

DIVISION

§30. Definitions.

Answer all you can orally

1. A man owes a debt of $12 which he is to pay by work at $2 per day. How much will he owe at the end of the first day? Of the second day? Of the third day? Of the fourth day? The fifth? The sixth?

2. How long will it take to cancel the debt?

3. How many times may 2 be subtracted from 12, leaving no remainder? How many 2's are there in 12?

4. A man buys a horse for $90 and is to pay $15 a month until the horse is paid for. How much does the man owe after the first payment? After the second? The third? Fourth? Fifth? Sixth?

5. How many months will it take to pay for the horse?

6. How many times may 15 be subtracted from 90, leaving no remainder? How many 15's in 90?

7. What is one of the 6 equal parts of 12? Of 90?

Use pencil and paper only when needed

8. Find by successive subtractions how many 88's there are in 616. What is one of the 7 equal parts of 616?

DIVISION is a short way of finding how many times one number may be subtracted from another.

9. A horse traveled 72 mi. in 8 hr.; find the average number of miles traveled per hour.

10. 4 bu. of potatoes cost $2.40. What was the price per bushel?

11. The two factors of a number are 13 and 7; what is the number?

12. The product of two numbers is 91 and one of the numbers is 13; what is the other number?

13. The product of two numbers is 240 and one of the numbers is 20; what is the other?

DIVISION is a way of finding one of two numbers when their product and the other number are given.

The DIVIDEND is the product.

The DIVISOR is the given number.

The QUOTIENT is the required number.

The product of two factors is 490. One factor is 7. What is the other?

Divisor $7)\overline{490}$ Dividend

$\underline{70}$ Quotient

We may also say that the dividend is the number to be divided, the divisor is the number by which the dividend is measured or divided, and the quotient is the measure.

The sign of division, ÷, is read "divided by," as 60 ÷ 12 = 5.

Other ways of writing division are:

$12)\overline{60}$ $\quad$ $12)\overline{60}(5$ $\quad$ $\frac{60}{12}=5$ $\quad$ $60/12=5$

$\underline{5}$ $\quad$ $\underline{60}$

The first and second are in common use in division.

In the third, the line placed between the two numbers shows that the number above it is to be divided by the number below it; as in $\frac{1}{3}$, 1 is the dividend and 3 the divisor. The fraction itself is the quotient. The line between the two numbers is the division sign.

14. $20 \times x = 240$; what number does x stand for?

15. Tell what number the letter stands for in each of these equations:

$12y=96$; $8x=56$; $7a=56$; $9b=72$; $10a=150$; $24m=240$.

SHORT DIVISION

§31. Small Divisors.

1. How many gallons are there in 296 pints?

As there are 8 pt. in 1 gal. there are as many gallons in 296 pt. as there are 8's in 296.

$$8)\overline{296}\quad 37$$

2. How many 12-hour days in 120 hours?

3. At \$8 per ton, how many tons of coal can be bought for \$240?

4. 156 marbles are distributed equally among a certain number of boys. Each boy has 13 marbles. How many boys are there?

5. How much will a dealer get for 540 marbles if he sells 6 for a cent?

6. 4 bbl. of lime cost \$2.60. What did one bbl. cost?

7. I bought 8 lb. of butter for \$2.16. What was the price per pound?

8. A dressmaker used 84 yd. of cloth for 6 dresses, allowing the same amount for each dress. How many yards were used in each dress?

9. A train ran 225 mi. in 9 hours. What was the average number of miles per hour?

Answer all you can orally

10. What does x stand for in each of these equations?

(1) $63 \div 7 = x$	(4) $45 \div 3 = x$	(7) $120 \div 3 = x$
(2) $630 \div 9 = x$	(5) $96 \div 8 = x$	(8) $108 \div 9 = x$
(3) $48 \div 4 = x$	(6) $960 \div 8 = x$	(9) $72 \div 6 = x$

When dividend and divisor are small numbers, the quotient is easily seen. We say it is obtained by *inspection*.

When the divisor is a large number, the quotient is not readily found by inspection.

§32. Exercises and Easy Problems.

1. Divide:

264 by 11, by 21.
528 by 12, by 22.
377 by 13, by 23.
434 by 14, by 24.
675 by 15, by 25.
352 by 16, by 26.

CONVENIENT FORM

$$21\overline{)264}$$
$$12\tfrac{12}{21}$$

2. A man paid \$31.46 for eggs at 13¢ a dozen. How many dozen did he buy?

3. A tank holds when full 300 barrels of water. When it is $\frac{1}{15}$ full, how many barrels does it hold?

4. In a flock of 1736 sheep, $\frac{1}{14}$ of the number were lambs. How many lambs were there?

5. A man bought 20 feet of piping for \$6.40. How much per foot did it cost him?

6. A man bought 18 yards of cloth for \$36.72. How much did it cost a yard?

7. A grocer sold 19 pounds of butter for which he received \$4.37. How much per pound did he get?

8. A boy can ride 15 miles an hour on his bicycle. At the same rate how many hours would it take him to ride 5136 miles?

9. The schoolroom floor contains 1100 square feet and is 25 feet wide. How long is it?

§33. Applications.

Use pencil and paper only when needed

1. In 51,448 qt. how many pecks are there?

Since there are 8 qt. in 1 peck, in 51,448 qt. there are as many pecks as there are 8 qt. in 51,448 quarts.
In 51,448 qt, there are 6431 pecks.

$$8\overline{)51448}$$
$$6431$$

Check: $6431 \times 8 = 51{,}448$.

2. There are 5280 feet in a mile. How many yards are there in 1 mile? How many yards are there in 2 miles? In 3 miles?

3. If limestone weighs 160 lb. per cubic foot, how many cubic feet are there in a piece of limestone weighing 6400 pounds?

4. If marble weighs 170 lb. per cubic foot, find the number of cubic feet in a piece of marble weighing 5100 pounds.

5. If sand weighs 120 pounds per cubic foot, what is the number of cubic feet in a load of sand weighing 4800 pounds?

6. A train of 12 sleeping cars is 840 ft. long from end to end. If the cars are all the same length, how long is each car?

7. A steel rail 30 ft. long weighs 720 lb.; what is its weight per yard of length?

8. An iron beam 24 feet long weighs 1080 pounds. How many pounds does a piece of the beam 1 foot long weigh?

9. A steel girder (or beam) weighs 780 lb. Each foot of length weighs 30 lb. How long is it?

10. There were 487,918 immigrants to the United States in the year 1901. What was the average number per month? Per day? (30 da. = 1 month.)

11. During 1900 there were 448,572 immigrants. Find the average number per month.

12. Find the average number of immigrants per month for 1891, 1892, 1893, 1895, and 1897, the numbers for these years being in succession 560,319; 623,084; 502,917; 258,536, and 230,832.

13. In Albany, N. Y., in a certain year there were 30 mi. of street railway, operated by 600 men. What was the average number of employees per mile?

14. From the data here given answer the same question for these cities:

City	Miles	Employees
St. Joseph, Mo	35	175
Memphis, Tenn	70	490
Oakland, Cal	80	560
Hartford, Conn	33	660
Worcester, Mass	43	453
Peoria, Ill	50	275

15. Find the value of x in the following:

(1) $3264 \div 6 = x$ (4) $89{,}765 \div 5 = x$ (7) $24{,}568 \div 8 = x$
(2) $9432 \div 12 = x$ (5) $78{,}870 \div 11 = x$ (8) $45{,}960 \div 12 = x$
(3) $2247 \div 7 = x$ (6) $75{,}699 \div 9 = x$ (9) $1{,}241{,}196 \div 11 = x$

16. How can you prove the correctness of your work in division?

17. How many weeks are there in 2184 days?

18. How many months of 30 days each are there in 720 days?

Divide first by 10; then by 3.

19. How many 24-hour days are there in 1080 hours?

20. The product of two factors is 540, and one of the factors is 15. Find the other factor.

21. Find what x equals in these equations:

(1) $\frac{80}{x} = 10$ (3) $\frac{6300}{x} = 9$ (5) $\frac{490}{x} = 49$ (7) $\frac{640}{x} = 20$

(2) $\frac{63}{x} = 9$ (4) $\frac{5600}{x} = 56$ (6) $\frac{810}{x} = 90$ (8) $\frac{990}{x} = 66$

LONG DIVISION

§34. Arrangement of Work and Checking.

When dividend and divisor are both large numbers, there are several distinct steps to the work.

1. 14,487 ÷ 33 = ?

CONVENIENT FORM

```
          439  Quotient
       ------
Divisor 33)14487  Dividend
          132
         ----
          128  2d Partial Dividend
           99
         ----
          297  3d Partial Dividend
          297
         ----
            0  Remainder
```

FIRST STEP

Use 30, the nearest multiple of 10 to 33, as a *trial* divisor.

30 does not go into 14, but it goes into 144 about 4 times. Write 4 as the first quotient figure *just above the last dividend figure used.*

Now multiply 33 by 4, getting 132, and take 132 from 144. Write the next dividend figure, 8, beside the 12, getting 128 for a 2d partial dividend.

SECOND STEP

30 goes into 128 about 4 times; but as the real divisor is 33, and as $4\times33=132$, which is larger than 128, take the quotient figure 1 unit smaller, that is 3.

$3\times33=99$, and $128-99=29$. Write the next dividend figure, 7, beside the 29 giving 297 as the 3d dividend.

THIRD STEP

30 goes into 297 about 9 times. Write 9 as the 3d quotient figure.

$9\times33=297$. $297-297=0$.

As the *remainder* is 0, the complete quotient is 439.

Check: $439\times33=14{,}487$.

When a zero appears in the quotient proceed as follows:

2. 18,722 ÷ 46 = ?

```
          407
       ------
46)18722
         184
        ----
          322
          322
        ----
```

Begin as above. 46 is contained in 187, 4 times, with the remainder 3. Bring down the first 2 in the dividend, giving 32. 46 is not contained in 32 a whole number of times. Write 0 in tens' place in the quotient and bring down the next 2 of the dividend, giving 322. 46 is contained in 322, 7 times. Write the 7 in units' place in the quotient.

Complete these equations and check your work:

3. $3580 \div 45 =$ 5. $18{,}144 \div 56 =$ 7. $33{,}768 \div 72 =$

4. $15{,}552 \div 64 =$ 6. $20{,}088 \div 72 =$ 8. $35{,}096 \div 82 =$

§35. Exercises and Problems.

1. Solve the following and check:

(1) $21\overline{)1302}$ (4) $34\overline{)7242}$ (7) $75\overline{)24300}$ (10) $121\overline{)9882}$

(2) $21\overline{)1575}$ (5) $37\overline{)4514}$ (8) $81\overline{)9801}$ (11) $124\overline{)10584}$

(3) $22\overline{)5302}$ (6) $37\overline{)8621}$ (9) $86\overline{)18404}$ (12) $214\overline{)18490}$

2. A field of 120 acres of corn yielded 4560 bushels. What was the average yield per acre?

3. A man paid $8400 for a farm that cost him $75 per acre. How many acres were there in the farm?

4. A man paid $11,200 for town lots, at the average price of $175 a lot. How many lots did he buy?

5. A field is 140 rods long and contains 11,900 square rods. How many rods wide is the field?

6. How many cars that hold 48 tons each will be required to hold 6144 tons of coal?

7. There are 144 square inches in a square foot. How many square feet are there in 19,008 square inches?

8. How many bushels are there in a load of corn that weighs 3458 lb., counting 56 lb. to the bushel?

9. Divide these numbers by 13, 14, 29, 38, 57, 74, 97:

(1)	(2)	(3)	(4)	(5)	(6)
213	237	248	253	269	9012
3123	3234	3345	3456	3567	8123
4678	5788	6879	7890	7243	6542

10. Solve and check the following, writing the remainders over the divisors, as is shown in the first:

(1) $27\overline{)1170}$ $43\frac{9}{27}$ (4) $123\overline{)15293}$ (7) $280\overline{)18840}$ (10) $480\overline{)78060}$

(2) $64\overline{)3120}$ (5) $165\overline{)12595}$ (8) $275\overline{)30910}$ (11) $300\overline{)19410}$

(3) $123\overline{)9307}$ (6) $208\overline{)19864}$ (9) $424\overline{)47223}$ (12) $375\overline{)45600}$

Check of (1): $27\times43=1161$; $1161+9=1170$. Multiply the divisor by the integral part of the quotient and add the remainder.

11. My friend is 1872 weeks old. How many years old is he?

12. There are 160 sq. rd. in 1 A. and a farmer pays 75¢ per acre for cutting and binding wheat. How much will it cost him to cut and bind the wheat on a field 68 rd. by 80 rods?

13. A farmer paid a man \$21.00 to shock his wheat, at \$1.50 per day. How many days did it take the man to do the work?

14. In 1880, 25 farm wagons sold for \$2250 and in 1900 15 such wagons sold for \$855. How much less was the average cost of a farm wagon in 1900 than in 1880?

15. In 1880 a Minnesota farmer paid \$3900 for 12 twine binders and in 1900, 14 twine binders cost him \$1680. How much had the average price of twine binders fallen during these 20 years?

16. In 25 da. a man earned \$56.25 husking corn at 3¢ per bushel. How many bushels per day did he average?

17. A city lot 175 ft. long, containing 8750 sq. ft., sold for \$6000. If the short side fronts the street what was the price per foot of frontage?

SPECIAL FORMS OF DIVISOR

Important advantages are sometimes gained by special ways of dividing.

§36. Division by Multiples of 10.

Answer all you can orally

1. Multiply each of these numbers by 10:

568 1268 306 $86.50 $8.65

2. Multiply each of the same numbers by 100; by 1000.

3. Divide each of these numbers mentally by 10:

5680 12,680 3060 $865.00 $86.50

4. Make a rule for dividing numbers quickly by 10; by 100; by 1000; by 1 with any number of zeros after it.

5. Name these quotients orally:

$60 \div 10 = ?$	$6 \div 2 = ?$	$60 \div 20 = ?$
$860 \div 10 = ?$	$86 \div 2 = ?$	$860 \div 20 = ?$
$\$64.20 \div 10 = ?$	$\$6.42 \div 2 = ?$	$\$64.20 \div 20 = ?$

Examining your answers to the questions just asked make a rule for dividing numbers quickly by 20; by 200; by 2000.

6. $180 \div 10 = ?$	$18 \div 3 = ?$	$180 \div 30 = ?$
$630 \div 10 = ?$	$63 \div 3 = ?$	$630 \div 30 = ?$

From these answers make a rule for dividing numbers quickly by 30; by 300; by 3000.

7. Make a rule for dividing numbers quickly by 40; by 400; by 4000; by 50; by 800; by 1200; by 1500.

8. Make a rule for dividing numbers quickly by any whole number of tens, as 40, 70, 90, 160; by any whole number of hundreds; of thousands.

9. Cutting off one zero from the right of a number has what effect on the number? Cutting off 2 zeros? 3 zeros?

$$165 \div 10 = 16\tfrac{5}{10}$$
$$75 \div 100 = \tfrac{75}{100}$$
$$478 \div 100 = 4\tfrac{78}{100}$$

10. Using first 10, and then 100, as a divisor show the quotients of the following:

400	500	2200	3300
460	790	4280	4860
287	439	9647	5732

11. There are 10 pk. in a barrel. How many barrels in 1488 pecks?

12. A bushel of potatoes weighs 60 lb. How many bushels are there in 486 pounds of potatoes?

13. 100 lb. = 1 cwt. How many hundredweight are there in 825 pounds?

14. 200 lb. of pork = 1 bbl. How many barrels are there in 7624 pounds?

15. How many minutes in 4260 seconds?

When there are ciphers at the right of both dividend and divisor, cut off an equal number of ciphers from each and divide.

$$60)\overline{4260}\quad 71$$

16. How many barrels will be needed for packing 4800 lb. of beef, allowing 200 lb. to the barrel?

§37. Shortening Division by Cancellation.

1. Remembering that the dividends stand above the line and the divisors below, solve these problems:

$$\frac{324}{81} = ? \qquad \frac{108}{27} = ? \qquad \frac{36}{9} = ? \qquad \frac{12}{3} = ?$$

How do the quotients compare? The dividends? The divisors?

2. How do the quotients, the dividends, and the divisors compare in these problems:

$$\frac{256}{128}=? \qquad \frac{64}{32}=? \qquad \frac{16}{8}=? \qquad \frac{4}{2}=?$$

3. To divide 324 by 81 what smaller numbers may be used to get the same quotient? How may these smaller numbers be obtained from 324 and 81?

We see that, as $324 = 27 \times 12$ and $81 = 27 \times 3$, we may write

$$\frac{12 \times 27}{3 \times 27} = \frac{12}{3} = 4.$$

This can be indicated thus: Striking out or removing 27 is the same as dividing both above and below the dividing line by 27.

$$\frac{12 \times \cancel{27}}{3 \times \cancel{27}} = \frac{12 \times 1}{3 \times 1} = 4$$

Removing these factors is called cancellation. It can often be used to simplify the division of products.

4. Answer questions like those of problem 3 for $256 \div 144$.

These problems show that any factor of both dividend and divisor may be dropped from both and the remaining factors divided.

5. Solve these problems by cancellation:

(1) $\frac{34\times16}{62\times16}=?$ (2) $\frac{6\times8\times3}{2\times8\times3}=?$ (3) $\frac{15\times21\times846}{3\times\ 7\times846}=?$

(4) $\frac{18\times3\times4\times67}{2\times4\times3\times67}=?$ (5) $\frac{625}{25}=?$ (6) $\frac{324}{189}=?$ (7) $\frac{85}{34}=?$

To use cancellation effectively, methods of finding factors of numbers are necessary.

§38. Tests of Divisibility.

1. Which of these numbers are exactly divisible (can be exactly divided) by 2?

12 24 36 23 45 18 37 40 59 61

What are the last digits of the numbers which 2 will exactly divide?

TEST FOR THE FACTOR 2.

If a number ends in 0, 2, 4, 6, or 8, it can be exactly divided by 2.

2. Which of the numbers, 24, 30, 45, 50, 700, 640, 83, 765, 6400, are exactly divisible by 10?

What is the last digit of the numbers which 10 exactly divides?

TEST FOR THE FACTOR 10.

If a number ends in 0, 10 exactly divides it.

3. Make a rule for testing whether 100 divides a number.

4. Which of the numbers, 16, 18, 35, 25, 60, 20, 28, 65, 460, 672, 1260, does 5 exactly divide?

What are the last digits of the numbers which 5 will exactly divide?

TEST FOR THE FACTOR 5.

If a number ends in 0 or 5, 5 exactly divides it.

5. Which of the numbers, 12, 17, 24, 81, 27, 93, 64, 75, 126, 324, 185, does 3 exactly divide?

Of those which 3 exactly divides, will 3 also exactly divide the sum of the digits? Of the numbers 3 does not exactly divide, is the sum of the digits exactly divisible by 3?

TEST FOR THE FACTOR 3.

If 3 exactly divides the sum of the digits of a number it exactly divides the number also.

6. Of the numbers 126, 368, 453, 729, 819, 639, 2358, what ones does 9 exactly divide?

7. Pick out the numbers of this list which are exactly divisible by 2:

81 65 86 129 9864 8643 7986 16,835 29,860

8. Pick out those in the list of problem 7 which can be exactly divided by 3; by 5; by 10.

§39. Applications of Cancellation.

In all the problems in this section indicate your divisions and multiplications, then apply the tests of divisibility, and cancel the factors found in both dividend and divisor. Multiply the uncancelled factors in the dividend together and divide this product by the product of the uncancelled factors of the divisor.

ILLUSTRATION.—A speed of 102 mi. per hour equals how many feet per second? Indicate the work thus:

$$\frac{\overset{17}{\cancel{102}}\times\overset{44}{\cancel{5280}}}{\underset{5}{\cancel{60}}\times\underset{1}{\cancel{60}}}=\frac{17\times44}{5}=\frac{748}{5}=149\tfrac{3}{5}.\quad \textit{Ans.}\ 149\tfrac{3}{5}\text{ ft. per second.}$$

Show that the 440 and the 5 arise by dividing 12 out of factors of numerator and denominator. Show how the 10 and the 17 arise.

1. It took 225,280 lb. of steel rails to lay 1 mi. of single track railroad. What was the average weight of the rails per yard for the track?

	LB. PER CU. FT.	CU. FT. PER T.
Granite	170	
Limestone	160	
Marble	170	
Sandstone	140	
Slate	170	
Cast Iron	450	
Steel	480	

2. Find the number of cu. ft. in a ton of 2000 lb. for each substance given in the table.

3. The average speed of American express trains is about 35 mi. per hour for long distances. Find the speed per second.

Put work in this form:

$$\frac{\overset{7}{\cancel{35}}\times\overset{22}{\cancel{5280}}}{\underset{12}{\cancel{60}}\times\underset{3}{\cancel{60}}}=\frac{154}{3}=51\tfrac{1}{3}.\quad \textit{Ans.}\ 51\tfrac{1}{3}\text{ ft. per second.}$$

4. The average speed of express trains for short distances is about 40 mi. per hour. What is the speed per second?

5. For long distances the average speed of English express trains is about 43 mi. per hour. Find the speed in feet per second.

6. In 1893 an express train in the United States ran 1 mi. at the rate of 98 mi. per hour. What is the speed in feet per second?

7. A railroad train ran for 1 min. at the speed of 130 mi. per hour. Find the number of feet per second which it traveled.

8. Sound travels in air at the rate of about 750 mi. per hour. Find the speed in feet per second.

9. Cannon balls have been thrown at a speed of 810 mi. per hour for a few seconds. What is the speed in feet per second?

10. The moon moves around the earth at a speed of about 1,840,000 mi. in 30 da. What is the speed in feet per second?

11. In 365 da. the earth moves around the sun through a distance of about 584,000,000 mi. What is the earth's speed in miles per second?

12. The earth, by turning on its axis, carries a place on its equator about 25,000 mi. in 24 hours. How many feet is it carried per second?

§40. Checking Division.

Division may be checked, or tested, by multiplying the divisor by the quotient and adding the remainder to the product. If the sum equals the dividend, the work is checked.

Another check is to divide by the factors of the divisor successively, and note whether the final quotient is the same as that given by the complete divisor.

REVIEW OF FUNDAMENTAL PROCESSES

§41. Exercises.

1. Add, prove, and note the time you take:

3	4	6	9	6	6	8	9	4	5	5	5
3	2	3	1	3	7	6	6	5	4	6	5
3	6	2	3	4	2	1	8	6	6	7	7
2	7	4	2	5	1	4	1	7	2	8	7
5	5	7	6	2	3	2	2	8	3	9	6

2. Add, prove, and note the time you take:

2	1	2	2	4	5	9	7	7	8	9	7
2	2	3	4	3	8	8	6	7	7	8	3
2	3	4	6	1	2	3	2	5	6	7	8
3	4	5	3	2	3	2	3	4	5	6	2
7	5	2	8	8	6	1	5	3	4	5	6

3. Add, prove, and note the time you take:

42	75	308	638	6847
37	28	109	98	391
31	47	647	124	4076
68	38	274	574	878
42	29	635	839	6325
54	82	998	608	49

4. Subtract, and note the time you take:

268	321	463	301	400	600	700	809	410
129	148	375	199	298	378	509	645	198

1506	2008	5000	4308	3864	50001	12001
329	639	2899	2609	2495	28973	9896

5. Multiply, prove, and note the time you take:

28	68	75	68	47
32	42	37	18	26
43	59	68	75	86
47	28	55	56	38
6834	2849	4107	4862	3906
63	56	81	402	209
6070	6839	7603	9065	8365
430	450	487	729	843

6. Divide, and note time taken:

4)176	6)738	6)1386	8)1880	9)3249
21)4242	23)2553	30)1620	40)2400	50)2700
32)768	41)1435	53)2226	46)1776	59)10679
36)8676	125)4250	127)2921	201)184518	
725)291450	413)124313	365)112420	936)379080	

7. Prove the results of exercise 4, by adding the *difference* to the subtrahend in each case and comparing the sum with the minuend. Note time.

8. Prove the results of 6 by multiplying quotient by divisor and comparing the product with the dividend.

§42. Miscellaneous Problems.

1. If one breathes 18 times a minute and at each breath consumes 2 cubic inches of oxygen more when moving about than when sitting, how many gallons less oxygen does one consume in 8 hours if he sits than if he is active?

The following table is an extract from the milk record of a herd of milk-cows:

WEIGHT OF MORNING (A.M.) AND EVENING (P.M.) MILKINGS IN OUNCES

1901 November	Queen		Beauty		Beech-wood		Spot		Rose		Totals
	A.M.	P.M.	A.M.	P.M.	A.M.	P.M.	A.M.	P.M.	A.M.	P.M.	
1.....	104	104	176	166	200	179	192	184	176	154	
2.....	90	96	176	134	176	176	184	168	181	141	
3.....	101	96	173	152	187	184	192	166	179	157	
4.....	96	104	200	163	202	208	184	184	176	157	
5.....	96	120	182	163	192	182	186	181	176	163	
6.....	106	109	192	160	128	186	200	189	187	160	
7.....	118	136	173	198	192	187	192	200	186	192	
8.....	125	117	179	186	211	195	184	181	184	162	
9.....	120	102	186	166	192	186	192	195	171	160	
10.....	125	115	176	166	192	192	184	136	189	170	
11.....	109	117	195	176	218	184	144	160	197	160	
12.....	125	123	200	192	208	186	192	202	195	170	
13.....	122	128	214	184	216	208	178	208	189	179	
14.....	112	128	192	189	224	203	200	203	184	165	
15.....	133	120	208	171	240	192	224	198	197	144	
16.....	144	112	200	176	243	219	203	197	226	136	
17.....	128	112	200	168	224	208	160	144	192	168	
18.....	130	123	214	174	224	184	200	184	176	146	
19.....	112	112	184	182	218	176	195	187	178	160	
20.....	120	104	202	168	203	192	189	182	184	152	
21.....	112	122	192	163	219	181	208	176	170	154	
22.....	128	120	195	190	235	179	198	171	181	152	
23.....	120	144	195	176	230	192	205	173	181	166	
24.....	112	114	216	170	221	184	208	171	192	166	
25.....	118	128	200	179	224	168	203	187	192	176	
26.....	104	134	192	189	192	192	211	184	184	168	
27.....	120	136	219	189	224	208	208	203	182	168	
28.....	115	147	192	192	218	221	210	200	187	168	
29.....	104	144	216	202	240	216	163	202	200	202	
30.....	115	128	187	203	227	195	198	184	203	165	
Totals.....											

For the next 5 problems see how quickly you can get correct results

2. How many ounces of milk did Queen give during Nov., 1901, at the morning milkings? At the evening milkings?

3. Answer similar questions for Beauty; for each of the other cows.

4. Without rewriting, find the total quantities of milk for each day of the month. Write totals in a column on a separate slip. Time yourself.

5. Find the totals for each cow for the whole month. Time yourself.

6. Find the total for all the cows for the entire month. Compare the result with that of problem 5.

7. The total tonnage of the commerce of the Great Lakes in 1907 was 83,387,919 tons, 40,727,972 tons of which was iron ore. What tonnage was not iron ore?

8. The south-bound traffic through the Detroit River in 1907 amounted to 46,967,166 tons, and the north-bound traffic amounted to 20,329,311 tons. Find the total traffic. Find the difference in tonnage of the north-bound and the south-bound traffic.

9. The United States Government paid pensions amounting in 1893 to $158,155,342; in 1898, to $145,-748,865, and in 1905, to $142,099,286. Find the decrease in amount paid for pensions from 1893 to 1898; from 1898 to 1905; from 1893 to 1905.

10. In the year 1897, 558 tons of rosewood was imported into Great Britain. One half of it sold at 5¢ and the rest at 10¢ a pound. Find the value in Great Britain of this rosewood.

11. How much cork will a tree yield in 150 years at the rate of 2000 lb. every 8 years?

12. The area of the United States is 3,600,000 sq. mi. (1 sq. mi. = 640 A) and it contains 450,000,000 acres of woodland. How many acres in the United States are not woodland?

13. A speed of 60 miles an hour is equivalent to how many feet per minute? Per second?

14. Find the cost of plastering the walls and ceiling of a room 27 ft. long by 24 ft. wide by 13 ft. high at 27 cents per square yard, making no allowance for openings.

15. In 1901 the total imports of tea into the United States were 83,395,924 lb. and of coffee, 511,041,459 lb. If the average cost of tea was 37¢ per lb., and of coffee, 18¢ per lb., what was the total cost of both?

16. What will it cost to plaster the four walls and the ceiling of a hall 60 ft. long, 36 ft. wide, and 18 ft. high, at $22\frac{1}{2}$¢ per square yard, after deducting 486 sq. ft. for door and window openings?

§43. Problems on Flooring and Lighting of Rooms.

Fig. 1 is a drawing or plan, of the second story of a house. The scale is 1 inch to 16 feet, i.e., 1″: 16′.

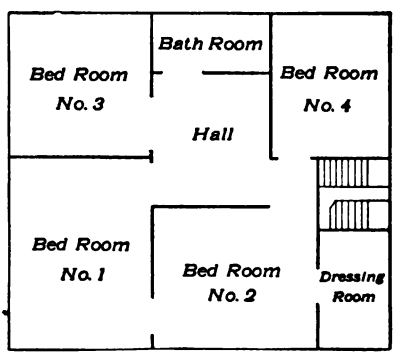


FIGURE 1

1. How long a distance in the house does a line of the drawing represent, that is 2 in. long? $1\frac{1}{2}$ in. long? $\frac{1}{2}$ inch long? $\frac{1}{4}$ inch? $\frac{1}{8}$ inch? $\frac{1}{16}$ inch?

2. Any line, or distance, in the drawing represents a line, or distance, how many times as long in the house?

The scale of the drawing might also be written 1: 192.

3. With a foot-rule marked to 16ths of an inch, measure and give the width and length:

(1) Of the whole house
(2) Of bed room No. 1
(3) Of bed room No. 2
(4) Of bed room No. 3
(5) Of the hall
(6) Of the bath room
(7) Of the dressing room
(8) Of the space for stair steps.

4. It costs 8¢ a square foot to floor the rooms with soft wood; 15¢ a sq. ft. to floor with hard maple; 24¢ a sq. ft. with oak, and 58¢ a sq. ft. with tile. What would it cost to floor bed room No. 3 and bed room No. 4 with soft wood flooring? What would it cost with hard maple flooring? With oak flooring?

5. It was decided to put oak floors in the hall, in bed room No. 1 and bed room No. 2, and in the dressing room; to put hard maple in bed room No. 3; soft wood in bed room No. 4, and tile in the bath room. Find the total cost for these rooms.

6. A room is said to be well-lighted as a living room when the area of the window surface that lets in light is at least $\frac{1}{6}$ of the floor area of the room. Bed room No. 1 is lighted by 3 windows, each containing 8 panes of glass, 16 inches by 20 inches. Is this room well lighted as a living room?

7. Bed room No. 2 is lighted by 2 windows, one containing 8 panes of glass, 16 inches by 20 inches and one containing 12 panes of glass, 16 by 20 inches. Is it well enough lighted for a bed room? For a living room?

8. Bed room No. 3 has 3 windows, 2 of which contain 8 panes of glass each, 16 inches by 20 inches. The third window contains 8 panes of glass, 12 inches by 20 inches. Is it well lighted for a room? For a living room?

9. My study is a room 12 ft.$\times$17 ft. and it is lighted by 3 windows, 2 of which contain 2 panes of glass each, 30 by 34 inches, and 1 of which contains 2 panes, each 30 by 40 inches. Is my study well lighted? Is it well lighted when the shades are half drawn?

10. Test whether your schoolroom is well lighted, supposing the total window surface that admits light should be at least $\frac{1}{5}$ of the floor area.

§44. The Rod.

1. Measure these two lines. Suppose them to be drawn to the scale of 1 inch to 3 feet. What does the short line represent?

2. The long line is how many times the short line?

3. If the short line represents 1 yard, how many yards does the long line represent?

4. If the short line represents 3 feet, how many feet does the long line represent?

5. Call the distance the long line represents 1 rod. How many yards make a rod? How many feet?

6. How many feet make a yard?

7. How many inches make a foot? 320 times the distance the long line represents is a mile.

8. How many yards in a mile? Feet in a mile?

9. How many rods in a mile?

10. A certain block is 22 rods wide and 33 rods long. How many yards is it around the block? How many feet?

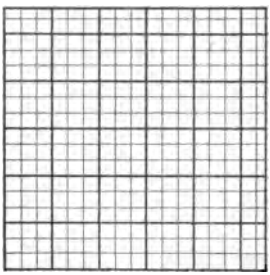
11. A boy lives 64 rods from his school. If he makes 4 trips each day how many yards does he go in 5 days?

12. A man lives $2\frac{1}{2}$ miles from his office. How many rods does he travel from Monday morning until Saturday night assuming that he goes to his office every day?

13. A lot is $82\frac{1}{2}$ feet wide and 165 feet long. What is the cost of fencing it at 18¢ a yard? At 75¢ a rod?

14. The rails on a railroad track are 2 rods long. How many rails are there in a mile of single track?

§45. The Square Rod.



Plan of a square floor, drawn to a scale of $\frac{1}{4}$ inch to 1 yard.

1. Measure the figure. What is the length? The width?

2. How many feet long is the floor? How many feet wide? How many yards wide?

3. Count the number of square yards. How many?

4. If a square were 320 times as long and wide as the square that the figure represents, how long and wide would it be? What is such a square figure called?

5. Draw a 6-inch square. Divide it into half-inch squares. How many squares are there in one row? How many rows are there? How many squares are there in all?

6. How many square inches are there in a square foot?

7. How many square feet in a square yard?

8. How many square yards in a square rod?

§46. Applications.

1. How many square yards in 2 square rods?

2. How many square yards are there in one-half of a square rod?

3. How many square yards in $2\frac{1}{2}$ square rods?

4. A garden contained $2\frac{1}{2}$ square rods and was 1 rod wide. How long was it?

5. A lot contains 20 square rods. How many square yards are there in it?

6. A lot containing 80 square rods was sold for $240. What was the price per square rod?

7. A lot containing 160 square rods was sold at the rate of $3 a square rod. How much was received for the entire lot?

8. A street in a village was 6 rods wide and 40 rods long. How many square rods did it contain? How many square yards?

9. Ten equal blocks front on a street 250 rods long. How many rods long is each block? How many yards long is each block? How many feet?

10. Find the cost at $2 a square yard of paving 60 yards of length of a street with a pavement 3 rods wide. Find the cost of a strip 1 yard wide reaching across the street.

11. A street is 28 blocks long. Each block, is 15 rods long and 5 rods wide. How many square rods does the street contain?

12. An acre is 160 square rods. How many acres does a pasture 20 rods by 40 rods contain?

13. How many acres are there in a piece of ground 10 rods by 16 rods? 5 rods by 32 rods? 8 rods by 20 rods? 4 rods by 40 rods? 2 rods by 80 rods?

14. How many acres are there in a field 40 rods square? 40 rods by 80 rods? 80 rods square?

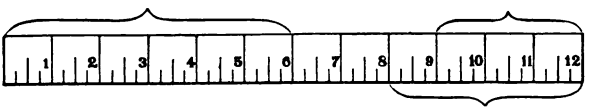
Notice the difference between the expressions "40 rods square" and "40 square rods."

15. A strip of plowed ground is 6 rods by 80 rods. How many acres does the strip contain?

16. How many acres are there in a farm that is 160 rods square?

COMMON FRACTIONS

§47. Fractions in Measuring.



FOOT-RULE DRAWN TO A SCALE OF 3 TO 12, OR $\frac{1}{4}$

Answer all you can orally

1. What part of a foot is 6 inches? 3 inches? 4 inches? 9 inches? 2 inches?

2. What part of a foot is 1 inch? 8 in.? 10 in.? 5 in.? 7 in.?

3. What part of a yard is a foot? 2 ft.? 3 ft.? 4 ft.? 6 ft.?

4. What part of a gallon is a quart? 2 quarts? 3 quarts? 4 quarts? 5 quarts?

5. What part of a bushel is a peck? 2 pk.? 3 pk.? 4 pk.? 5 pecks?

6. What part of a gallon is a pint? 2 pints? 3 pints? 4 pints? 5 pints? 6 pints?

7. What part of a peck is a quart? 2 quarts? 3 quarts? 4 quarts? 5 quarts? 6 quarts?

8. What part of a yard is 6 inches? 3 inches? 2 inches? 1 inch? 12 inches? 24 inches?

9. What part of 36 is 6? 3? 2? 12? 18? 4? 9?

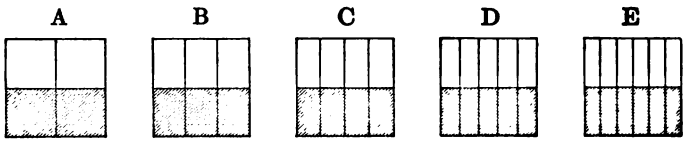
10. What part of a bushel is a quart? 2 quarts? 4 quarts? 8 quarts? 16 quarts?

11. What part of 32 is 2? 4? 6? 8? 16? 12?

12. What part of a bushel is a pint? 2 pints? 4 pints? 8 pints? 16 pints? 32 pints?

13. What part of 64 is 32? 16? 8? 4? 2? 24? 40? 48? 56?

§48. Fractions Pictured.



Answer all you can orally

1. What part of A is $\frac{1}{2}$ of $\frac{1}{2}$ of A?

2. What part of B is $\frac{1}{2}$ of $\frac{1}{3}$ of B? $\frac{1}{3}$ of $\frac{1}{2}$ of B? $\frac{1}{2}$ of $\frac{2}{3}$ of B? $\frac{2}{3}$ of $\frac{1}{2}$ of B?

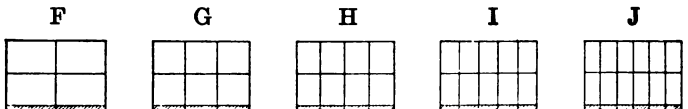
3. What part of C is $\frac{1}{2}$ of $\frac{1}{4}$ of C? $\frac{1}{4}$ of $\frac{1}{2}$ of C? $\frac{1}{2}$ of $\frac{3}{4}$ of C? $\frac{3}{4}$ of $\frac{1}{2}$ of C?

4. What part of D is $\frac{1}{2}$ of $\frac{1}{5}$ of D? $\frac{1}{5}$ of $\frac{1}{2}$ of D? $\frac{1}{2}$ of $\frac{3}{5}$ of D? $\frac{3}{5}$ of $\frac{1}{2}$ of D?

5. What part of E is $\frac{1}{2}$ of $\frac{1}{6}$ of E? $\frac{1}{6}$ of $\frac{1}{2}$ of E? $\frac{1}{2}$ of $\frac{5}{6}$ of E? $\frac{5}{6}$ of $\frac{1}{2}$ of E?

6. Show from A, B, C, D, and E that:

$\frac{1}{2}=\frac{2}{4}$ $\frac{1}{2}=\frac{3}{6}$ $\frac{1}{2}=\frac{4}{8}$ $\frac{1}{2}=\frac{5}{10}$ $\frac{1}{2}=\frac{6}{12}$



7. What part of F is $\frac{1}{3}$ of $\frac{1}{2}$ of F? $\frac{1}{2}$ of $\frac{1}{3}$ of F?

8. What part of G is $\frac{1}{3}$ of $\frac{1}{3}$ of G?

9. What part of H is $\frac{1}{3}$ of $\frac{1}{4}$ of H? $\frac{1}{4}$ of $\frac{1}{3}$ of H? $\frac{1}{3}$ of $\frac{3}{4}$ of H? $\frac{3}{4}$ of $\frac{1}{3}$ of H?

10. What part of I is $\frac{1}{3}$ of $\frac{1}{5}$ of I? $\frac{1}{5}$ of $\frac{1}{3}$ of I? $\frac{1}{3}$ of $\frac{2}{5}$ of I? $\frac{2}{5}$ of $\frac{1}{3}$ of I?

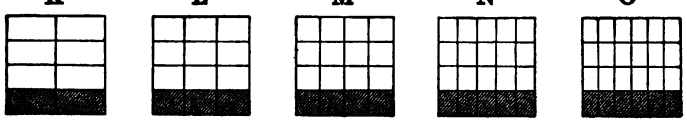
11. What part of J is $\frac{1}{3}$ of $\frac{1}{6}$ of J? $\frac{1}{6}$ of $\frac{1}{3}$ of J? $\frac{1}{3}$ of $\frac{5}{6}$ of J? $\frac{5}{6}$ of $\frac{1}{3}$ of J?

12. Show from F, G, H, I, and J that:

$\frac{1}{3}=\frac{2}{6}$ $\frac{1}{3}=\frac{3}{9}$ $\frac{1}{3}=\frac{4}{12}$ $\frac{1}{3}=\frac{5}{15}$ $\frac{1}{3}=\frac{6}{18}$

$\frac{2}{3}=\frac{4}{6}$ $\frac{2}{3}=\frac{6}{9}$ $\frac{2}{3}=\frac{8}{12}$ $\frac{2}{3}=\frac{10}{15}$ $\frac{2}{3}=\frac{12}{18}$

K L M N O



13. What part of K is $\frac{1}{4}$ of $\frac{1}{2}$ of K? $\frac{1}{2}$ of $\frac{1}{4}$ of K?

14. What part of L is $\frac{1}{4}$ of $\frac{1}{3}$ of L? $\frac{1}{3}$ of $\frac{1}{4}$ of L? $\frac{1}{4}$ of $\frac{2}{3}$ of L? $\frac{2}{3}$ of $\frac{1}{4}$ of L?

15. What part of M is $\frac{1}{4}$ of $\frac{1}{4}$ of M?

16. What part of N is $\frac{1}{4}$ of $\frac{1}{5}$ of N? $\frac{1}{5}$ of $\frac{1}{4}$ of N? $\frac{1}{4}$ of $\frac{3}{5}$ of N? $\frac{3}{5}$ of $\frac{1}{4}$ of N?

17. What part of O is $\frac{1}{4}$ of $\frac{1}{6}$ of O? $\frac{1}{6}$ of $\frac{1}{4}$ of O? $\frac{1}{4}$ of $\frac{5}{6}$ of O? $\frac{5}{6}$ of $\frac{1}{4}$ of O?

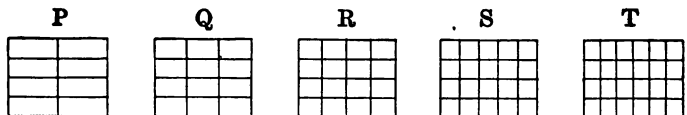
18. Show from K, L, M, N, and O that:

$\frac{1}{4}=\frac{2}{8}$ $\frac{1}{4}=\frac{3}{12}$ $\frac{1}{4}=\frac{4}{16}$ $\frac{1}{4}=\frac{5}{20}$ $\frac{1}{4}=\frac{6}{24}$

$\frac{1}{2}=\frac{4}{8}$ $\frac{1}{2}=\frac{6}{12}$ $\frac{1}{2}=\frac{8}{16}$ $\frac{1}{2}=\frac{10}{20}$ $\frac{1}{2}=\frac{12}{24}$

$\frac{3}{4}=\frac{6}{8}$ $\frac{3}{4}=\frac{9}{12}$ $\frac{3}{4}=\frac{12}{16}$ $\frac{3}{4}=\frac{15}{20}$ $\frac{3}{4}=\frac{18}{24}$

P Q R S T



19. What part of P is $\frac{1}{5}$ of $\frac{1}{2}$ of P? $\frac{1}{2}$ of $\frac{1}{5}$ of P?

20. What part of Q is $\frac{1}{5}$ of $\frac{1}{3}$ of Q? $\frac{1}{3}$ of $\frac{1}{5}$ of Q?

21. What part of R is $\frac{1}{5}$ of $\frac{1}{4}$ of R? $\frac{1}{4}$ of $\frac{1}{5}$ of R?

22. What part of S is $\frac{1}{5}$ of $\frac{1}{5}$ of S?

23. What part of T is $\frac{1}{5}$ of $\frac{1}{6}$ of T? $\frac{1}{6}$ of $\frac{1}{5}$ of T?

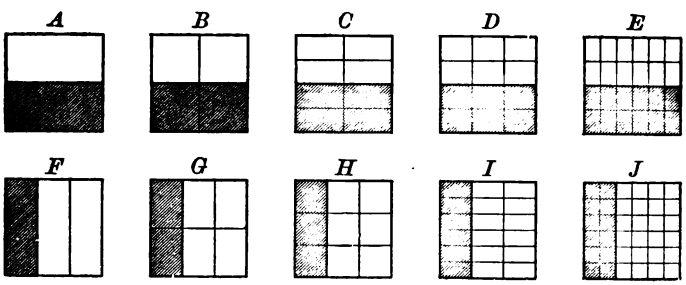
24. Show from P, Q, R, S, and T that:

$\frac{1}{5}=\frac{2}{10}$	$\frac{1}{5}=\frac{3}{15}$	$\frac{1}{5}=\frac{4}{20}$	$\frac{1}{5}=\frac{5}{25}$	$\frac{1}{5}=\frac{6}{30}$
$\frac{2}{5}=\frac{4}{10}$	$\frac{2}{5}=\frac{6}{15}$	$\frac{2}{5}=\frac{8}{20}$	$\frac{2}{5}=\frac{10}{25}$	$\frac{2}{5}=\frac{12}{30}$
$\frac{3}{5}=\frac{6}{10}$	$\frac{3}{5}=\frac{9}{15}$	$\frac{3}{5}=\frac{12}{20}$	$\frac{3}{5}=\frac{15}{25}$	$\frac{3}{5}=\frac{18}{30}$
$\frac{4}{5}=\frac{8}{10}$	$\frac{4}{5}=\frac{12}{15}$	$\frac{4}{5}=\frac{16}{20}$	$\frac{4}{5}=\frac{20}{25}$	$\frac{4}{5}=\frac{24}{30}$

§49. Reducing Fractions to Higher or Lower Terms.

1. Show from figures *B*, *G*, *C*, *D*, and *E* that:

$\frac{1}{2}=\frac{2}{4}$ $\frac{1}{2}=\frac{3}{6}$ $\frac{1}{2}=\frac{4}{8}$ $\frac{1}{2}=\frac{6}{12}$ $\frac{1}{2}=\frac{12}{24}$



2. How could halves be expressed as 4ths without a picture? As 6ths? As 8ths? As 12ths?

3. Show from figures *G*, *H*, *D*, *I*, and *J* that:

$\frac{1}{3}=\frac{2}{6}$	$\frac{1}{3}=\frac{3}{9}$	$\frac{1}{3}=\frac{4}{12}$	$\frac{1}{3}=\frac{6}{18}$	$\frac{1}{3}=\frac{12}{36}$
$\frac{2}{3}=\frac{4}{6}$	$\frac{2}{3}=\frac{6}{9}$	$\frac{2}{3}=\frac{8}{12}$	$\frac{2}{3}=\frac{12}{18}$	$\frac{2}{3}=\frac{24}{36}$

4. How could thirds be expressed as 6ths without a picture? As 9ths? As 12ths? As 18ths?

5. Draw figures and show that:

$\frac{1}{4}=\frac{2}{8}$	$\frac{1}{4}=\frac{3}{12}$	$\frac{1}{4}=\frac{4}{16}$	$\frac{1}{4}=\frac{5}{20}$	$\frac{1}{4}=\frac{6}{24}$
$\frac{3}{4}=\frac{6}{8}$	$\frac{3}{4}=\frac{9}{12}$	$\frac{3}{4}=\frac{12}{16}$	$\frac{3}{4}=\frac{15}{20}$	$\frac{3}{4}=\frac{18}{24}$

6. How could fourths be expressed as 8ths without using a picture? As 12ths? As 16ths?

7. Tell what numbers should stand in place of the letter x, in the following:

$\frac{1}{5}=\frac{x}{10}$ $\frac{1}{5}=\frac{x}{15}$ $\frac{1}{5}=\frac{x}{20}$ $\frac{1}{5}=\frac{x}{25}$ $\frac{1}{5}=\frac{x}{30}$

$\frac{2}{5}=\frac{x}{10}$ $\frac{2}{5}=\frac{x}{15}$ $\frac{3}{5}=\frac{x}{20}$ $\frac{2}{5}=\frac{x}{25}$ $\frac{3}{5}=\frac{x}{30}$

$\frac{3}{5}=\frac{6}{x}$ $\frac{3}{5}=\frac{9}{x}$ $\frac{3}{5}=\frac{12}{x}$ $\frac{3}{5}=\frac{15}{x}$ $\frac{2}{5}=\frac{12}{x}$

$\frac{4}{5}=\frac{8}{x}$ $\frac{4}{5}=\frac{12}{x}$ $\frac{4}{5}=\frac{20}{x}$ $\frac{4}{5}=\frac{24}{x}$ $\frac{4}{5}=\frac{28}{x}$.

8. Express as sixths: $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$.

9. Express as twelfths: $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{5}{6}$.

10. Express as eighteenths: $\frac{1}{9}$ $\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{5}{9}$ $\frac{7}{9}$.

11. Express as halves $\frac{2}{4}$ $\frac{3}{6}$ $\frac{5}{10}$ $\frac{7}{14}$ $\frac{8}{16}$ $\frac{10}{20}$.

12. Express as thirds: $\frac{2}{6}$ $\frac{4}{6}$ $\frac{3}{9}$ $\frac{6}{9}$ $\frac{4}{12}$

$\frac{8}{12}$ $\frac{12}{12}$ $\frac{5}{15}$ $\frac{10}{15}$ $\frac{15}{15}$.

13. How may 6ths be expressed as 3ds? 9ths as 3ds? 12ths as 3ds? 15ths as 3ds?

14. Express as 4ths: $\frac{2}{8}$ $\frac{4}{8}$ $\frac{6}{8}$ $\frac{3}{12}$ $\frac{6}{12}$ $\frac{9}{12}$

$\frac{12}{12}$ $\frac{4}{16}$ $\frac{12}{16}$ $\frac{10}{20}$ $\frac{5}{20}$ $\frac{15}{20}$.

15. How may 8ths be expressed as 4ths? 12ths as 4ths? 16ths as 4ths? 20ths as 4ths?

THE NUMERATOR (meaning numberer) **of a fraction is the number above the line.**

THE DENOMINATOR (meaning namer) **is the number below the line.**

The numerator and denominator, when spoken of together, are called the TERMS of the fraction.

It has been seen that multiplying (or dividing) both terms of a fraction by the same number does not change the value of the fraction.

Problems 11 to 15 show that *dividing* both terms *by the same number* reduces to lower terms but does not change the value of the fraction.

16. Show, from Fig. 1, how to express $\frac{1}{2}$ as 4ths; as 8ths; 16ths.

17. Express $\frac{1}{3}$ as 6ths; 9ths; 12ths; 18ths.

18. Express as 3ds: $\frac{4}{6}$ $\frac{6}{9}$ $\frac{14}{21}$ $\frac{20}{30}$.

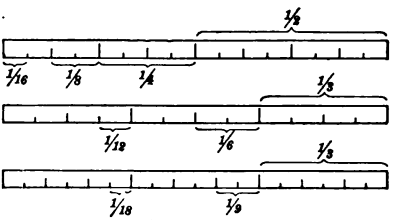


FIGURE 1

19. Express as 7ths: $\frac{6}{21}$ $\frac{8}{14}$ $\frac{4}{28}$ $\frac{35}{49}$.

20. Compare the values of the following fractions after reducing each to its lowest terms:

$\frac{3}{4}$ $\frac{6}{8}$ $\frac{9}{12}$ $\frac{60}{80}$ $\frac{75}{100}$ $\frac{300}{400}$.

21. Express $\frac{2}{5}$ as 10ths; 15ths; 20ths; 35ths; 50ths; 100ths.

22. By what must the numerator of $\frac{3}{4}$ be multiplied to make the numerator the same as that of $\frac{6}{8}$? By what must the denominator of $\frac{3}{4}$ be multiplied to make the denominator the same as that of $\frac{6}{8}$? Sketch a square and divide it to show the comparative sizes of $\frac{3}{4}$ and $\frac{6}{8}$.

23. By what must each term of $\frac{3}{4}$ be multiplied to obtain $\frac{6}{8}$? $\frac{9}{12}$? $\frac{21}{28}$? $\frac{75}{100}$? $\frac{300}{400}$? What, then, is the *value* of each fraction of problem 20?

24. What is the value of all the fractions in each of the following sets?

(1) $\frac{2}{3}$ $\frac{6}{9}$ $\frac{8}{12}$ $\frac{10}{15}$ $\frac{12}{18}$ $\frac{14}{21}$ $\frac{20}{30}$.

(2) $\frac{4}{5}$ $\frac{8}{10}$ $\frac{12}{15}$ $\frac{16}{20}$ $\frac{20}{25}$ $\frac{40}{50}$.

25. How may the value of a fraction be expressed in higher terms?

A fraction may be expressed in higher terms, without changing its value, by multiplying both terms by the same number.

26. Express the following fractions in higher terms:

$\frac{3}{8}$ $\frac{6}{7}$ $\frac{5}{9}$ $\frac{13}{15}$ $\frac{12}{24}$ $\frac{6}{12}$ $\frac{20}{50}$.

27. How can the terms of $\frac{3}{4}$ be obtained from the terms of $\frac{6}{8}$? Of $\frac{9}{12}$? Of $\frac{21}{28}$? Of $\frac{75}{100}$?

28. What fraction is obtained by dividing each term of $\frac{4}{28}$ by 4? Compare the value of $\frac{4}{28}$ with the value of the fraction $\frac{1}{7}$. (See Fig. 1.)

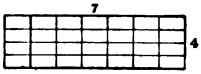


FIGURE 1

29. By using a rectangle, divided as in Fig. 1, show that $\frac{4}{28}$ of the rectangle equals $\frac{1}{7}$ of it.

30. In what lower terms can $\frac{20}{30}$ be expressed? $\frac{40}{100}$? $\frac{30}{42}$? Show that $\frac{20}{30}=\frac{2}{3}$ by means of a properly divided rectangle.

31. What effect on the *value* of a fraction does dividing each of its terms by the same number have?

32. How may the value of a fraction be expressed in lower terms?

A fraction may be expressed in lower terms, without changing its value by dividing both terms by the same whole number.

Fractions are most easily used when their numerators and their denominators are the smallest possible whole numbers.

Fractions are in their lowest terms when their terms are the smallest whole numbers that give a fraction of the same value.

Among the many ways in which $\frac{12}{24}$ can be written are these:

$\frac{12}{24}$ $\frac{6}{12}$ $\frac{3}{6}$ $\frac{2}{4}$ $\frac{1}{2}$

33. Show how $\frac{1}{2}$ is obtained from $\frac{12}{24}$ without a picture, that is, by using numbers only.

§50. Problems.

Express in their lowest terms the fractions of the answers to the following problems:

1. A man works 48 da. out of 64 days. What part of 64 da. does he work?

2. A grocer bought 24 boxes of oranges and sold 16 boxes. What part of his purchase did he sell?

3. Out of 56 bu. of potatoes a huckster sold 49 bu. What part of his potatoes did he sell?

4. A street car on one line makes a weekly average of 72 trips. A car on another line makes an average of 144 trips. What part of the number of trips made by the second street car, is the number of trips made by the first car?

5. A man pays \$50 for wood and \$125 for coal. What part of the cost of the coal is the cost of the wood?

6. Out of 1000 ft. of lumber purchased, 450 ft. were used for the flooring of two rooms. What part of the amount purchased was used for flooring?

7. Out of 126 bu. of oats bought, a livery man fed 63 bu. in a week. What part of the number of bushels bought were fed during this one week?

8. In 644 lb. of sandy loam there were 364 lb. of sand and gravel. What part of the soil, by weight, was sand and gravel?

9. A boy spent $\frac{2}{5}$ of his money and gave away $\frac{1}{3}$ of it; what part of it had he left? (Use a divided rectangle.)

To solve such problems as 9 more easily it is necessary to know how to add, and to subtract fractions.

ADDITION AND SUBTRACTION OF FRACTIONS

§51. Fractions Having a Common Fractional Unit (a Common Denominator).

The fraction $\frac{3}{4}$ means 3 of the $\frac{1}{4}$ths (one-fourths) of a number. $\frac{1}{4}$ is called the fractional unit of $\frac{3}{4}$.

1. What are the fractional units of:

$\frac{4}{7}$? $\frac{5}{9}$? $\frac{2}{3}$? $\frac{11}{12}$? $\frac{8}{15}$? $\frac{19}{26}$? $\frac{32}{75}$?

2. What is the unit of a fraction having 6 for its denominator? 7? 18? 25? 30? 50? 100?

3. Complete these equations:

$\frac{4}{9}$ yd. + $\frac{3}{9}$ yd. = $\frac{3}{6}$ gal. + $\frac{2}{6}$ gal. =

\$$\frac{5}{10}$ + \$$\frac{8}{10}$ = $\frac{5}{17}$ + $\frac{6}{17}$ =

$\frac{3}{7}$ wk. + $\frac{2}{7}$ wk. = $\frac{7}{12}$ + $\frac{1}{12}$ =

$\frac{3}{5}$ bu. + $\frac{1}{5}$ bu. = $\frac{5}{a}$ + $\frac{6}{a}$ =

$\frac{5}{a}$ is read "5 divided by a."

4. Make a rule for adding two fractions having a common denominator (or fractional unit).

5. Complete these equations:

$\frac{4}{9} - \frac{3}{9} =$ $\frac{8}{9} - \frac{5}{9} =$ $\frac{7}{15} - \frac{3}{15} =$

$\frac{7}{10} - \frac{4}{10} =$ $\frac{5}{7} - \frac{2}{7} =$ $\frac{10}{12} - \frac{3}{12} =$

6. Make a rule for finding the difference between two fractions with a common denominator, that is, whose denominators are the same number.

The sum, or the difference, of any two fractions having a common denominator equals the sum, or the difference, of their numerators, divided by the common denominator.

§52. Fractions Easily Reduced to a Common Fractional Unit.

1. $\frac{1}{4}$ equals how many 12ths? 8ths? 16ths? 20ths? 64ths? 100ths?

2. $\frac{7}{12}$ hr. $+ \frac{1}{4}$ hr. $= x$ hr. What is the value of x?

$\frac{7}{12}$ hr. $+ \frac{8}{12}$ hr. $= \frac{10}{12}$ hr. $= \frac{5}{6}$ hr. $\quad x = \frac{5}{6}$.

3. $\frac{7}{12}$ hr. $- \frac{1}{4}$ hr. $= y$ hr. What is y?

4. Find these sums and differences:

$\frac{2}{5} + \frac{3}{10} = ?$	$\frac{6}{7} - \frac{3}{14} = ?$	$\frac{9}{11} + \frac{3}{22} + \frac{2}{66} = ?$
$\frac{1}{2} + \frac{3}{8} = ?$	$\frac{3}{8} - \frac{3}{16} = ?$	$\frac{13}{32} + \frac{5}{16} + \frac{3}{8} = ?$
$\frac{7}{9} - \frac{1}{3} = ?$	$\frac{7}{15} - \frac{1}{5} = ?$	$\frac{7}{14} + \frac{3}{28} - \frac{5}{56} = ?$
$\frac{5}{12} + \frac{2}{3} = ?$	$\frac{7}{12} - \frac{1}{3} = ?$	$\frac{8}{30} + \frac{2}{15} - \frac{1}{5} = ?$
$\frac{11}{12} - \frac{2}{3} = ?$	$\frac{10}{21} - \frac{2}{7} = ?$	$\frac{13}{24} - \frac{1}{12} + \frac{5}{6} = ?$

§53. Whole and Mixed Numbers as Fractions.

Thus far we have had to do with (1) whole numbers, or integers, and (2) fractional numbers, or fractions.

A PROPER FRACTION is a fraction whose numerator is less than its denominator: as $\frac{2}{3}$, $\frac{3}{5}$, $\frac{3}{7}$.

An IMPROPER FRACTION is a fraction whose numerator is equal to, or greater than, its denominator: as $\frac{3}{3}$, $\frac{8}{2}$, $\frac{7}{5}$, $\frac{6}{2}$.

A MIXED NUMBER is a number that is expressed as an integer and a fraction: as $2\frac{1}{2}$, $12\frac{3}{4}$, $7\frac{5}{6}$.

1. Name the proper and the improper fractions, the mixed and the whole numbers in the following:

$\frac{3}{4}$ $\quad \frac{3}{8}$ $\quad \frac{1}{16}$ $\quad 5$ $\quad \frac{12}{5}$ $\quad \frac{8}{2}$ $\quad \frac{2}{8}$ $\quad 1\frac{7}{8}$ $\quad 3\frac{1}{15}$ $\quad \frac{3}{15}$ $\quad 2$

2. Give an example of each class.

3. How many times is 1 contained in 3? In any number?

We may write this in symbols, thus:

$\frac{3}{1} = 3 \qquad \frac{18}{1} = 18 \qquad \frac{25}{1} = 25 \qquad \frac{a}{1} = a$

Any whole number may be written in the fractional form by writing 1 for its denominator.

4. Write the following numbers in fractional form:

$$12 \quad 24 \quad 32 \quad 68 \quad a \quad x$$

5. How many halves are there in 1? In 2? In 5? In $2\frac{1}{2}$? In $5\frac{1}{2}$?

6. How many 5ths are there in 1? In 2? In 4? In 6? In $6\frac{2}{5}$? In $7\frac{3}{5}$?

7. Change these whole numbers into 6ths:

$$2 \quad 5 \quad 16 \quad 120.$$

8. Change into 6ths:

$$1\tfrac{5}{6} \quad 3\tfrac{1}{6} \quad 8\tfrac{1}{6} \quad 16\tfrac{5}{6} \quad 15\tfrac{1}{6}.$$

9. Change into 7ths:

$$5 \quad 6\tfrac{2}{7} \quad 10\tfrac{5}{7} \quad 15\tfrac{6}{7} \quad 20\tfrac{3}{7}.$$

10. Change into 12ths:

$$2 \quad 8 \quad 12 \quad 20 \quad 56 \quad 128.$$

11. How may any whole number be changed into 12ths? Into 9ths? Into 24ths? Into a fraction having any given denominator?

Any whole number may be expressed in the form of a fraction having a given denominator by multiplying the whole number by the given denominator and writing the product over that denominator.

12. Express 6, 18, 17, 22, 39, 28 as 4ths; as 7ths; as 10ths; as 100ths.

13. Change into 6ths: $2 \quad 4\frac{1}{6} \quad 24\frac{5}{6} \quad 37\frac{5}{6}$.

14. Change the following numbers into improper fractions having 7 for a denominator:

$$3\tfrac{1}{7} \quad 24\tfrac{2}{7} \quad 106\tfrac{3}{7} \quad 234\tfrac{6}{7} \quad 648.$$

15. How can you change any mixed number into an improper fraction with the same denominator as that of the given fraction?

A mixed number may be expressed as an improper fraction by multiplying the whole number by the denominator, adding the numerator to the product, and writing the sum over the given denominator.

16. Reduce to improper fractions:

$2\frac{1}{5}$ $6\frac{1}{3}$ $12\frac{1}{2}$ $18\frac{3}{4}$ $25\frac{6}{7}$ $328\frac{5}{9}$ $609\frac{11}{13}$

17. Express $\frac{12}{6}$ as a whole number. Change $\frac{13}{6}$ to a mixed number.

18. Change to whole or mixed numbers the following improper fractions:

$\frac{7}{2}$ $\frac{12}{7}$ $\frac{14}{7}$ $\frac{16}{7}$ $\frac{24}{3}$ $\frac{25}{9}$ $\frac{36}{11}$ $\frac{36}{12}$ $\frac{45}{9}$ $\frac{20}{7}$

19. How may any improper fraction be changed to a whole number? A mixed number?

An improper fraction may be changed to a whole or mixed number by making the indicated division.

20. Reduce to whole or mixed numbers the following:

$\frac{4}{2}$ $\frac{16}{7}$ $\frac{28}{14}$ $\frac{706}{7}$ $\frac{95}{7}$ $\frac{343}{100}$ $\frac{1682}{13}$

§54. Adding and Subtracting Fractions in General.

1. From the river, R, to the bend, A, the road is $1\frac{1}{4}$ miles long. From the bend, A, to the bend, B, is $\frac{1}{2}$ mile, and from bend, B, to the cross-road, C, is $\frac{3}{4}$ mile. How far is it, by the road, from R to C?

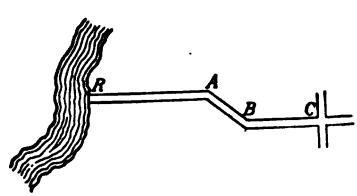
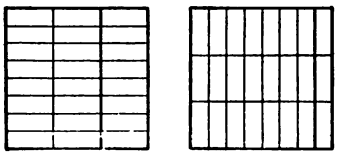


FIGURE 1

2. A man owns $\frac{2}{3}$ of an acre in one block and $\frac{7}{8}$ of an acre in another block. How much land does he own?

This problem requires us to add $\frac{2}{3}$ and $\frac{7}{8}$.



(*a*) $\frac{2}{3}=\frac{16}{24}$ (*b*) $\frac{7}{8}=\frac{21}{24}$

FIGURE 1

Fig. 1 (*a*) is divided by the vertical lines into 3 equal parts and by the horizontal lines into 8 equal parts. Into how many equal parts do the 2 sets of parallel lines divide square (*a*)?

Show by Fig. 1 (*a*) that $\frac{2}{3}=\frac{16}{24}$.

Show by Fig. 1 (*b*) that $\frac{7}{8}=\frac{21}{24}$.

$\frac{2}{3}+\frac{7}{8}=\frac{16}{24}+\frac{21}{24}=\frac{37}{24}=1\frac{13}{24}$. *Ans.* $1\frac{13}{24}$ acres.

3. Find the sums: $\frac{2}{5}+\frac{3}{4}$; $\frac{1}{3}+\frac{4}{7}$; $\frac{2}{3}+\frac{3}{5}$.

§55. Least Common Multiple.

A more convenient way of adding or subtracting fractions in general than the way first used will now be given.

The phrase "exactly divides" means "divides without a remainder," or "goes into a whole number of times."

The multiples of 3, are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, and all other numbers that 3 exactly divides.

The multiples of 5, are 5, 10, 15, 20, 25, 30, and all other numbers that 5 exactly divides.

The numbers that are *common* multiples of 3 and 5 are 15, 30, 45, etc.

The least of these common multiples of 3 and 5 is called the *least* common multiple of 3 and 5.

The LEAST COMMON MULTIPLE of two or more numbers is the least number that every one of the numbers exactly divides. The letters l. c. m. stand for least common multiple.

1. To find the l. c. m. of 6, 12, and 18.

Since the l. c. m. of 6, 12, and 18 must contain every one of these numbers *as a factor*, it must contain also 2 and 3; 4 and 3; and 9 and 2.

Write the numbers in factored form, thus:

$6 = 2 \times 3$; $12 = 2 \times 2 \times 3$; $18 = 2 \times 3 \times 3$

The number $2 \times 2 \times 3 \times 3$, or 36, made by taking the *different* factors of 6, 12, and 18, each factor the greatest number of times it is in any one of the numbers, is the required l. c. m. of 6, 12, and 18.

Hence, *to find the l. c. m. of two or more numbers, multiply together all the different prime factors, using each factor the greatest number of times it is to be found in any one of the given numbers.*

Two numbers, 3 and 5, that have no common factor but 1, are said to be *prime to each other.*

2. Show that the l. c. m. of two numbers that are prime to each other is their product.

3. Find the l. c. m. of the following:

(1) 12 and 18 (3) 9, 18, and 20 (5) 12, 21, and 36
(2) 24 and 30 (4) 15, 20, and 24 (6) 18, 56, and 84

4. Find the l. c. m. of the denominators (1) of $\frac{5}{21}$ and $\frac{7}{24}$; (2) of $\frac{11}{30}$ and $\frac{12}{35}$; (3) of $\frac{18}{49}$ and $\frac{19}{56}$; (4) $\frac{7}{22}$, $\frac{5}{33}$, and $\frac{17}{99}$.

The l. c. m. of the denominators of two or more fractions is called the LEAST COMMON DENOMINATOR of the fractions. The letters l. c. d. stand for least common denominator.

5. What is the least common denominator of $\frac{2}{3}$ and $\frac{7}{8}$? What, then, is the sum of $\frac{2}{3}$ and $\frac{7}{8}$? $\frac{2}{3} = \frac{?}{24}$? $\frac{7}{8} = \frac{?}{24}$?

$$\frac{2}{3} + \frac{7}{8} = \frac{16}{24} + \frac{21}{24} = \frac{16+21}{24} = \frac{37}{24} = 1\frac{13}{24}. \quad \textit{Ans.}\ 1\frac{13}{24}.$$

6. Add: $\frac{3}{4}$ and $\frac{2}{3}$; $\frac{4}{5}$ and $\frac{3}{7}$; $\frac{1}{4}$ and $\frac{5}{6}$; $\frac{7}{8}$ and $\frac{11}{12}$.

7. In 1900, the New England states produced about $\frac{11}{24}$, the Middle Atlantic states about $\frac{9}{20}$, and the Central states about $\frac{1}{12}$ of all the maple sugar produced in the United States. About what part of the maple sugar did they all produce?

8. In 1900, the New England states produced about $\frac{1}{8}$ of the maple syrup of the country; the Middle Atlantic

states about $\frac{2}{7}$, and the Central states about $\frac{4}{7}$ of it. About what part of the maple syrup of the country did the three groups together produce?

9. A boy had $\$4\frac{4}{5}$ and spent $\$\frac{1}{2}$. What part of a dollar did he have left.?

The least common denominator is $2 \times 5 = 10$.

$\frac{4}{5} = \frac{8}{10}$; $\frac{1}{2} = \frac{5}{10}$. Then, $\frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$. *Ans.* $\frac{3}{10}$.

Show this solution also, using a divided rectangle.

10. William is $4\frac{2}{3}$ ft. tall and James is $4\frac{5}{12}$ ft. tall. Who is the taller and by how much?

11. A man owned $\frac{15}{16}$ A. of land and sold $\frac{3}{7}$ A. How much land did he then own?

12. From $\frac{5}{6}$ lb. of loam $\frac{2}{7}$ lb. of sand was removed. How much of the loam remained?

13. A man having $\$4\frac{1}{5}$, paid a debt of $\$2\frac{3}{4}$. How much money had he after paying the debt?

$\frac{1}{5} = \frac{4}{20}$, $\frac{3}{4} = \frac{15}{20}$. As $\frac{4}{20}$ is less than $\frac{15}{20}$, write $4\frac{4}{20}$ in the form $3\frac{24}{20}$. The problem is then $3\frac{24}{20} - 2\frac{15}{20} = 1\frac{9}{20}$.

14. Solve these exercises:

(1) $13\frac{1}{8} - 7\frac{2}{5} = ?$ (3) $128\frac{4}{5} - 97\frac{15}{16} = ?$

(2) $30\frac{1}{20} - 16\frac{7}{8} = ?$ (4) $1217\frac{5}{8} - 989\frac{11}{15} = ?$

15. A tree $72\frac{3}{4}$ ft. high is broken off $28\frac{7}{15}$ ft. from the top. How high is the stump?

16. The 5-cent piece weighs $73\frac{1}{6}$ grains and the quarter-dollar $96\frac{9}{20}$ grains. How much heavier is the quarter than the 5-cent piece?

17. The dime weighs $38\frac{7}{12}$ grains. Before 1853 it weighed $41\frac{1}{4}$ grains. By how much was the weight of the dime reduced in 1853?

18. A man owned $\frac{3}{4}$ of a vessel and sold $\frac{2}{5}$ of the vessel. What part did he still own?

19. There were $3\frac{4}{5}$ A. in a tract of land, and $1\frac{7}{8}$ A. were laid out in town lots. How much was not laid out in lots?

20. Write out rapidly the values of these sums and differences:

(1) $\frac{1}{2}+\frac{1}{3}=?$ (6) $\frac{5}{8}-\frac{3}{5}=?$ (11) $\frac{6}{11}+\frac{2}{3}=?$

(2) $\frac{1}{4}-\frac{1}{5}=?$ (7) $\frac{4}{5}-\frac{1}{4}=?$ (12) $\frac{3}{8}-\frac{2}{9}=?$

(3) $\frac{2}{3}-\frac{1}{2}=?$ (8) $\frac{6}{7}+\frac{3}{4}=?$ (13) $\frac{4}{11}-\frac{3}{20}=?$

(4) $2\frac{3}{8}+1\frac{5}{6}=?$ (9) $2\frac{8}{9}-1\frac{3}{4}=?$ (14) $3\frac{6}{11}-1\frac{3}{7}=?$

(5) $3\frac{1}{5}+1\frac{3}{8}=?$ (10) $4\frac{3}{5}-2\frac{1}{9}=?$ (15) $6\frac{7}{12}-4\frac{2}{7}=?$

21. In a sample of soil weighing $46\frac{3}{8}$ ounces, $29\frac{5}{6}$ ounces were sandy loam. How many ounces of the soil were not sandy loam?

22. Six apples weighed $18\frac{3}{4}$ ounces before paring and coring, and $12\frac{3}{10}$ ounces after paring and coring. How many ounces were removed by paring and coring?

23. Find the following sums and differences:

(1) $\frac{3}{5}+\frac{2}{7}+\frac{3}{10}$ (4) $\frac{7}{8}-\frac{2}{3}+\frac{1}{5}$ (7) $\frac{6}{11}+\frac{5}{7}+\frac{8}{33}$

(2) $\frac{1}{3}+\frac{3}{5}+\frac{5}{12}$ (5) $\frac{3}{7}+\frac{5}{8}-\frac{7}{14}$ (8) $\frac{5}{9}+\frac{3}{8}+\frac{7}{10}$

(3) $\frac{3}{4}+\frac{1}{6}-\frac{2}{15}$ (6) $\frac{5}{6}+\frac{7}{8}-\frac{2}{15}$ (9) $\frac{8}{15}-\frac{3}{7}+\frac{3}{4}$

MULTIPLYING FRACTIONS

§56. Multiplying a Fraction by a Whole Number.

1. 6 times 3 mi. = how many miles?

2. 8×4 yd. = how many yards?

3. 8×4 fifths = how many fifths?

4. $5 \times \frac{4}{7} = \frac{x}{7}$. What is the value of x?

5. $9 \times \frac{3}{11} = \frac{x}{11}$. What is the value of x?

6. A line is divided into 11 equal parts, and 5 of them are taken. What fraction represents the part of the whole which is taken?

7. What fraction would represent 3 times this part?

8. Replace the letter in each of these equations by its correct value. The sign (×) should be read "times" in these problems.

(1) $6 \times \frac{2}{3} = \frac{x}{3}$ (5) $5 \times \frac{1}{6} = \frac{x}{6}$ (9) $2 \times \frac{3}{11} = \frac{x}{11}$
(2) $4 \times \frac{1}{2} = \frac{x}{2}$ (6) $3 \times \frac{5}{6} = \frac{x}{6}$ (10) $6 \times \frac{1}{7} = \frac{x}{7}$
(3) $3 \times \frac{1}{4} = \frac{x}{4}$ (7) $4 \times \frac{1}{9} = \frac{x}{9}$ (11) $5 \times \frac{2}{13} = \frac{x}{13}$
(4) $2 \times \frac{3}{4} = \frac{x}{4}$ (8) $2 \times \frac{4}{7} = \frac{x}{7}$ (12) $2 \times \frac{5}{12} = \frac{x}{12}$

In multiplying these fractions by the whole numbers what change was made in the numerator to get the product?

9. Make a rule for multiplying a fraction by a whole number.

10. Compare your rule with the following:

To multiply a fraction by a whole number, place the product of the numerator by the whole number, over the denominator.

11. Apply the rule to these problems, reducing to whole or mixed numbers all the products that are improper fractions:

(1) $3 \times \frac{4}{5} = ?$ (5) $11 \times \frac{3}{11} = ?$ (9) $6 \times \frac{7}{8} = ?$
(2) $5 \times \frac{3}{7} = ?$ (6) $9 \times \frac{1}{12} = ?$ (10) $8 \times \frac{5}{12} = ?$
(3) $7 \times \frac{2}{9} = ?$ (7) $4 \times \frac{5}{6} = ?$ (11) $10 \times \frac{5}{4} = ?$
(4) $9 \times \frac{5}{6} = ?$ (8) $6 \times \frac{4}{9} = ?$ (12) $9 \times \frac{11}{12} = ?$

In such exercises as (7) use cancellation, thus:

$$\overset{2}{\cancel{4}} \times \frac{5}{\underset{3}{\cancel{6}}} = \frac{10}{3} = 3\tfrac{1}{3}$$

12. Solve these problems and make a rule for multiplying a fraction quickly by its denominator:

(1) $6 \times \frac{1}{6} = ?$ (4) $7 \times \frac{5}{7} = ?$ (7) $15 \times \frac{13}{15} = ?$
(2) $9 \times \frac{1}{9} = ?$ (5) $12 \times \frac{5}{12} = ?$ (8) $13 \times \frac{135}{13} = ?$
(3) $5 \times \frac{3}{5} = ?$ (6) $16 \times \frac{12}{16} = ?$ (9) $25 \times \frac{46}{25} = ?$

In such problems use cancellation wherever it shortens the work.

13. Compare your rule with:

To multiply a fraction by its denominator merely omit the denominator.

§57. Multiplying a Mixed Number by a Whole Number.

1. What will be the cost of 12 yd. of muslin at $8\frac{2}{3}$ cents?

If 1 yd. costs $8\frac{2}{3}$¢ what will 12 yards cost? $12\times8\frac{2}{3}$ means $12\times8+12\times\frac{2}{3}=96+8=104$. *Ans.* \$1.04.

In dealing with the expression $12\times8+12\times\frac{2}{3}$, *or similar expressions, which contain both signs,* (×) *and* (+), *the indicated multiplications should be performed first, and then the additions.*

2. $13\times4\frac{5}{9}=?$

$13\times4\ =52$

$13\times\ \frac{5}{9}=\ 7\frac{2}{9}$

$13\times4\frac{5}{9}=59\frac{2}{9}$

Multiply the whole numbers in the usual way.

$13\times\frac{5}{9}=\frac{6}{9}^{5}=7\frac{2}{3}$.

3. Solve the following, employing cancellation whenever it shortens the work:

(1) $5\times\frac{11}{15}=?$ (5) $9\times\frac{17}{36}=?$ (9) $2\times\frac{13}{4}=?$

(2) $3\times\frac{5}{9}=?$ (6) $9\times\frac{8}{9}=?$ (10) $3\times6\frac{7}{8}=?$

(3) $5\times4\frac{3}{4}=?$ (7) $9\times\frac{7}{12}=?$ (11) $9\times\frac{5}{12}=?$

(4) $2\times2\frac{1}{2}=?$ (8) $9\times\frac{4}{5}=?$ (12) $8\times\frac{9}{11}=?$

4. Find the cost of 25 yd. of cloth at $\$\frac{2}{5}$ per yard.

5. A earns $\$1\frac{3}{4}$, and B earns 4 times as much. B earns $\$x$. Find x.

6. A grocer sold $\frac{3}{4}$ bu. of apples to one customer, and 5 times that quantity to another. He sold y bu. to the second customer. Find y. If apples were \$.40 a peck what amount of money did the grocer receive from the first customer? From the second?

7. If a man can cut $\frac{7}{8}$ of a cord of wood in one day, how much can he cut in 6 days?

8. One boy lives $\frac{5}{16}$ mi. from school and another 4 times as far. What is the distance from school to the second boy's home?

9. Make and solve problems based upon the following:
An iron tube weighs $\frac{7}{8}$ lb. per foot.
$\frac{8}{9}$ of the weight of water is oxygen.
The circumference of a bicycle wheel is $6\frac{17}{21}$ feet.
A west wind moved $47\frac{4}{5}$ mi. per hour.

§58. Multiplying a Whole Number by a Fraction.

1. $\frac{5}{7} \times 14 = ?$

$\frac{5}{7} \times 14$ or, $\frac{5}{7}$ of 14 means 5 of the $\frac{1}{7}$ths of 14. $\frac{1}{7}$ of $14 = 2$. $\frac{5}{7}$ of $14 = 5 \times 2 = 10$.

2. Solve these problems, reading the sign ($\times$) "times":

(1) $\frac{3}{5} \times 10 = ?$ (4) $\frac{2}{3} \times 15 = ?$ (7) $\frac{4}{9} \times 45 = ?$
(2) $\frac{4}{5} \times 25 = ?$ (5) $\frac{3}{7} \times 28 = ?$ (8) $\frac{8}{13} \times 26 = ?$
(3) $\frac{7}{8} \times 32 = ?$ (6) $\frac{6}{11} \times 33 = ?$ (9) $\frac{7}{11} \times 47 = ?$

In (9), $\frac{1}{11}$ of $47 = 4\frac{3}{11}$, and $7 \times 4\frac{3}{11} = 28\frac{21}{11} = 28 + 1\frac{10}{11} = 29\frac{10}{11}$. *Ans.* $29\frac{10}{11}$.

3. Solve the following:

(1) $\frac{9}{16} \times 50 = ?$ (3) $\frac{17}{25} \times 105 = ?$ (5) $\frac{5}{23} \times 110 = ?$
(2) $\frac{11}{18} \times 39 = ?$ (4) $\frac{25}{27} \times 220 = ?$ (6) $6\frac{7}{9} \times 28 = ?$

$6\frac{7}{9} \times 28$ means $6 \times 28 + \frac{7}{9} \times 28 = 168 + 21\frac{7}{9} = 189\frac{7}{9}$. See statement in italics under problem 1, §57.

§59. Multiplying a Fraction by a Fraction.

1. How many square feet are there in a rectangle $\frac{1}{2}'$ by $8'$? $\frac{3}{4}'$ by $12'$? $\frac{7}{8}'$ by $16'$? $\frac{9}{10}'$ by $30'$?

2. What is the area of a rectangle $\frac{2}{3}'' \times \frac{4}{5}''$? See Fig. 1.

The sign $\times$, when written between dimensions is read "by."

ABCD is a rectangle $\frac{4}{5}$ in. long and $\frac{2}{3}$ in. wide.

AEGH is a square inch.

What part of a square inch is ABKH? What part of ABKH is ABCD? Point out on the figure the part that represents $\frac{2}{3}$ of $\frac{4}{5}$ of 1 square inch.

Into how many small rectangles, such as *a*, do the dotted lines divide the square inch?

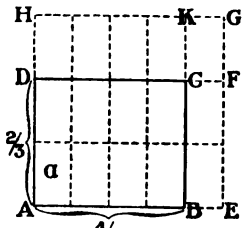


FIGURE 1

What part of the square inch is one of the small rectangles, *a*? How many of these are there in the given rectangle ABCD? The area of ABCD is then what part of a square inch? We may write the two expressions for the area equal, thus:

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

Fractions may be multiplied by merely writing the product of the numerators over the product of the denominators with the dividing line between.

3. Draw a square inch and indicate within it the areas of the following rectangles:

(1) $\frac{2}{3}'' \times \frac{3}{4}'' = ?$ (3) $\frac{1}{9}'' \times \frac{5}{8}'' = ?$ (5) $\frac{7}{8}'' \times \frac{3}{4}'' = ?$

(2) $\frac{3}{4}'' \times \frac{2}{3}'' = ?$ (4) $\frac{3}{8}'' \times \frac{5}{6}'' = ?$ (6) $\frac{3}{4}'' \times \frac{7}{8}'' = ?$

4. Find the same products without the picture, using only the given numbers. Use cancellation.

5. By drawing and dividing a square inch, as in Fig. 1, show that the following are correct:

(1) $\frac{6}{7} \times \frac{5}{11} = \frac{6}{7}$ of $\frac{5}{11}$ (2) $\frac{5}{11} \times \frac{6}{7} = \frac{5}{11}$ of $\frac{6}{7}$

To find $\frac{1}{7}$ of $\frac{5}{11}$, find first $\frac{1}{7}$ of $\frac{1}{11}$.

Then find $5 \times \frac{1}{77} = \frac{5}{77}$.

But $\frac{6}{7}$ of $\frac{5}{11} = 6 \times \frac{1}{7}$ of $\frac{5}{11} = 6 \times \frac{5}{77} = \frac{30}{77}$.

As $\frac{6}{7} \times \frac{5}{11} = \frac{6}{7}$ of $\frac{5}{11}$, $\frac{6}{7} \times \frac{5}{11} = \frac{30}{77}$.

Similar reasoning would show also that $\frac{5}{11} \times \frac{6}{7} = \frac{30}{77}$. Now we recall that the numerator (30) of the product ($\frac{5}{11} \times \frac{6}{7}$) was obtained by multiplying the numerators (6 and 5) of the factors ($\frac{5}{11}$ and $\frac{6}{7}$). How was the denominator (77) of the product obtained?

Always cancel common factors from numerators and denominators, when possible.

6. Solve *as rapidly as you can work accurately:*

(1) $\frac{3}{8}\times\frac{1}{2}=?$ (4) $\frac{2}{5}\times\frac{1}{2}=?$ (7) $\frac{2}{3}\times\frac{5}{8}=?$

(2) $\frac{1}{2}\times\frac{3}{4}=?$ (5) $\frac{3}{7}\times\frac{2}{3}=?$ (8) $\frac{2}{7}\times\frac{3}{5}=?$

(3) $\frac{3}{5}\times\frac{5}{9}=?$ (6) $\frac{6}{7}\times\frac{3}{4}=?$ (9) $\frac{5}{7}\times\frac{2}{9}=?$

The product of any two fractions is a fraction whose numerator is the product of the numerators and whose denominator is the product of the denominators.

7. Answer as rapidly as you can work correctly:

(1) $\frac{3}{4}\times\frac{3}{5}=?$ (4) $\frac{3}{7}\times\frac{4}{5}=?$ (7) $\frac{4}{7}\times\frac{6}{1}=?$

(2) $\frac{4}{5}\times\frac{6}{7}=?$ (5) $\frac{5}{8}\times\frac{3}{7}=?$ (8) $\frac{7}{9}\times\frac{5}{1}=?$

(3) $\frac{2}{3}\times\frac{4}{9}=?$ (6) $\frac{2}{1}\times\frac{3}{5}=?$ (9) $\frac{11}{12}\times\frac{7}{1}=?$

8. Solve as rapidly as you can work correctly:

(1) $\frac{2}{3}\times\frac{6}{7}=?$ (5) $\frac{2}{9}\times\frac{27}{16}=?$ (9) $\frac{6}{7}\times\frac{21}{12}=?$

(2) $\frac{3}{4}\times\frac{7}{9}=?$ (6) $\frac{7}{8}\times\frac{8}{7}=?$ (10) $\frac{10}{11}\times\frac{22}{20}=?$

(3) $\frac{4}{5}\times\frac{25}{16}=?$ (7) $\frac{8}{11}\times\frac{33}{16}=?$ (11) $\frac{11}{12}\times\frac{36}{44}=?$

(4) $\frac{3}{7}\times\frac{14}{27}=?$ (8) $\frac{7}{12}\times\frac{24}{21}=?$ (12) $\frac{13}{16}\times\frac{48}{65}=?$

§60. Multiplying Mixed Numbers.

In §57, prob. 2, $13\times4\frac{5}{9}=13\times4+13\times\frac{5}{9}=52+7\frac{2}{9}=59\frac{2}{9}$.

But $4\frac{5}{9}\times13$ means $4\times13+\frac{5}{9}$ of $13=52+7\frac{2}{9}=59\frac{2}{9}$.

As with whole numbers, so also in multiplying a whole number and a fraction, either factor may be called the multiplier.

To multiply $4\frac{3}{4}$ by $5\frac{1}{2}$ or to find $4\frac{3}{4}\times5\frac{1}{2}$, the numbers may be reduced to fractions and then multiplied, thus:

$$4\tfrac{3}{4}=\tfrac{19}{4};\quad 5\tfrac{1}{2}=\tfrac{11}{2}$$

$$4\tfrac{3}{4}\times5\tfrac{1}{2}=\tfrac{19}{4}\times\tfrac{11}{2}=\tfrac{209}{8}=26\tfrac{1}{8}$$

$$\text{or, } 5\tfrac{1}{2}\times4\tfrac{3}{4}=\tfrac{11}{2}\times\tfrac{19}{4}=\tfrac{209}{8}=26\tfrac{1}{8}$$

Of course, the last step is to simplify the product. Again either factor may be used as the multiplier. Make the following multiplications:

(1) $\frac{4}{5} \times 25$	(5) $3\frac{1}{4} \times 4\frac{2}{3}$	(9) $20\frac{3}{8} \times 6\frac{1}{5}$
(2) $21 \times \frac{6}{7}$	(6) $5\frac{1}{2} \times 3\frac{2}{5}$	(10) $4\frac{3}{7} \times 7\frac{1}{4}$
(3) $24 \times \frac{5}{9}$	(7) $8\frac{3}{4} \times 2\frac{3}{8}$	(11) $16\frac{2}{3} \times 5\frac{3}{16}$
(4) $\frac{7}{9} \times 32$	(8) $3\frac{3}{5} \times 10\frac{2}{3}$	(12) $28\frac{3}{7} \times 32\frac{1}{2}$

DIVIDING FRACTIONS

§61. Dividing a Fraction by a Whole Number.

Do as much of this work orally as you can

1. What is $\frac{1}{8}$ of 40 A.? 40 A. divided by 8 equals what?

2. $\frac{1}{7}$ of 35 lb. = ? 35 lb. $\div 7$ = ?

3. $\frac{1}{10}$ of 80 = ? $80 \div 10$ = ?

4. $\frac{1}{3}$ of 3 fourths = ? 3 fourths $\div 3$ = ?

5. $\frac{1}{5}$ of $\frac{5}{6}$ A. = ? $\frac{5}{6}$ A. $\div 5$ = ?

6. $\frac{1}{3}$ of $\frac{6}{7}$ ft. = ? $\frac{6}{7}$ ft. $\div 3$ = ?

Unit fractions, or fractional units are fractions whose numerators are 1.

7. Compare these unit fractions:

$\frac{1}{4}$ is what part of $\frac{1}{2}$? $\frac{1}{12}$ is what part of $\frac{1}{4}$?

$\frac{1}{6}$ is what part of $\frac{1}{3}$? $\frac{1}{15}$ is what part of $\frac{1}{5}$?

$\frac{1}{8}$ is what part of $\frac{1}{4}$? $\frac{1}{8}$ is what part of $\frac{1}{2}$?

$\frac{1}{10}$ is what part of $\frac{1}{5}$? $\frac{1}{12}$ is what part of $\frac{1}{3}$?

8. Divide $\frac{1}{2}$ by 2 $\frac{1}{3}$ by 2 $\frac{1}{4}$ by 2 $\frac{1}{5}$ by 2

9. Divide $\frac{1}{4}$ by 3 $\frac{1}{5}$ by 3 $\frac{1}{2}$ by 3 $\frac{1}{3}$ by 3

10. How is the size of a unit fraction changed by multiplying its denominator by 2? By 3? By 4? By 10? By 100? By *a*?

11. By what must the first fraction in each of the following pairs be multiplied to get the second?

(1) $\frac{2}{9}$ and $\frac{2}{3}$ (4) $\frac{8}{45}$ and $\frac{8}{9}$ (7) $\frac{37}{200}$ and $\frac{37}{50}$

(2) $\frac{3}{16}$ and $\frac{3}{4}$ (5) $\frac{13}{75}$ and $\frac{13}{15}$ (8) $\frac{a}{12b}$ and $\frac{a}{b}$.

(3) $\frac{6}{14}$ and $\frac{6}{7}$ (6) $\frac{25}{96}$ and $\frac{25}{32}$ (9) $\frac{a}{12b}$ and $\frac{a}{12}$.

12. Divide (1) $\frac{2}{3}$ by 3 (2) $\frac{3}{4}$ by 4 (3) $\frac{6}{7}$ by 2

$\frac{8}{9}$ by 5 $\frac{13}{15}$ by 5 $\frac{25}{96}$ by 3.

13. Make a rule for dividing a fraction by 3 without changing its numerator; by 5; by 28; by a.

Use pencil and paper only when needed

14. What number should stand for x in these exercises?

(1) $\frac{12}{17} \div 3 = \frac{x}{17}$ (4) $\frac{64}{19} \div 16 = \frac{x}{19}$ (7) $\frac{27}{17} \div 9 = \frac{x}{17}$

(2) $\frac{75}{7} \div 15 = \frac{x}{7}$ (5) $\frac{50}{36} \div 25 = \frac{x}{36}$ (8) $\frac{ac}{b} \div a = \frac{x}{b}$

(3) $\frac{196}{15} \div 14 = \frac{x}{15}$ (6) $\frac{144}{13} \div 12 = \frac{x}{13}$ (9) $\frac{mc}{n} \div c = \frac{x}{n}$

15. Solve, by finding what x stands for:

(1) $\frac{5}{6} \div 3 = \frac{5}{x}$ (3) $\frac{15}{16} \div 8 = \frac{15}{x}$ (5) $\frac{24}{13} \div 3 = \frac{24}{x}$

(2) $\frac{7}{8} \div 4 = \frac{7}{x}$ (4) $\frac{13}{3} \div 25 = \frac{13}{x}$ (6) $\frac{19}{12} \div 15 = \frac{19}{x}$

16. In what two ways may a fraction be divided by a whole number? When would you use the one method? The other?

A fraction may be divided by a whole number, (1) *by dividing its numerator by the whole number, without changing its denominator; or* (2) *by multiplying its denominator by the whole number, without changing its numerator.*

The usefulness of cancellation consists in making these two processes balance (undo) each other.

17. Divide $\frac{4}{5}$ lb. of shot equally among 5 boys, and give the weight of one share.

18. $\frac{7}{9}$ lb. of maple sugar is equally divided among 3 children. What part of a pound does each receive?

19. $\frac{4}{5}$ of a piece of work can be done in 8 days. What part of it can be done in 1 day?

20. Make and solve problems based upon the following items, showing work in each case:

$\$\frac{2}{2}\frac{1}{5}$ to buy lace costing \$5 a yard
$\$\frac{1}{2}\frac{1}{0}$ to buy apples costing \$3 per barrel
$\frac{3}{7}$ of an estate divided among 4 heirs.

21. If $\frac{5}{8}$ lb. of coffee is divided into 5 equal portions, what part of a pound is there for each portion?

22. If 3 lb. of butter cost $\$\frac{3}{4}$, 1 lb. costs what part of \$1?

§62. Dividing a Mixed Number by a Whole Number.

1. 7 strips of carpet of equal width cover a room $5\frac{1}{4}$ yd. wide. What is the width, in yards, of one strip?

$5\frac{1}{4}$ yd $= \frac{2}{4}\frac{1}{}$ yard.
$\frac{2}{4}\frac{1}{}$ yd. $\div 7 = \frac{3}{4}$ yd., the width of one strip.
$5\frac{1}{4}$ is what kind of number?
Before dividing what change was made? How was the division performed?

2. A boy earned $\$4\frac{1}{2}$ in 6 days. What part of a dollar did his earnings average per day?

$$4\tfrac{1}{2} = \tfrac{9}{2} \qquad \tfrac{9}{2} \div 6 = \frac{\overset{3}{\cancel{9}}}{2 \times \underset{2}{\cancel{6}}} = \tfrac{3}{4} \qquad \textit{Ans.}\ \tfrac{3}{4}$$

How is the dividing done in this case?

3. $36\frac{2}{3}$ yd. of cloth were made up into 6 ladies' suits. If the suits contain the same number of yards, how many yards are there in each suit?

4. Make a rule for dividing a mixed number by a whole number.

To divide a mixed number by a whole number first change the mixed number to an improper fraction and then proceed as in the division of a fraction by a whole number.

5. A ceiling 16 ft. long contains $233\frac{1}{3}$ square feet. Find the width.

$$233\tfrac{1}{3} = \frac{700}{3}$$

$$\frac{700}{3} \div 16 = \frac{\overset{175}{\cancel{700}}}{3 \times \cancel{16}_{4}} = \frac{175}{12} = 14\tfrac{7}{12}$$

$14\frac{7}{12}$ ft. $= 14'7''$, the width of the room.

6. Solve the following:

(1) $14\frac{3}{4}$ bu. $\div$ 8 = ?
(2) $9\frac{7}{12}$ yd. $\div$ 6 = ?
(3) $16\frac{2}{3}$ hr. $\div$ 12 = ?
(4) $44\frac{9}{16}$ lb. $\div$ 9 = ?
(5) $39\frac{5}{8}$ mi. $\div$ 7 = ?
(6) $\$12\frac{5}{8} \div 5 = ?$
(7) $45\frac{9}{16} \div 5 = ?$
(8) $\frac{224}{5} \div 4 = ?$
(9) $14\frac{7}{21} \div 7 = ?$
(10) $66\frac{2}{3} \div 8 = ?$

7. Find the lengths of the surfaces with areas and widths as follows:

	Areas	Widths	Lengths
(1)	$212\frac{1}{2}$ sq. ft.	17 ft.	——
(2)	$261\frac{1}{2}$ sq. ft.	14 ft.	——
(3)	$406\frac{1}{4}$ sq. ft.	25 ft.	——
(4)	$164\frac{3}{4}$ sq. ft.	13 ft.	——
(5)	$457\frac{5}{7}$ sq. ft.	18 ft.	——
(6)	$202\frac{15}{16}$ sq. ft.	15 ft.	——

§63. Dividing by a Fraction.

1. How many hair ribbons $\frac{2}{3}$ yd. long can be made from 8 yd. of ribbon?

How many thirds are there in 1? In 8?

$$\frac{\frac{24}{3}}{\frac{2}{3}} = \frac{24 \text{ thirds}}{2 \text{ thirds}} = \frac{24}{2} = 12 \qquad \textit{Ans.}\ 12$$

2. How many $\frac{2}{5}$ths are there in $\frac{4}{5}$?

3. How many $\frac{3}{8}$ths are there in $\frac{9}{8}$ths?

4. How many $\frac{2}{3}$ds are there in $\frac{10}{3}$?

5. How many $\frac{2}{3}$ds are there in $\frac{7}{8}$?

The common unit for thirds and eighths is twenty-fourths.

$\frac{2}{3}=\frac{16}{24}$; $\frac{7}{8}=\frac{21}{24}$

$\frac{\frac{7}{8}}{\frac{2}{3}}=\frac{\frac{21}{24}}{\frac{16}{24}}=\frac{21}{16}=1\frac{5}{16}$ *Ans.* $1\frac{5}{16}$

We have seen that when the denominators are different, both fractions may be expressed in a common unit, and the numerators are then to be divided.

6. Answer the following:

(1) $\frac{3}{4}\div\frac{1}{4}=?$
(2) $\frac{7}{8}\div\frac{1}{8}=?$
(3) $\frac{6}{11}\div\frac{2}{11}=?$
(4) $\frac{10}{11}\div\frac{5}{11}=?$
(5) $\frac{11}{13}\div\frac{6}{13}=?$
(6) $\frac{15}{17}\div\frac{13}{17}=?$
(7) $\frac{2}{3}\div\frac{3}{5}=?$
(8) $\frac{5}{7}\div\frac{3}{4}=?$
(9) $\frac{6}{11}\div\frac{5}{12}=?$
(10) $\frac{11}{12}\div\frac{7}{15}=?$
(11) $\frac{12}{17}\div\frac{9}{11}=?$
(12) $\frac{14}{15}\div\frac{12}{17}=?$

Whole numbers may be written in fraction form by writing 1 as their denominators. For example, 3, 8, 24, may be written fractionally, thus: $\frac{3}{1}$, $\frac{8}{1}$, $\frac{24}{1}$.

Whole numbers may then be divided by fractions just as fractions are divided by fractions. Thus, to divide 6 by $\frac{3}{5}$, we write:

$$6\div\frac{3}{5}=\frac{30}{5}\div\frac{3}{5}=\frac{30}{3}=10.$$

7. Make a rule for dividing any number by a fraction.

8. Compare your rule with the following:

To divide any number by a fraction, reduce the dividend and divisor to a common denominator and divide the numerators.

9. Answer the following questions:

(1) $2 \div \frac{3}{4} = ?$ (7) $15 \div \frac{4}{5} = ?$ (13) $16 \div \frac{8}{5} = ?$

(2) $4 \div \frac{2}{5} = ?$ (8) $20 \div \frac{5}{8} = ?$ (14) $32 \div \frac{16}{7} = ?$

(3) $7 \div \frac{3}{8} = ?$ (9) $24 \div \frac{7}{8} = ?$ (15) $64 \div \frac{8}{3} = ?$

(4) $7 \div \frac{3}{7} = ?$ (10) $25 \div \frac{7}{5} = ?$ (16) $81 \div \frac{9}{7} = ?$

(5) $12 \div \frac{4}{5} = ?$ (11) $25 \div \frac{4}{15} = ?$ (17) $72 \div \frac{12}{5} = ?$

(6) $16 \div \frac{5}{9} = ?$ (12) $36 \div \frac{9}{10} = ?$ (18) $96 \div \frac{12}{5} = ?$

§64. Dividing Mixed Numbers.

Mixed numbers as, $6\frac{2}{3}$ and $3\frac{1}{2}$, may also be expressed as fractions and then divided as above. Thus:

$$6\frac{2}{3} = \frac{20}{3}, \text{ and } 3\frac{1}{2} = \frac{7}{2}$$

Both thirds and halves may be expressed as 6ths.

$$\frac{20}{3} = \frac{40}{6} \text{ and } \frac{7}{2} = \frac{21}{6}$$

Thus $6\frac{2}{3} \div 3\frac{1}{2} = \frac{40}{6} \div \frac{21}{6} = 40 \div 21 = \frac{40}{21} = 1\frac{19}{21}$.

Also $3\frac{1}{2} \div 6\frac{2}{3} = \frac{21}{6} \div \frac{40}{6} = 21 \div 40 = \frac{21}{40}$.

1. Make the following divisions:

(1) $7 \div \frac{2}{3}$ (12) $6\frac{1}{4} \div 12\frac{1}{2}$ (23) $16\frac{2}{3} \div 8\frac{1}{3}$

(2) $7\frac{1}{2} \div 5$ (13) $3\frac{3}{5} \div 9$ (24) $33\frac{1}{3} \div 2$

(3) $6\frac{2}{3} \div 3\frac{1}{3}$ (14) $9 \div 2\frac{1}{4}$ (25) $66\frac{2}{3} \div 16\frac{2}{3}$

(4) $8\frac{3}{4} \div 2\frac{3}{8}$ (15) $3\frac{3}{5} \div \frac{2}{5}$ (26) $37\frac{1}{2} \div 18\frac{3}{4}$

(5) $12\frac{1}{2} \div 6\frac{1}{4}$ (16) $\frac{2}{5} \div 3\frac{3}{5}$ (27) $75 \div 18\frac{3}{4}$

(6) $22\frac{1}{2} \div 45$ (17) $6\frac{2}{9} \div \frac{7}{9}$ (28) $87\frac{1}{2} \div 18\frac{3}{4}$

(7) $18\frac{2}{3} \div 6\frac{2}{9}$ (18) $28\frac{7}{8} \div 4\frac{1}{8}$ (29) $62\frac{1}{2} \div 12\frac{1}{2}$

(8) $27\frac{1}{2} \div 5\frac{1}{2}$ (19) $32\frac{5}{6} \div 6\frac{17}{30}$ (30) $56\frac{1}{4} \div 6\frac{1}{4}$

(9) $16\frac{1}{4} \div 32\frac{1}{2}$ (20) $30\frac{1}{4} \div 5\frac{1}{2}$ (31) $37\frac{1}{2} \div 9\frac{3}{8}$

(10) $8\frac{1}{3} \div 33\frac{1}{3}$ (21) $33 \div 16\frac{1}{2}$ (32) $87\frac{1}{2} \div 37\frac{1}{2}$

(11) $13\frac{1}{3} \div 53\frac{1}{3}$ (22) $53\frac{1}{3} \div 6\frac{2}{3}$ (33) $106\frac{1}{4} \div 37\frac{1}{2}$

DECIMAL FRACTIONS

§65. Writing Decimals.

1. What part of a dollar is 50¢? 25¢? 10¢? 1¢?

2. How do you write 1¢ as a fraction of a dollar? How do you write it with the dollar sign ($)?

3. How do you write 10¢ as a fraction of a dollar? With the dollar sign?

4. How do you write 25¢ as a fraction of a dollar? With the dollar sign? 50¢ as a fraction of a dollar? With the dollar sign?

5. Write as hundredths (100ths) of a dollar:

$.01 $.07 $.15 $.25 $.39 $.75 $.87

6. Write the following as 100ths, and as tenths (10ths) of a dollar:

$.10 $.30 $.50 $.70 $1.10 $1.50 $2.60

The period used between the figures in writing dollars and cents is called the DECIMAL POINT.

7. Write each of the following as a common fractional part of a dollar, and also as a decimal part of a dollar:

1¢ 10¢ 25¢ 50¢ 75¢ 120¢ 150¢

Numbers when written with the decimal point are read exactly as when written as fractions.

$\frac{3}{100}$ of a dollar is read *three-hundredths* of a dollar.
.03 of a dollar is read *three-hundredths* of a dollar.
$\frac{15}{100}$ of a dollar is read *fifteen-hundredths* of a dollar.
.15 of a dollar is read *fifteen-hundredths* of a dollar.
$\frac{25}{100}$ of a dollar is read *twenty-five-hundredths* of a dollar.
.25 of a dollar is read *twenty-five-hundredths* of a dollar.
$\frac{5}{10}$ of a dollar is read *five-tenths* of a dollar.
.5 of a dollar is read *five-tenths* of a dollar.
$\frac{7}{10}$ of a dollar is read *seven-tenths* of a dollar.
.7 of a dollar is read *seven-tenths* of a dollar.

8. Read the following numbers as parts of a dollar:

.05	.18	.45	.06	.23	.75	.3	.02
.6	.72	.93	.15	.8	.9	.44	.12

9. Write each of the decimal fractions in problem 8 in the form of a common fraction.

10. Write the decimal equivalents of the following:

$$\frac{11\frac{1}{9}}{100} \quad \frac{22\frac{2}{9}}{100} \quad \frac{33\frac{1}{3}}{100} \quad \frac{44\frac{4}{9}}{100} \quad \frac{55\frac{5}{9}}{100} \quad \frac{66\frac{2}{3}}{100} \quad \frac{77\frac{7}{9}}{100}$$

11. A farmer sold .3 of a flock of 350 sheep. How many did he sell?

12. A piece of cloth 10 yards long was sold to two customers, one buying .5 of the piece, the other buying the rest. How many yards did each of the customers buy?

13. A boy traveled 100 miles, going .2 of the way by boat. How many miles did he go by boat?

14. A man bought .75 of a quantity of grain measuring 400 bushels. How many bushels of grain did he buy?

$4\frac{7}{10}$, or 4.7 is read four *and* seven tenths.

$204\frac{9}{100}$, or 204.09 is read two hundred four *and* nine hundredths.

$234\frac{69}{100}$, or 234.69 is read two hundred thirty-four *and* sixty-nine hundredths.

15. What name is given to any figure in the first place to the right of the decimal point? In the second place to the right?

16. Read the following:

8.3	27.32	30.03	7.07	201.05
14.17	643.24	500.5	93.94	330.33

§66. Problems and Exercises in Decimals.

1. A man bought a ton of hard coal for $6.25, a ton of soft coal for $4.50, and a load of kindling for $1.50. How much did the coal and kindling cost him?

2. James lives .75 mi. north, and Fred, .35 mi. south of the schoolhouse. How far apart do the boys live?

3. Find the cost of 5 barrels of flour at $4.75 a barrel.

4. A bushel of potatoes contains 47.34 lb. water, 10.8 lb. starch, .54 lb. ash, 1.26 lb. proteid, and .06 lb. fat. How much does a bushel of potatoes weigh?

47.34 lb.	Begin on the right and add as with
10.80 "	whole numbers.
.54 "	Place the decimal point (.) in the sum
1.26 "	just beneath the points in the num-
.06 "	bers to be added (addends).

5. One street car line has 4.85 miles of track; another 6.55 miles, and a third 7.5 miles. How many miles of track are there in all three?

6. A farm is divided as follows: cornfield, 66.75 acres; oatfield, 42.6 acres; meadow, 24.5 acres; grove, 20.25 acres; vegetable garden, 4.4 acres; building lot, 1.5 acres. How many acres are there in the farm?

7. Add:

(1)	(2)	(3)	(4)
623.21	4561.38	685.329	365.250
68.75	28.17	27.602	.168
90.66	191.76	7.001	90.027
8.32	9.68	.868	106.995
4.00	3125.88	12.789	78.200
67.75	.98	1.896	9.375

TABLE SHOWING FOOD PARTS IN 10 LB.

Food	Water	Fat	Ash	Proteid	Starch
Oatmeal	.72 lb.	.73 lb,	.19 lb,	1.56 lb.	6.80 lb.
Corn meal	1.29 "	.22 "	.07 "	.89 "	7.53 "
Rye meal	1.27 "	.09 "	.08 "	.71 "	7.85 "
Rice	1.23 "	.03 "	.04 "	.78 "	7.92 "
Macaroni	1.08 "	.16 "	.30 "	1.17 "	7.29 "

8. How much does the water in the 5 foods weigh? The fat? The ash? The proteid? The starch?

9. How much does the oatmeal weigh? The corn meal? The rye meal? The rice? The macaroni?

10. What is the weight of all the parts of problem 8?

11. Ten pounds of apples contain 8.46 pounds of water and 10 pounds of bananas contain 7.53 pounds of water. How much more water is there in 10 pounds of apples than in 10 pounds of bananas?

8.46 lb.
7.53 "

Subtract as with whole numbers. Place the decimal point in the difference just beneath the points in the minuend and subtrahend.

12. Subtract:

(1)	(2)	(3)	(4)	(5)	(6)
86.3	60.9	35.16	30.16	201.06	108.08
7.9	14.7	21.87	3.95	99.98	69.89

(7)	(8)	(9)	(10)	(11)	(12)
476.00	806.30	36.004	1008.902	101.010	600.006
96.99	89.92	9.735	86.967	90.909	589.099

GENERAL REVIEW PROBLEMS

§67. Industrial Applications for Review.

1. In 1909 there were in Indiana 67 automobile factories, 134 canning factories, and 201 furniture factories. The number of automobile factories is what part of the number of canning factories? Of furniture factories?

2. A furniture factory employed on the average $8\frac{2}{7}$ wage earners. How many wage earners were employed by all the 201 furniture factories?

3. If each wage earner in a furniture factory supported 4 persons, how many persons in all were supported from the earnings of laborers in Indiana furniture factories?

4. There were in 1909 in Indiana 39 farm implement factories employing 4758 wage earners. Find the average number of wage earners in an implement factory.

5. If 5 persons were supported on the wages of each of these 4758 wage earners, how many persons were supported by these wage earners? What was the average number of persons supported on the wages earned in one of these factories?

6. There were in 1909 in Indiana about 756 factories making bread and bakery goods, and employing on the average $3\frac{2}{3}$ wage earners per factory. How many persons were employed in this industry?

7. In the same year 221 carriage and wagon factories employed about 8840 wage earners. What was the average number of wage earners to a factory?

8. The value of the year's product of the 221 carriage and wagon factories was about $21,658,000. What was the average value of the year's product per factory?

9. These 221 factories added to the value of the raw materials by manufacture about $9,193,600. Find the average value added by manufacture per factory.

10. The raw material used by the 67 automobile factories of Indiana was worth $14,994,869 and the manufactured product was worth $23,764,070. Find the value added by manufacture. Find to the nearest dollar the average value added per factory.

11. Calling the value of the raw material of the last problem $15,000,000 and the value added by manufacture $9,000,000, what part of the value of the raw material was added to it by manufacturing it into automobiles?

12. In an Indiana town there are 32 factories with a combined payroll of $25,000 a week. What is the average weekly payroll of a factory?

13. The payroll of a factory was $781.30 a week. What was the average pay per week for one of the 65 persons whose names were on the payroll?

14. Two men bought a steam laundry for $7500, one paying $3500, the other $4000. They agreed that each should receive a salary of $1200 a year, and that the rest of the profits should be divided between them proportionally to their investments. The rest of the profits the first year was $375. How much of this should each receive?

15. The running expenses of the laundry for the first year amounted to $7600. How much must the laundry have earned this year to pay the two $1200 salaries, the running expenses, and the $375 additional?

16. The second year the earnings exceeded the salaries and the running expenses by $600. To what part of the $7500 investment is this profit equal?

17. In 1909 there were employed in the 311 brick and tile factories of Indiana 273 salaried officers, who received as salaries for the year a total of $251,220. Find the average salary of these officials per year; per month.

18. In the same factories 3788 wage earners received a total of $1,837,180 in wages for the year. What was the average wage of this class of workers per year? Per month?

19. The amount of capital invested in the 311 brick and tile factories was $7,454,981. Find the average amount of money invested in one factory.

20. If the total earnings for the year of the 311 brick and tile factories were $1,343,520, what was the average earning for the year of one factory?

21. The 51 salaried officers of the Indiana tool and cutlery factories were paid $57,000 in salaries in 1904 and $62,000 in 1909. What change occurred during the 5 years in the average salary in these factories?

22. The 639 wage earners of these factories were paid a total of $286,059 in wages in 1904, and the 572 wage earners of 1909 were paid $305,019. Find the average wage for each year and the change of average from 1904 to 1909.

23. In an Indiana town having a population of 7744 there are 484 persons engaged in the retail business. What part of the population is engaged in this business?

24. If each person who is engaged in the retail business in this town supports 6 persons, what part of the total population is supported by the retail business?

25. The retail merchants of this town occupy 124 store-rooms having a total rental value of $60,760 a year. What is the average rental value of one of these retail store-rooms?

26. A certain factory has built 30,000 locomotives during the last 24 years. Find the average number per year.

§68. Applications to Farm Life.

1. How many cubic yards of loam are required to cover a lawn 120 ft. by 150 ft., 2 in. deep?

2. If it takes an hour and a half to mow a lawn and it must be mowed 20 times during the summer, how many hours of time are required? How much is this time worth at 20¢ per hour?

3. How far would one walk in mowing a lawn 120 ft. by 150 ft., with a lawn mower that cuts a swath 18 in. wide?

4. How many trees will be required to set 10 rows 40 rods long,—trees $8\frac{1}{4}$ feet apart in the row?

5. How many sq. ft. of screen wire are required to make a screen for a window 36 in. by 66 in.? What would the wire cost at 3¢ per sq. ft.?

6. What is the cost of screen wire at 3¢ per sq. ft. for a door 3 ft. and 4 in. by 7 ft. 6 in.?

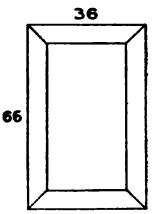


FIGURE 1

7. Screen window frames are usually made out of lumber 1 in. thick and 3 in. wide. How much lumber is required to make a full sized screen frame for a window 36 in. by 66 in.? What is the lumber worth at $36 per thousand feet?*

8. Screen door frames are usually made of lumber $1\frac{1}{2}$ in. thick and $3\frac{1}{4}$ in. wide. How much lumber is required to make a screen door frame 3 ft. 4 in. by 7 ft. 6 in.? What is the cost of the lumber at $36 per thousand feet?

---

*Lumber is sold by the board foot. A board foot is a board 1 foot long, 1 foot wide, and 1 inch or less thick. Thicknesses between 1 and 2 inches are rated as though 2 inches thick.

9. What is the cost of material to make screen frames for a house having 4 doors and 20 windows? (Use size and prices given above.)

10. Dishes must be washed in the average home three times per day, 365 days in the year. If 40 minutes are required after each meal to wash the dishes, how many hours are required in a year?

11. If by the addition of such conveniences as a sink, hot water, etc., 15 minutes could be saved each time the dishes were washed, how much time would be saved in a year?

12. If 12 hours are spent each week in washing and ironing, how many hours of manual work would be saved in a year by having the washing and ironing done by a machine which saves 11 of the 12 hours formerly required per week?

13. A person needs 24 cubic feet of air per hour. How much does he need in a bed room for 8 hours? How much air will 2 people need while sleeping in a room 8 hours?

14. If one child needs 24 cubic feet of air in an hour, how much will 30 children need in 6 hours? How much will the children in your schoolroom need?

15. How many cu. ft. of air are there in a bed room 12 ft. long, 10 ft. wide, and 8 ft., 6 in. high?

16. A cow requires about 120 cu. ft. of air per hour. How much does she require per day?

17. If a cow requires 120 cu. ft. of air per hour, how many cu. ft. of air do 17 cows need in 24 hours?

18. A barn 40 ft. by 60 ft. will shelter 40 cows. If the ceiling is 12 ft. high and each cow requires 120 cu. ft. of air per hour, how long will it take the 40 cows to breathe all of the air in the barn once?

19. If strawberries yield 3600 quarts per acre, how many quarts should 10 rows, 60 ft. long, and 3 ft. apart, yield?

20. If one plants 10 rows of strawberries 60 ft. long, with 2 plants per hill 3 ft. apart in the row, how many plants are required?

21. How much ground is occupied by 10 60-foot rows of strawberries, rows 6 ft. apart?

22. If raspberries yield 3000 qt. per acre, how many quarts should one get from a patch 4 rods by 8 rods?

23. If apple trees are set 25 feet apart each way, how many trees can be set in one acre?

24. If trees are set 25 ft. apart each way and each tree produces 5 bu. of apples, how many bushels per acre will be produced? How much are the apples worth at present price of apples?

25. Each of 120 apple trees produces 4.5 bu. of apples. How much are the apples worth at present prices?

26. If 40 quarts of fresh berries worth 12¢ per quart are required to make 20 quarts of canned berries, how much do the berries cost for one quart of canned berries?

27. If it takes three hours to can the 20 quarts of berries, what is the cost of the labor per quart if time is worth 20¢ per hour?

28. If 20 pounds of sugar cost \$1, and $\frac{7}{8}$ pound of sugar is required for 1 quart of canned berries, how much does the sugar cost for each quart?

29. What is the total cost of 20 quarts of canned berries? (See three preceding problems.)

PART II

GENERAL REVIEW

FUNDAMENTAL OPERATIONS

§69. Problems of the Grocery Clerk.

A grocery boy made the following transactions (sales and purchases), including the filling of the orders, and returned the correct change all in one hour. What change did he return to each customer?

1. The first customer bought:

Celery	$0.05	Potatoes	$0.15
Eggs	.22	Sugar, 10 lb. @	.05½

The sign @ is often used instead of the word "at."

and paid the clerk with a $1.00 bill.

2. The second customer bought:

Salt	$0.10	Apples	$0.05
Flour	.55	Breakfast food	.13
Yeast	.02	Rice, 3 lb. @	.08⅓
Milk	.07		

and paid with $1.50.

3. The third customer bought:

Tomatoes	$0.20	Beans, 4 qt. @	$0.07
Soap	.25	Tea, 2 lb. @	.60
Butter, 3 lb. @	.27	Cauliflower 3 heads @	.18
Coffee, 3 lb. @	33⅓		

and paid with a $5.00 bill.

4. The fourth customer bought:

Eggs	$0.22	Lampwicks	$0 10
Bacon	.18	Cheese	.16
Peas	.13		

and paid with a $5.00 bill.

5. The fifth customer bought:

Cornmeal	$0.25	Melon	$0.25
Sweet potatoes	.27	Pineapple	.20
Candy	.10	Lard, 3 lb. @	.13⅓
Grapes	.18		

and paid with a $2.00 bill.

6. The sixth customer was a farmer, who sold to the grocer:

Eggs, 6 doz. @	18¢	Tomatoes, 2 bu. @	75¢
Butter, 15 lb. @	26¢	Sugar corn, 6 doz. @	10¢
Potatoes, 5 bu. @	35¢		

and bought of the grocer:

Sugar, 20 lb. @	5½¢	Tea, 2 lb. @	55¢
Coffee, 3 lb. @	25¢	Flour, 1 sack,	$1.15
Cheese, 2 lb. @	18¢	Rice, 6 lb. @	8⅓¢

How much money should the grocer pay the farmer to balance the account?

7. A grocer's daily sales for 4 weeks are given below. Without rewriting the numbers find the total sales (*a*) for each week; (*b*) for the 4 weeks; (*c*) the average daily sales for the month.

Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Weekly Totals
$28.75	$37.25	$35.18	$68.12	$20.13	$86.58	.....
18.62	9.68	21.83	40.28	37.60	75.76	.....
30.18	29.95	23.61	9.61	10.84	68.94	.....
19.27	39.13	28.16	38.19	5.63	98.56	.....

§70. Capacity and Volume.

How long is a cubic inch? How wide? How high? What is a cubic foot? (Fig. 1) A cubic yard?

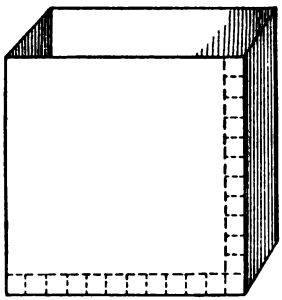


FIGURE 1

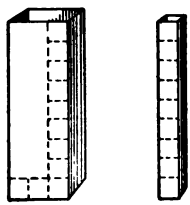


FIGURE 2

A tin box 3 in. square on the bottom and 9 in. high (Fig. 2) was used by a school as a rain-gauge. A second tin box 1 in. square on the bottom and 9 in. high was used to measure the depth of water in the rain-gauge. After the water was poured from the rain-gauge into the small box, the depth of water in the small box was measured with a thin stick and a foot-rule.

1. The rain-gauge was placed one evening where the rain could fall freely into it. During the night it rained and the next morning the water was poured from the gauge into the small box. It completely filled the small box. How deep did the water fill the gauge? How many cubic inches of water were caught in the gauge?

One cu. in. of water for every sq. in. of surface is what is meant by 1 in. of rainfall.

2. How many cu. in. of water are there in a layer 1 in. deep in the large box (Fig. 3)? 2 in. deep? 5 in. deep?

3. How many cu. in. of water fall on 1 sq. ft. of ground during a rainfall of 1 in. (Fig. 3)? Of 6 in.?

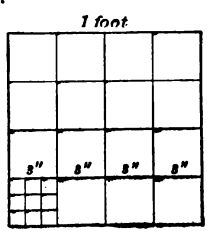


FIGURE 3
BOTTOM OF BOX

The number of cubic units (cu. in., cu. ft., cu. yd., etc.), a vessel holds, when full, is called its "capacity."

4. What is the capacity of a square-cornered box $3'' \times 3'' \times 9''$ (Fig. 2, page 109)?

5. What was the depth of rainfall during a shower if the gauge (Fig. 2, page 109) caught enough water to fill the small box half full? To a depth of 3 in.? Of 6 in.? Of 1 in.? Of 2 in.? Of 7 inches?

6. How many cu. in. of water fell on 1 sq. ft. of the ground during a shower giving 1 in. of rainfall? 2 in.? $\frac{1}{2}$ inch?

7. During a rainfall of 1 in. how many sq. ft. of ground received enough water to make 1 cu. ft.? 2 cu. ft.? 12 cubic feet?

8. During June, 1902, the rainfall in the vicinity of Chicago was $6\frac{1}{2}$ inches. During this month how many cu. in. of water fell on 1 sq. in. of ground? On 1 sq. ft.? On 12 sq. ft.? On 1 sq. yd.? On 30 sq. yd.? On $\frac{1}{4}$ sq. yd.? On $30\frac{1}{4}$ square yards?

9. Just before a shower set an uncovered bucket where the rain may fall freely into it. After the shower measure the depth of the water in the bucket to find the depth of rainfall. Find how many cu. in., or cu. ft. of water fell on each sq. ft.? Each sq. yd.? Each sq. rd. of the ground?

The bucket used must have the same size at the top and bottom with straight sides. Why?

10. A gallon is 231 cubic inches. During a rainfall of $\frac{1}{2}$ inch how many gallons of water fell on a lawn 44 ft. by 56 feet?

11. During a rainfall of $\frac{1}{2}$ inch how many 40 gallon barrels of water fell on a plot of ground 10 rods square?

12. Counting a barrel to hold $4\frac{1}{5}$ cu. ft. how many barrels of water fell on a garden 48 ft. by 140 ft. during a rainfall of $1\frac{1}{2}$ inches?

§71. Bills and Accounts.

1. Explain the meaning and how the totals are found in the following copies of bills:

CHICAGO, June 1, 1908

Albert Miller

Bought of JOHN SAMPLE, BOOKSELLER

3	Books	@ .35	1	05
2	Tablets	@ .10		20
4	Pencils	@ .05		20
6	Drawing Books	@ .10		60
8	Rulers	@ .02		16
		Total	2	21

CHICAGO, Dec. 23, 1907

Miss Edna Lane

Bought of PRINCE'S TOY STORE

2	Dolls	@ .69	1	38
3	Sets Toy Dishes	@ .87	2	61
1	Music Box		1	35
$1\frac{1}{2}$	Doz. Christmas Cards	@ .54		81
1	Set Building Blocks			75
		Total	6	90
	Received Payment			
	Prince's Toy Store			

2. Copy, find amounts due on the following accounts, and receipt:

J. MANNING,

In account with D. L. PARMER, Debtor.

19____

Jan.	2	To 75 lb. Rice	@	$ .05		
"	2	" 330 lb. Sugar	@	.05½		
"	7	" 50 lb. Java Coffee	@	.32		
"	7	" 45 lb. Tea	@	.60		
		Amount due - - - - - - - - - - - -				

JAMES GILMAN,

In account with GEO. JOHNSON, Debtor.

19____

May	6	To 5 Days' Work	@	$2.50		
"	13	" 12 lb. Nails	@	.03		
"	14	" 7 Panes of Glass	@	.40		
June	11	" 10 gal. Paint	@	1.00		
"	18	" Job Work on House			275	00
		Amount due - - - - - - - - - - - -				

Make bills for the following:

3. Feb. 4, Mrs. J. K. Brown bought of White, Jones & Co.

10 lb. Sugar	@	$0.05½
20 " Flour	@	.04
2 " Tea	@	.60

4. Jan. 11, L. B. Hall bought of Smith Bros.:

20	pr. Boys' Boots	@	$1.75
15	" Slippers	@	1.50
25	" Ladies' Shoes	@	2.57
25	" Rubbers	@	.50

5. June 4, Messrs. Black & Co. bought of Marshall Field & Co.:

3	bolts Velvet	@	$100.00
3	" Muslin	@	37.00
2	" Calico	@	10.00

6. May 5, L. French bought of Browne, Steele & Co.:

5	yd. Silk	@	$1.75
3	" Ribbon	@	.75
7	" Velvet	@	4.50
4	papers Pins	@	.07
2	" Needles	@	.08

7. Sept. 1, J. C. Hill bought of Birch & Son:

8	Histories	@	$1.25
15	Spellers	@	.25
9	Readers	@	.40
12	Grammars	@	.60
20	Arithmetics	@	.55

8. Oct. 21, O. F. Horn bought of Taylor & Co.:

5	Coal Stoves	@	$20.00
10	Oil Stoves	@	8.00
25	lb. Nails	@	.04

Make out bills, supplying names, find the amounts, and receipt:

9.

9	lb. Ham	@	$0.16
8	" Veal	@	.12
12	" Mutton	@	.12½
16	" Beef	@	.14
4	" Pork	@	.08⅓

10.	13 lb. Dried Beef	@	\$0.12
	25 " Codfish	@	.11
	16 " Mackerel	@	.09¼
	18 " Bacon	@	.12½
11.	30 yd. Cassimere	@	\$1.75
	70 Spools Thread	@	.03½
	64 yd. Sheeting	@	.12
	45 " Calico	@	.04
12.	112 bbl. Flour	@	\$6.20
	108 T. Hay	@	14.00
	250 bu. Wheat	@	.92
	130 " Corn	@	.32½
	75 " Barley	@	.85
13.	12 doz. Eggs	@	\$0.22½
	12 lb. Rice	@	.07
	48 " Coffee	@	.33⅓
	15 " Butter	@	.27
	32 " Cheese	@	.14

14. Pupils may take market reports from newspapers and make problems like the above, using names of local merchants or of classmates.

§72. Weekly Account for Work.

	Mon.	Tu.	Wed.	Th.	Fri.	Sat.	Wages	
	hours	hours	hours	hours	hours	hours	$	¢
Adams.....	7½	8	8	8½	9	8¾		
Benson.....	8	8½	8¼	8¾	8½	9		
Boyd.....	0	4	6	6	6½	7		
Claussen....	8	8½	8½	8½	9	8½		
Denning....	8½	9	8½	8¼	9	9		
Doan.....	8	8	8	8	8	8		

1. Find the number of hours each man worked.

2. Find the number of hours all the men worked.

3. At 30¢ an hour, find the daily wage of each man.

4. Find the amount due each man at the close of the week.

5. Find the amount due to the six men at the close of the week.

6. Which man worked the greatest number of hours? The smallest number?

7. Counting 9 hr. to the day, what is the full amount any one man would earn in a week at 30¢ an hour?

8. Counting 8 hr. to the day, how much did each man work over time?

9. How many extra hours did four of the men work, counting 8 hours to the day?

10. If double wages were paid for "overtime," how much did each man earn during the week? How much did all earn?

11. How many hours did the third man lose counting 8 hours to a day?

§73. Problems Partly Original.

In these problems, 30¢ per hour or any other hourly wage may be used.

1. Adams paid $4 on a doctor's bill, and put aside $2 for rent.

2. Benson has a wife and two children. Average the amount received for each one in the family.

3. How much does Boyd's loss of time reduce his wages for the week?

4. Claussen paid $10 for clothing and $3.50 for the week's board.

5. Denning saves $1 a week to pay the premium (yearly charge) on a life insurance policy; pays $3.50 a week for board, and 35 cents for laundry.

§74. Coal Mining.

1. In 1909 there were in Indiana 86 coal mines operated by individuals, 47 operated by firms, and 90 operated by corporations. How many coal mines in all?

2. The first of these kinds of mine employed 685 men, the second 337, and the third 21,335. How many men in all were employed in coal mining?

3. If each of these men supported a wife and four children, how many persons in all got their living from coal mining in Indiana in 1909?

4. The total value of the coal mined that year was, for the first kind of mine, $368,784; for the second, $247,352; and for the third, $14,402,017. What was the total value of the coal mined by all?

5. How many more mines were there of the first two classes together, than of the third class? (See prob. 1).

6. From problem 2 find how many more men were employed with the third kind of mine than with the other two together.

7. From problem 4 find how much more the coal mined by the third kind of mine was worth than that of the other two together.

8. If the 86 mines operated by individuals had employed 688 men, what would have been the average number of men employed per mine? Check by multiplying.

9. If the 47 mines operated by firms had employed 329 men, what would have been the average number of men per mine? Check by multiplying.

10. If the 90 mines run by corporations had employed 21,330 men, what would have been the average number of men employed per mine? Check by multiplying.

11. If the total value of the coal mined by the 86 mines of the first kind had been $368,768, what would have been the average value of the coal per mine? Check.

12. If the value of the coal mined by the 47 mines of the second kind had been $247,361, what would have been the average value of the coal per mine for this kind of mine?

13. If the total value of the coal of the 90 mines of the third kind had been $14,401,980, what would have been the average value of the coal per mine for this kind of mine?

14. If the value of the coal of the first kind of mine had been $524 for each of the 685 men employed, how much would it have been?

15. If the value of the coal of the second kind of mine had been $734 for each of the 337 men employed, what would have been the total value of the product?

FRACTIONS

§75. Simplifying and Reducing Fractions.

1. Reduce to lowest terms, doing all you can orally:

(1) $\frac{3}{6}$ (7) $\frac{30}{45}$ (13) $\frac{72}{81}$

(2) $\frac{4}{12}$ (8) $\frac{35}{40}$ (14) $\frac{99}{121}$

(3) $\frac{15}{25}$ (9) $\frac{48}{56}$ (15) $\frac{84}{132}$

(4) $\frac{12}{60}$ (10) $\frac{49}{63}$ (16) $\frac{120}{144}$

(5) $\frac{21}{56}$ (11) $\frac{54}{99}$ (17) $\frac{420}{660}$

(6) $\frac{40}{56}$ (12) $\frac{81}{99}$ (18) $\frac{396}{484}$

2. Reduce to whole or mixed numbers, working all you can orally:

(1) $\frac{10}{5}$ (6) $\frac{18}{5}$ (11) $\frac{75}{4}$
(2) $\frac{3}{2}$ (7) $\frac{26}{12}$ (12) $\frac{50}{40}$
(3) $\frac{6}{4}$ (8) $\frac{36}{9}$ (13) $\frac{75}{15}$
(4) $\frac{8}{3}$ (9) $\frac{49}{8}$ (14) $\frac{250}{3}$
(5) $\frac{15}{4}$ (10) $\frac{33}{2}$ (15) $\frac{200}{13}$

3. Reduce to improper fractions, all you can orally:

(1) $2\frac{1}{2}$ (7) $33\frac{1}{3}$ (13) $15\frac{3}{4}$
(2) $3\frac{1}{3}$ (8) $16\frac{2}{3}$ (14) $41\frac{7}{8}$
(3) $5\frac{1}{2}$ (9) $13\frac{5}{6}$ (15) $35\frac{5}{7}$
(4) $16\frac{1}{2}$ (10) $20\frac{2}{3}$ (16) $28\frac{5}{8}$
(5) $12\frac{1}{2}$ (11) $37\frac{1}{2}$ (17) $65\frac{5}{11}$
(6) $18\frac{3}{4}$ (12) $66\frac{2}{3}$ (18) $126\frac{4}{5}$

§76. Addition and Subtraction of Fractions.

See §§51-55.

1. Find the following sums and differences, working all you can orally:

(1) $\frac{3}{5}+\frac{3}{10}$ (7) $\frac{6}{7}-\frac{2}{3}$ (13) $\frac{16}{9}-\frac{7}{18}$
(2) $\frac{3}{4}-\frac{2}{5}$ (8) $\frac{6}{7}+\frac{4}{5}$ (14) $\frac{12}{5}+\frac{3}{10}$
(3) $\frac{5}{6}-\frac{3}{7}$ (9) $\frac{7}{9}-\frac{3}{5}$ (15) $\frac{17}{3}-\frac{8}{11}$
(4) $\frac{5}{6}+\frac{1}{3}$ (10) $\frac{10}{11}-\frac{3}{5}$ (16) $\frac{18}{5}-\frac{12}{5}$
(5) $\frac{7}{8}+\frac{1}{16}$ (11) $\frac{8}{9}+\frac{2}{7}$ (17) $3\frac{3}{5}-2\frac{2}{5}$
(6) $\frac{3}{8}+\frac{5}{9}$ (12) $\frac{11}{16}-\frac{3}{32}$ (18) $8\frac{3}{4}+2\frac{1}{4}$

To add mixed numbers as $16\frac{1}{2}$ and $8\frac{2}{3}$, add first the whole numbers, $(16+8=24)$, then the fractions $(\frac{1}{2}+\frac{2}{3}=1\frac{1}{6})$, then add the two sums, $(24+1\frac{1}{6}=25\frac{1}{6})$.

To subtract mixed numbers as $8\frac{2}{3}$ from $16\frac{1}{2}$, proceed thus:

Minuend	$16\frac{1}{2}$	$=15\frac{3}{2}=15\frac{9}{6}$
Subtrahend	$8\frac{2}{3}=$	$8\frac{2}{3}=\underline{8\frac{4}{6}}$
		$7\frac{5}{6}$

That is, subtract first the fractions, combining one of the whole units with the minuend-fraction, if needed, then subtract the whole numbers.

2. Find the following sums and differences:

(1) $3\frac{1}{3}-1\frac{1}{3}$ (7) $8\frac{3}{4}-2\frac{1}{4}$ (13) $24\frac{3}{8}-10\frac{7}{8}$

(2) $3\frac{2}{3}-2\frac{1}{3}$ (8) $3\frac{2}{3}+5\frac{2}{3}$ (14) $24\frac{3}{8}+12\frac{7}{8}$

(3) $6\frac{2}{5}-4\frac{1}{5}$ (9) $8\frac{3}{8}+6\frac{5}{8}$ (15) $33\frac{1}{3}+66\frac{2}{3}$

(4) $6\frac{4}{5}-3\frac{3}{5}$ (10) $12\frac{1}{2}-8\frac{3}{4}$ (16) $66\frac{2}{3}-12\frac{1}{2}$

(5) $8\frac{5}{6}-6\frac{1}{6}$ (11) $16\frac{1}{2}-5\frac{3}{5}$ (17) $37\frac{1}{2}-16\frac{2}{3}$

(6) $7\frac{3}{5}-5\frac{4}{5}$ (12) $28\frac{7}{8}-18\frac{5}{8}$ (18) $3\frac{1}{5}+1\frac{2}{3}+6\frac{9}{12}$

§77. Multiplication of Fractions.

1. Give the following products, working as many as you can orally: See §§56-59.

(1) $\frac{2}{5}\times\frac{3}{7}$ (7) $\frac{8}{9}\times\frac{9}{16}$ (13) $\frac{5}{8}\times\frac{12}{20}$

(2) $\frac{3}{4}\times\frac{5}{7}$ (8) $\frac{9}{10}\times\frac{30}{45}$ (14) $\frac{11}{13}\times\frac{39}{24}$

(3) $\frac{4}{5}\times\frac{5}{7}$ (9) $\frac{12}{16}\times\frac{8}{9}$ (15) $\frac{15}{26}\times\frac{39}{60}$

(4) $\frac{4}{7}\times\frac{3}{8}$ (10) $\frac{10}{11}\times\frac{33}{40}$ (16) $\frac{12}{15}\times\frac{34}{30}$

(5) $\frac{2}{3}\times\frac{5}{6}$ (11) $\frac{7}{11}\times\frac{44}{49}$ (17) $\frac{16}{27}\times\frac{80}{49}$

(6) $\frac{6}{7}\times\frac{5}{9}$ (12) $\frac{15}{22}\times\frac{33}{35}$ (18) $\frac{25}{36}\times\frac{75}{108}$

2. Give the following products, working as many as possible orally:

(1) $6\times\frac{3}{25}$ (7) $\frac{7}{9}\times12$ (13) $\frac{4}{9}\times54$

(2) $8\times\frac{3}{16}$ (8) $\frac{3}{11}\times24$ (14) $\frac{5}{14}\times70$

(3) $\frac{4}{5}\times10$ (9) $\frac{9}{10}\times20$ (15) $\frac{9}{17}\times68$

(4) $\frac{3}{4}\times12$ (10) $\frac{12}{13}\times52$ (16) $\frac{35}{48}\times240$

(5) $\frac{5}{8}\times24$ (11) $\frac{12}{17}\times51$ (17) $63\times\frac{13}{21}$

(6) $12\times\frac{3}{4}$ (12) $18\times\frac{5}{24}$ (18) $75\times\frac{31}{25}$

§78. Multiplying by a Mixed Number.

1. Give the following products: See §60.

(1) $18 \times 2\frac{2}{9}$ (7) $24 \times 4\frac{1}{6}$ (13) $84 \times 7\frac{1}{12}$

(2) $3\frac{3}{4} \times 24$ (8) $7\frac{2}{7} \times 210$ (14) $16\frac{2}{3} \times 51$

(3) $5\frac{2}{3} \times 18$ (9) $12\frac{3}{4} \times 20$ (15) $30\frac{5}{6} \times 24$

(4) $8\frac{1}{3} \times 12$ (10) $37\frac{1}{2} \times 40$ (16) $20\frac{7}{8} \times 32$

(5) $14 \times 5\frac{5}{7}$ (11) $36 \times 10\frac{5}{9}$ (17) $35 \times 10\frac{5}{7}$

(6) $22 \times 8\frac{4}{11}$ (12) $72 \times 12\frac{8}{9}$ (18) $12\frac{1}{2} \times 60$

2. Solve the following:

(1) $37\frac{1}{2} \times 58\frac{3}{4}$ (4) $23\frac{1}{3} \times 44\frac{4}{7}$

(2) $33\frac{1}{3} \times 48\frac{3}{4}$ (5) $62\frac{2}{5} \times 176\frac{5}{6}$

(3) $66\frac{2}{3} \times 87\frac{1}{2}$ (6) $165\frac{2}{5} \times 328\frac{3}{8}$

§79. Problems and Material for Original Problems.

1. Find the cost of $2\frac{1}{3}$ yd. of cloth at \$3 per yard.

2. A man had \$75; he spent $\frac{2}{3}$ of it for a bicycle, and $\frac{4}{5}$ of the remainder for clothing. Find the amount spent, and the amount he had left.

3. A gallon contains 231 cubic inches. How many cubic inches in $\frac{2}{3}$ gal.? In $\frac{5}{6}$ gallon?

4. A cubic foot of granite weighs 170 pounds. How many pounds do $5\frac{3}{4}$ cubic feet of granite weigh?

5. $\frac{7}{18}$ of a plot of ground containing 918 sq. rd. was fenced for a garden. The garden contained how many square rods?

6. Find the cost of $15\frac{3}{4}$ bu. of wheat at $88\frac{1}{2}$¢ per bushel.

7. A man worked $5\frac{1}{2}$ days at \$$1\frac{3}{4}$ per day. How much did he earn in that time?

8. How many square yards are there in $18\frac{7}{8}$ square rods?

9. Make and solve problems, based upon facts obtained by yourself, or based upon the data given in the list just below:

An avoirdupois pound of pure gold is worth about $\$348\frac{2}{5}$. A pupil in your class weighs 85 pounds.

A tank contains 126 gal. linseed oil. Price per gallon, $0.62.

Corn was quoted and sold at $50\frac{7}{8}¢$ a bushel.

Wheat was quoted and sold at $77\frac{5}{8}¢$ a bushel.

Oats were quoted and sold at $32\frac{5}{8}¢$ a bushel.

Beans were quoted and sold at $2.10 a bushel.

Apples were quoted and sold at $\$4\frac{1}{2}$ a barrel.

Potatoes were quoted and sold at $68\frac{3}{4}¢$ a bushel.

Calves were quoted and sold at $\$5\frac{3}{5}$ a hundredweight.

§80. Division of Fractions.

Look again at §§61–64, Part I.

Give the following quotients, working all you can orally:

(1) $1\frac{2}{3} \div 6$ (11) $6\frac{2}{3} \div 4$ (21) $24 \div \frac{8}{5}$

(2) $\frac{24}{5} \div 8$ (12) $12\frac{1}{2} \div 4$ (22) $25 \div \frac{9}{4}$

(3) $\frac{5}{6} \div 7$ (13) $13\frac{2}{3} \div 14$ (23) $25 \div 2\frac{1}{4}$

(4) $\frac{7}{8} \div 10$ (14) $16\frac{1}{2} \div 11$ (24) $\frac{4}{5} \div \frac{2}{3}$

(5) $\frac{6}{7} \div 9$ (15) $66\frac{2}{3} \div 10$ (25) $\frac{8}{33} \div \frac{4}{11}$

(6) $\frac{32}{45} \div 14$ (16) $8 \div \frac{4}{7}$ (26) $\frac{9}{17} \div \frac{5}{7}$

(7) $\frac{8}{5} \div 4$ (17) $10 \div \frac{4}{5}$ (27) $\frac{12}{5} \div \frac{2}{7}$

(8) $\frac{18}{7} \div 6$ (18) $14 \div \frac{4}{5}$ (28) $\frac{7}{8} \div \frac{7}{9}$

(9) $2\frac{4}{7} \div 6$ (19) $21 \div \frac{7}{8}$ (29) $\frac{11}{12} \div \frac{6}{11}$

(10) $1\frac{3}{5} \div 2$ (20) $36 \div \frac{9}{11}$ (30) $\frac{16}{3} \div \frac{7}{2}$

§81. General Exercises for Practice.

1. Find the following sums and differences as rapidly as you can work correctly:

(1) $\frac{7}{8}+\frac{5}{11}=?$
(2) $\frac{12}{13}-\frac{7}{8}=?$
(3) $\frac{11}{15}-\frac{5}{12}=?$
(4) $\frac{16}{27}+\frac{15}{32}=?$
(5) $\frac{25}{32}-\frac{16}{25}=?$
(6) $8\frac{8}{9}+4\frac{2}{27}=?$
(7) $9\frac{4}{9}-3\frac{14}{15}=?$
(8) $16\frac{2}{9}-10\frac{8}{15}=?$
(9) $210\frac{5}{12}+49\frac{9}{10}=?$
(10) $37\frac{1}{2}-18\frac{5}{21}=?$

2. Find the products and quotients as rapidly as you can work correctly, using cancellation when convenient:

(1) $\frac{7}{8}\times\frac{16}{21}=?$
(2) $\frac{6}{11}\times\frac{22}{27}=?$
(3) $\frac{14}{17}\times\frac{17}{21}=?$
(4) $\frac{16}{21}\div\frac{8}{21}=?$
(5) $\frac{12}{17}\div\frac{5}{18}=?$
(6) $18\frac{9}{10}\div16\frac{2}{3}=?$
(7) $21\frac{3}{8}\div14\frac{5}{16}=?$
(8) $8\frac{15}{28}\div12\frac{5}{7}=?$
(9) $15\frac{6}{7}\div3\frac{7}{8}=?$
(10) $26\frac{11}{12}\div80\frac{2}{3}=?$

3. Find the simplest results for these exercises:

(1) $\frac{3}{4}\times\frac{2}{5}\times\frac{5}{14}=?$
(2) $\frac{5}{8}\times\frac{9}{15}\times\frac{11}{12}=?$
(3) $\frac{5}{12}+\frac{9}{10}+\frac{11}{15}=?$
(4) $\frac{3}{14}\times\frac{7}{12}\times\frac{16}{21}=?$
(5) $\frac{3}{14}\times\frac{7}{12}+\frac{16}{21}=?$
(6) $12\frac{1}{2}\times8\frac{1}{3}\div6\frac{1}{4}=?$
(7) $9\frac{3}{7}\times4\frac{2}{5}\div3\frac{7}{8}=?$
(8) $16\frac{2}{3}+70\frac{5}{8}-87\frac{1}{2}=?$
(9) $9\frac{3}{8}\div6\frac{1}{4}\times6\frac{2}{3}=?$
(10) $16\frac{2}{3}\div37\frac{1}{2}\times6\frac{7}{8}=?$

4. Multiply mentally, writing only the products:

(1) 644×25
(2) 748×50
(3) $832\times12\frac{1}{2}$
(4) 862×11
(5) 386×250
(6) $279\times33\frac{1}{3}$
(7) $606\times66\frac{2}{3}$
(8) 328×125
(9) $245\times98.$

5. Solve mentally, writing only the quotients:

(1) $6400\div25$
(2) $6900\div300$
(3) $7500\div150$
(4) $35000\div500$
(5) $2600\div130$
(6) $8100\div90$
(7) $1640\div82$
(8) $1960\div98$
(9) $2180\div109.$

§82. Practical Applications.

Solve all you can orally

1. At the rate of 20 lb. for $1.00, what will 15 lb. of sugar cost?

Answer first: "15 lb. is what part of 20 lb.?" Then, what part of $1 will 15 lb. cost?

2. A boy buys 36 oranges at the rate of 3 for 5 cents, What do they all cost him? If 3 oranges cost 5 cents, what does 1 orange cost? What, then, do 36 oranges cost?

3. The boy sells his 36 oranges at the rate of 2 for 5 cents. How much does he receive for them?

4. A man walks 54 miles at the rate of 9 mi. in 2 hours. How long does it take him to walk the 54 miles?

The rate of 9 mi. in 2 hr. equals how many miles in 1 hour? How many hours, then, to walk 54 miles?

5. A boy buys 72 marbles at the rate of 6 marbles for 5 cents. How much do the 72 marbles cost him?

6. It takes $1\frac{1}{4}$ yards of ribbon to trim a hat. How many such hats can be trimmed with $37\frac{1}{2}$ yards of ribbon?

7. At the rate of 4 dollars for 5 dozen, what will 3 dozen cocoanuts cost?

8. In a class there are 3 girls to every 4 boys. There are 24 boys in the class. How many girls are in the class?

9. A man receives $\$6\frac{3}{4}$ for 3 days' work. At the same rate how much would he receive for 1 day? For 5 days?

10. What rate per day is the same as $\$12\frac{3}{8}$ for 3 days? $\$12\frac{3}{8}$ for $4\frac{1}{2}$ days?

11. It takes $2\frac{1}{3}$ yards of gingham to make an apron. How many such aprons will 42 yards of gingham make?

12. If $\frac{7}{9}$ of a ton of coal costs $4.81, what does 1 ton cost at the same rate?

13. At $6\frac{1}{4}$ cents a pound, how many pounds of sugar can be bought for \$1?

14. If the average column of a newspaper contains 1600 words, how much should a writer receive for 700 words at the rate of \$5 per column?

15. In going 19,725 feet the front wheel of a bicycle makes 1430 revolutions. How far would it go in making 1001 revolutions?

16. How long will it take one to read a book of 204 pages, reading at the rate of $12\frac{3}{4}$ pages a day?

17. How long must a piece of cloth $1\frac{1}{2}$ yd. wide be to contain the same number of square yards as a strip 1 yard wide and $4\frac{1}{2}$ yd. long?

18. A man sold $\frac{3}{8}$ of his farm and had 100 acres left. How many acres were there in the farm before the sale?

19. What would a fish weighing $3\frac{3}{4}$ lb. be worth at $12\frac{1}{2}$ cents a pound?

20. A man sold $\frac{3}{5}$ of his interest in a store for \$1485. At this rate what was his entire interest in the store worth?

21. A man owned $\frac{4}{5}$ of a store and sold $\frac{3}{5}$ of what he owned. The part of the store that he kept was worth \$1200. How much should he receive for the part he sold? At the same rate what was the whole store worth?

22. A man willed his estate to two children, giving $\frac{1}{2}$ of it to one, $\frac{1}{3}$ of it to the other, and the rest to charity. The sum willed to charity was \$4000. What was the value of the whole estate?

23. A and B together can husk a field of corn in 20 days. A husks twice as fast as B. In what time can each, working alone, husk the field of corn?

24. A can do a piece of work in $3\frac{1}{9}$ days that B can do in $7\frac{7}{9}$ days. In what time can both do it, working together?

§83. Fractions as Ratios.

A fraction, as $\frac{2}{3}$, is only another way of writing the ratio of 2 to 3, or 2:3.

The expression 4:2 is read 4 to 2.

1. Write as ratios: $\frac{2}{4}$, $\frac{5}{10}$, $\frac{6}{12}$, $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$, $\frac{12}{18}$.

2. Write as fractions: 3:6, 4:8, 8:12, 4:5, 7:12.

The ratio of 2:4 is $\frac{2}{4}$ or $\frac{1}{2}$; the ratio of 4:2 is $\frac{4}{2}$ or 2.

3. What is the ratio of the following pairs of numbers?

(1) 2:4	(7) 4:2	(13) 8:3	(19) 11:2
(2) 3:6	(8) 6:3	(14) 9:4	(20) 18:5
(3) 2:5	(9) 5:2	(15) 7:2	(21) 25:6
(4) 6:15	(10) 15:6	(16) 18:3	(22) 20:3
(5) 8:20	(11) 20:8	(17) 16:5	(23) 17:5
(6) 9:24	(12) 24:9	(18) 21:6	(24) 14:4

4. A drawing book cost 10¢ and a pencil 5¢. What is the ratio of the cost of the drawing book to the cost of the pencil? What is the ratio of the cost of the pencil to the cost of the drawing book?

5. A farmer paid $200.00 for a horse and $40.00 for a cow. What is the ratio of the cost of the cow to the cost of the horse?

6. A dwelling house cost $1200.00; a barn cost $400.00. What is the ratio of the cost of the house to the cost of the barn?

7. In a school of forty pupils there are 24 girls and 16 boys. What is the ratio of the number of boys to the number of girls? What is the ratio of the number of girls to the number of boys?

8. If three books cost $0.60, what will five books cost?

9. When sugar is sold at 16 lb. for $1.00, what will 24 lb. cost?

10. When eggs sell at $0.35 for two dozen, what will six dozen eggs cost?

MEASUREMENT

§84. Measuring Value.

Money is the common measure of the value of all articles that are bought and sold.

The unit on which United States money is based is the dollar. This unit is called the U. S. standard of value.

1. A dollar is worth as much as how many dimes? Nickels? Quarters? Cents?

To measure the value of one amount of money by another is to find how many times one of the amounts is as large as the other.

2. Measure \$1 by 50¢; by 25¢; by 20¢; by 40¢; by 75¢.

3. Measure \$10.50 by 50¢; by 75¢; by \$1.50; by \$5.25.

4. A farm is worth \$1000 and a city lot \$2000. Measure the value of the farm by the value of the lot.

5. Measure the value of a \$150 horse by the value of a \$15 pig.

6. Measure \$75 by \$5; by \$25; by \$15; by \$150; by \$225.

7. Measure the value of 160 A. of land worth \$75 per acre, by \$100; by \$500; by \$1000; by \$8000; by \$24,000.

8. Which is worth the more, 80 A. of land @ \$75 per acre, or 50 A. @ \$112 per acre? How much more?

9. Measure \$400 by the value of an \$8 calf.

§85. Measuring Length and Distance.

Answer these problems, first, by estimate, writing your estimates in a notebook; then answer by actual measurement. Finally, compare your estimates with the results of your measurements and give the difference.

1. How wide is the page of this book? How long? How wide are the margins?

2. How wide is a pane of glass in your schoolroom window? How long is it?

3. How wide is the top of your desk? How long? How high is the top of your desk from the floor?

4. How high is the bottom of the blackboard from the floor? How wide is your blackboard? How long is it?

5. How tall are you? How far can you reach by stretching both arms as far apart as possible?

6. How far is it around your waist? How far is it around your chest, just below your arms, when as much of the air as possible is exhaled from your lungs? What is your chest measurement when as much air as possible is drawn into your lungs?

7. The difference between these two chest measures is your chest expansion. How much is your chest expansion? How does your chest expansion compare with the average for the pupils of your room?

8. How many steps wide is your schoolroom? How many steps long?

9. How many feet long is your room? How wide?

10. How many yards long is it? How many yd. wide?

11. How many inches long is your step? How many feet long? How many yards long?

12. How many in. long and wide is your schoolroom?

13. How many feet long is your schoolhouse? How many yards long?

14. How many steps wide and how many steps long is your school yard? How many feet wide and how many feet long? How many yards wide and long?

15. How many steps is it from your home to the schoolhouse? How many feet? How many yards? Rods?

16. How far can you walk in 1 min.? How many miles could you walk in 1 hr. at the same rate?

17. How far is it from your home to a neighboring large city? Answer this by using a map and the scale given with it. How long would it take you to walk to that city at the rate of walking in problem 16?

18. How long will it take a train to run from your station to the nearest city at the rate of 24 mi. per hour?

19. Using the scale map of your state (see Geography) find the length and the width of your state; of your county; of the U. S.

TABLE OF EQUIVALENTS

1 foot	1 yard	1 rod	1 mile
12 inches	3 feet	5½ yards	320 rods
	36 inches	16½ feet	1760 yards
			5280 feet

20. What kind of unit do you need to measure and express long distances? Medium distances? Short distances?

§86. Measuring Surfaces.

A square unit is a square 1 unit long and 1 unit wide.

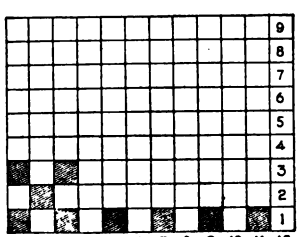


FIGURE 1

1. The side wall of a room is 9 ft. by 12 ft. What will it cost to lath and plaster the wall at 33¢ per square yard?

2. Using your own measures of the side and the end walls of your schoolroom, answer the same question for its walls and ceiling making no allowance for windows or doors.

3. How many sq. ft. of blackboard surface in your room?

4. How many square inches are there in a pane of glass in one of your windows? How many square feet of window surface admit light into your room?

5. This lighting surface should be at least as great as $\frac{1}{6}$ of the floor surface of your room. Is it?

6. $\frac{1}{16}$ of an inch in the drawing, Fig. 1, stands for one foot in the room. Measure and find the number of square feet of surface in the whole interior of the room, deducting the part covered by the baseboard and the floor.

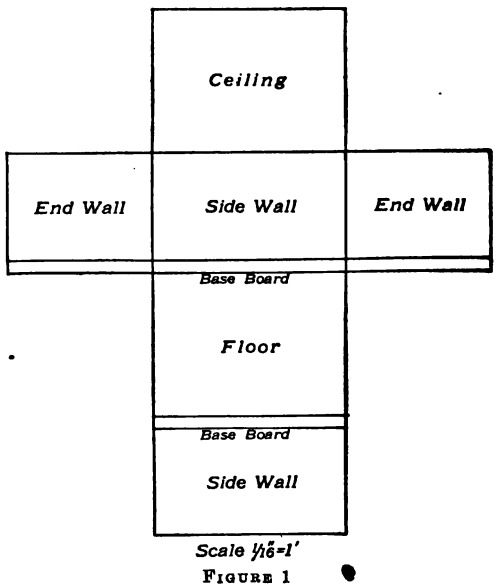


FIGURE 1

Such a representation of the room as that of Figure 1, showing the walls and the ceiling spread out on a flat surface, is called a development of the room.

7. Draw to scale, from your own measures, the development of your schoolroom.

8. A schoolroom is 24'* square and 15' high. How many sq. yd. of flooring are needed for the room? How many sq. yd. of plastering are needed for the ceiling?

* The mark (') means *foot* or *feet;* (") means *inch* or *inches.*

9. One side wall (problem 8) contains two windows, each $6' \times 8'$; the other has a window and a door, each $4' \times 8'$; an end wall contains a window $6\frac{1}{2}' \times 8'$. How many sq. yd. of window and door surface in the entire room?

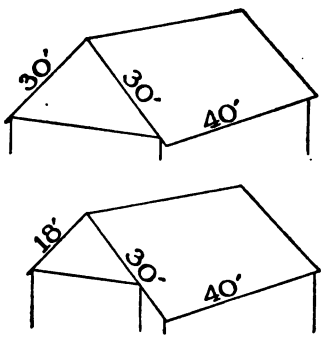


FIGURE 1

10. The rest of the wall surface is plastered. How many square feet of plastering are there in the room?

11. A barn roof is made up of two rectangles, each $30' \times 40'$. How many square feet are there in the roof? How many squares of roofing? (A square of roofing is a 10-ft. square = 100 sq. ft.)

12. A house roof is made up of two rectangles, one $18' \times 40'$ and the other $30' \times 40'$. How many squares of roofing in the whole roof? How many sq. yd. of roof?

13. How many acres (1 acre = 160 sq. rd.) are there in a field 40 rd. $\times$ 80 rd.? In a field 60 rd. $\times$ 80 rd.? 80 rd. $\times$ 80 rd.? 75 rd. $\times$ 120 rods?

14. What is the cost at \$2.20 a sq. yd. of paving a street 60′ wide and $\frac{5}{8}$ of a mile long?

15. What is the cost at 15¢ per sq. ft. of making a cement sidewalk, $1\frac{2}{3}$ yd. wide and $\frac{3}{8}$ of a mile long? $1\frac{3}{4}$ yd. wide and $\frac{5}{8}$ of a mile long?

16. The floor of an L-shaped basement is made of two rectangles, one of which is 9 yd. $\times$ 5 yd., and the other 5 yd. $\times$ 3 yd. How many square yards does it contain? How many square feet? What would it cost to cement it at 14¢ a square foot?

17. How many square feet of glass are there in a window containing 4 panes of glass, each $18'' \times 1$ yard?

18. A rug $9' \times 12'$ is placed in a room $14' \times 18'$, with its long edge parallel to the long side of the room and with its center at the center of the floor. How many sq. ft. of the floor remain uncovered along the sides of the room?

19. The streets of a city cut out a row of blocks like that of Fig. 1. The numbers along the sides are the lengths in feet. How many square feet are there in block *A*? In *B*? In *C*?

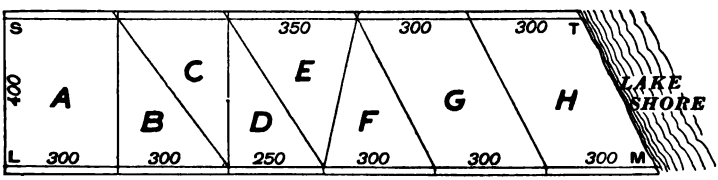


FIGURE 1

A four-sided figure, like *A*, having square corners, is called a rectangle.

A four-sided figure, like *G*, or *H*, having two pairs of parallel sides, whether the corners are square, or not, is a parallelogram.

20. What is the area of the rectangle of Fig. 1?

21. Examine the parallelograms and the rectangles beneath them and find the areas of the parallelograms.

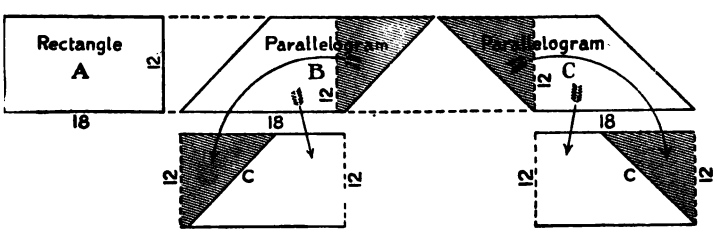


FIGURE 2

22. If the length of the base of a parallelogram and its distance square across (*altitude*) are given, how can we find a rectangle with the same area as the parallelogram?

23. Point out in Fig. 1 a rectangle that has the same area as the parallelogram *G*.

24. The length of a parallelogram is b ft. and the altitude is a feet. What is the area?

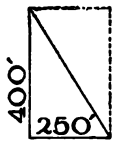


FIGURE 1

25. The streets, ST and LM, are parallel, Fig. 1, page 131. How many square feet are there in block G? In H? In E? In F?

26. Study Fig. 2, page 131, and find how many sq. ft. there are in D of Fig. 1, same page.

27. How can you find the area of a triangle when you know the length of its base and its altitude (shortest distance to the base from the opposite corner)?

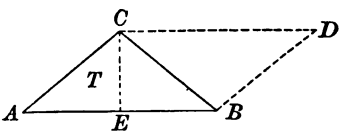


FIGURE 2

28. In the triangle, ABC (Fig. 2), call the area T, the altitude (CE) a, the base (AB), b. Write an equation to find T from a and b.

29. Find the area in square feet of the following triangles of Fig. 1, page 131: B; C; D.

30. A roof in the shape of a parallelogram has a length (*base*) of 18 ft. and a breadth (*altitude*) of 12 ft. What is the area of the parallelogram?

31. A city lot has the shape of a parallelogram of 400 ft. base and 300 ft. altitude. Give the area in square feet.

32. A field has the form of a triangle of 68 rd. base and 46 rd. altitude. What is the area in square rods?

33. Find the missing part (base, or altitude, or area) of the following parallelograms:

	BASE	ALTITUDE	AREA
(1)	28 ft.	18 ft.	———
(2)	40 rd.	———	960 sq. rd.
(3)	———	40 rd.	1600 sq. rd.
(4)	160 rd.	$60\frac{3}{4}$ rd.	———
(5)	———	88 rd.	28160 sq. rd.
(6)	m rd.	n rd.	———

TABLE OF SQUARE MEASURE

144	sq. in. (square inches)	= 1 sq. ft. (square foot).
9	sq. ft. (square feet)	= 1 sq. yd. (square yard).
30¼	sq. yd. (square yards)	= 1 sq. rd. (square rod).
160	sq. rd. (square rods)	= 1 A. (acre).
640	A. (acres)	= 1 sq. mi. (square mile).

§87. Measuring Volume (Bulk) and Capacity.

Answer all you can orally

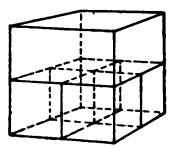


FIGURE 1

1. How many cubic inches are there in a 2-inch cube? A 4-in. cube? An 8-in. cube? A 16-in. cube?

2. How many cubic feet are there in a cubic yard? (See Fig. 2.) In a 6-ft. cube? In a room 3 yd. × 3 yd. × 3 yd.?

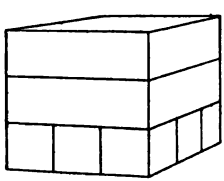


FIGURE 2

3. How many cubic inches are there in a 5-in. cube? In a 10-in. cube?

4. How many cubic inches are there in a 9-in. cube? In an 18-in. cube?

5. How many cubic inches are there in a cubic foot?

6. What is the capacity of a square-cornered box 3″ × 4″ × 6″ (see Fig. 3)? 3′ × 4′ × 6′? Of a room 3 yd. × 4 yd. × 5 yd.? *a* yd. × *b* yd. × *c* yards?

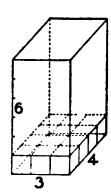


FIGURE 3

7. A box-car 8 feet wide and 34 feet long (inside measure) can be filled with wheat to a height of 5 ft. When thus filled how many cubic feet of grain does it hold?

8. If a bushel of wheat = $\frac{5}{4}$ cubic feet, how many bushels does the car hold when full?

9. A cubic foot of grain (not ear-corn) is $\frac{4}{5}$ of a bushel. Make a rule for finding the number of bushels a wagon-box or a granary will hold when full.

10. A wagon-box is $2' \times 4' \times 10'$. How many bushels of ear-corn will it hold if $\frac{9}{4}$ cu. ft. = 1 bu. of ear-corn?

11. How many rectangular solids $4'' \times 3'' \times 7''$, will fill a box $16'' \times 12'' \times 28''$? (See Fig. 1).

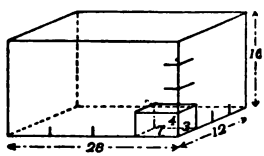


FIGURE 1

12. A box $30'' \times 24'' \times 12''$ will contain how many blocks $6'' \times 4'' \times 3$ inches?

13. 128 cu. ft. = 1 cord. How many cords in a straight pile of wood 80 ft. $\times$ 4 ft. $\times$ 4 feet?

14. The volume of a rectangular bin is 1500 cu. ft. If it is 25 ft. long and 6 ft. deep, what is its width?

15. How many bricks $8'' \times 4'' \times 2''$ are there in a regular pile $218'' \times 24'' \times 48$ inches?

16. Measure the length, the width, and the height of your schoolroom, and find how many cubic feet of air it contains.

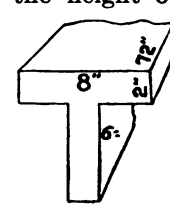


FIGURE 2

17. How many pounds does the steel T-beam of Fig. 2 weigh if 1 cu. in. of steel weighs $4\frac{1}{2}$ ounces.

18. A steel I-beam 24′ long has a cross section of the form and size shown in Fig. 3. How much does it weigh if steel weighs 486 lb. per cubic foot?

FIGURE 3

19. A water tank is $3' \times 6' \times 10'$. How many cubic feet of water does it hold when full?

20. There are 231 cu. in. in a gal.; how many gal. does the tank of problem 19 hold when full?

21. How many cubic inches of grain does a box hold if it is 8 in. square and $4\frac{1}{8}$ in. deep? How many gallons?

22. How many cubic inches does a box hold if it is 16 in. square and $8\frac{1}{2}$ in. deep? How many bushels (2150.42 cu. in. = 1 bushel)?

23. How many cubic inches does a box hold if it is $9'' \times 10'' \times 12''$? About what part of a bushel?

24. Answer similar questions for a box 10 in. square and $10\frac{3}{4}$ in deep; for a box $10'' \times 12'' \times 18''$.

TABLE OF CUBIC MEASURE

1728	cu. in. (cubic inches)	= 1 cu. ft. (cubic foot).
27	cu. ft. (cubic feet)	= 1 cu. yd. (cubic yard).
128	cu. ft.	= 1 cord (cd.)
231	cu. in.	= 1 gal.
$2150\frac{1}{2}$	cu. in. (nearly)	= 1 bu.

§88. Measuring Weight.

1. A 3-in. cube of soil weighs 20 ounces; how many pounds does 1 cu. ft. of the same soil weigh? How many pounds does a cubic yard weigh?

2. A 4-in. cube of sand weighs 48 ounces; how many pounds does 1 cu. ft. of the same sand weigh? How many pounds does a cubic yard weigh?

3. A cubic foot of water weighs 1000 ounces; how much does a 6-in. cube of water weigh? A 3-in. cube?

4. A cubic foot of iron weighs 480 pounds; how many ounces does 1 cu. in. weigh?

Notice that 1 cu. ft. $= 12 \times 12 \times 12$ cu. in., then use cancellation, thus:

$$\frac{\overset{10}{\cancel{480}} \times \overset{4}{\cancel{16}}}{\underset{3}{\cancel{12}} \times \underset{3}{\cancel{12}} \times \cancel{12}} = \frac{10 \times 4}{3 \times 3} = \frac{40}{9} = 4\tfrac{4}{9}.$$ Ans. 1 cu. in. weighs $4\frac{4}{9}$ oz.

5. A cubic foot of oak wood weighs 64 lb.; how many ounces does 1 cu. in. of oak wood weigh? How many cubic inches of oak weigh just 1 pound?

16 ounces (oz.) = 1 avoirdupois pound (av. lb.)
2000 pounds = 1 ton (T.)

6. A cubic foot of copper weighs 552 lb.; how many ounces does 1 cu. in. of copper weigh? How many cubic inches of copper weigh a pound?

7. A cubic foot of gold weighs 1200 lb. (av.); how many ounces does 1 cu. in. of gold weigh? How many cubic inches of gold weigh a pound?

8. A cubic foot of zinc weighs 436 lb.; how many ounces does 1 cu. in. of zinc weigh? How many cubic inches of zinc weigh a pound?

9. A bushel of coal weighs 80 lb., a short ton weighs 2000 lb., and a long ton weighs 2240 pounds. How many bushels of coal are there in a car load of 36 short tons? Of 36 long tons?

10. How many tons of timothy hay will fill a hay-mow, $18' \times 23' \times 40'$, counting 460 cu. ft. to a ton?

11. How many 100-lb. bales of hay, each $1\frac{1}{2}' \times 2' \times 3\frac{1}{2}'$, will it take to fill a car $8' \times 9' \times 42'$? Find the weight.

12. A granary $10' \times 11\frac{1}{2}' \times 16'$ is full of wheat. Allowing $1\frac{1}{4}$ cu. ft. to the bu., how many bu. are in the granary? How many 46-bu. loads in the granary?

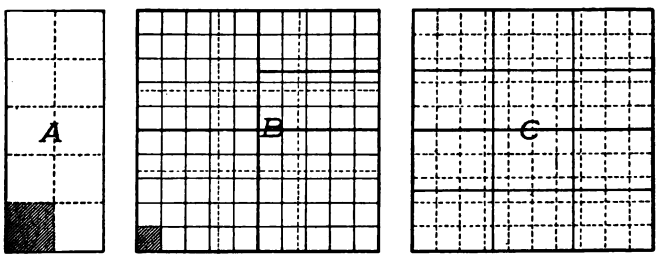
13. Allowing 60 lb. to the bu. find the weight of the 46-bu. load of problem 12. Find the weight of wheat in the granary of problem 12.

14. A crib $9\frac{1}{2}' \times 9\frac{1}{2}' \times 14'$ is full of ear corn, averaging $1\frac{3}{4}$ cu. ft. to the bu. How many bu. of corn in the crib?

15. If each bu. contains 56 lb. of shelled corn and 20 lb. of cobs, what is the weight of shelled corn and of cobs in the crib of problem 14?

DECIMAL FRACTIONS

§89. Common and Decimal Fractions.



1. What part of the rectangle, A, is shaded? Not shaded?

2. Show from A how many tenths (10ths) there are in $\frac{1}{2}$. In $\frac{2}{2}$. In $\frac{1}{5}$. In $\frac{2}{5}$. In $\frac{3}{5}$. In $\frac{4}{5}$. In $\frac{5}{5}$.

3. Write these fractions as 10ths: $\frac{1}{2}$ $\frac{2}{2}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{4}{5}$.

4. What part of the square, B, is shaded? Not shaded?

5. Show from B how many hundredths (100ths) there are in 1. In $\frac{1}{2}$. In $\frac{2}{2}$. In $\frac{1}{4}$. In $\frac{3}{4}$.

6. Show how many 100ths there are in: $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$.

7. Write as 100ths: $\frac{1}{2}$ $\frac{2}{2}$ $\frac{1}{10}$ $\frac{1}{5}$ $\frac{3}{4}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{7}{10}$.

Another way of writing $\frac{2}{10}$, $\frac{3}{10}$, $\frac{5}{10}$, is .2, .3, .5.

Another way of writing $\frac{25}{100}$, $\frac{45}{100}$, $\frac{75}{100}$, and so forth, is .25, .45, .75, and so forth.

When 10ths, or 100ths are written in the second way they are called *decimal fractions*, or *decimals*.

The point used in writing decimal fractions is called the DECIMAL POINT.

The decimal, .7, is read "7 *tenths*," .07 is read "7 *hundredths*," and .37 is read "37 *hundredths*."

8. Write these fractions decimally, and read them:

$\frac{8}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{12}{10}$ $\frac{7}{100}$ $\frac{20}{100}$ $\frac{35}{100}$ $\frac{50}{100}$ $\frac{66}{100}$ $\frac{78}{100}$

9. Write the following decimals as common fractions, and then read them:

.4 .9 .07 .13 .18 .26 .48 .67 .50 .25 .79

10. $\frac{1}{3}$ of the square, *B*, (page 137) equals how many 100ths of *B*? $33\frac{1}{3}$ hundredths is written $.33\frac{1}{3}$.

11. $\frac{2}{3}$ of the square, *B*, equals how many 100ths of *B*?

12. Write $66\frac{2}{3}$ hundredths without the word, *hundredths*.

13. How many 100ths of *B* equals $\frac{1}{8}$ of *B*? $\frac{2}{8}$ of *B*? $\frac{3}{8}$ of *B*? $\frac{5}{8}$ of *B*? $\frac{7}{8}$ of *B*?

14. Write the following common fractions as decimals:

$\frac{33\frac{1}{3}}{100}$ $\frac{66\frac{2}{3}}{100}$ $\frac{12\frac{1}{2}}{100}$ $\frac{37\frac{1}{2}}{100}$ $\frac{62\frac{1}{2}}{100}$ $\frac{87\frac{1}{2}}{100}$

15. $\frac{1}{6}$ of square *C* equals how many 100ths of *C*?

16. $\frac{5}{6}$ of the square *C*, equals how many 100ths of *C*?

17. Write $\frac{16\frac{2}{3}}{100}$, and $\frac{83\frac{1}{3}}{100}$ decimally.

18. Show from *C* how many 100ths $\frac{1}{12}$ equals; $\frac{5}{12}$; $\frac{7}{12}$; $\frac{11}{12}$.

19. Write the following decimals as common fractions:

.27 .33 $.67\frac{1}{2}$ $.72\frac{1}{3}$ $.22\frac{1}{2}$ $.44\frac{4}{9}$ $.77\frac{7}{9}$

20. Write as decimals, and then read:

$\frac{8\frac{1}{3}}{100}$ $\frac{14\frac{2}{7}}{100}$ $\frac{41\frac{2}{3}}{100}$ $\frac{42\frac{6}{7}}{100}$ $\frac{58\frac{1}{3}}{100}$ $\frac{71\frac{3}{7}}{100}$ $\frac{91\frac{2}{3}}{100}$

21. Show how many 100ths $\frac{1}{9}$ of the square, *B*, (page 137) equals.

22. How many 100ths of square B do the following fractional parts equal:

$\frac{1}{9}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{1}{3}$ $\frac{4}{9}$ $\frac{5}{9}$ $\frac{2}{3}$ $\frac{6}{9}$ $\frac{7}{9}$ $\frac{8}{9}$ $\frac{9}{9}$?

§90. Notation of Decimals.

Answer all you can orally

1. In \$1111 what does the 1st 1 on the right stand for? The 2d? The 3d? The 4th?

2. In \$6666 what does the 1st 6 on the left denote? The 2d? The 3d? The 4th?

3. In \$372.68 what is the unit of the 3? (*Ans.* \$100.) Of the 7? Of the 2? The 6? The 8?

4. In \$5555 how does the number denoted by each 5 compare with the number denoted by the 5 to its left? To its right?

5. How do the units of the places of a number change as we pass through the number from left to right?

6. Which unit is the fundamental unit out of which the other units are made? How is the place of this fundamental unit indicated in \$675.28?

A dot, called the decimal point, or point, is used to show the units' digit. The point stands, or is supposed to stand, to the right of the units' digit or place.

7. If the law of problem 5 holds in 444.444, the unit of the 1st 4 to the right of the point equals what part of the unit of the 1st 4 to the left of the point? The unit of the 2d 4 to the right equals what part of the unit of the 1st 4 to the right?

8. In any number what part of the unit of the 1st place to the left of the point equals the unit of the 1st place to the right? Of the 2d place to the right? Of the 3d? Of the 4th?

The unit of the 1st place, or digit, to the right is called the tenth; of the 2d place, or digit, the hundredth; of the 3d, the thousandth; of the 4th, the ten-thousandth and so on.

§91. Numeration of Decimals.

1. The number 444.444 might be read, "4 hundreds, 4 tens, 4 units, *and* 4 tenths, 4 hundredths, 4 thousandths"; but it is simpler to read it, "Four hundred forty-four *and* four hundred forty-four *thousandths*." Read 234.234 both ways. Which is the shorter?

The latter mode of reading is used in practice. It may be stated thus: Read the integral (whole) part of the number, pronounce *and* for the decimal point, then read the part of the number to the right of the point as though it were a whole number and, finally, name the unit of the last digit on the extreme right.

2. Read 675.328; 236.89; 7.65; 43.6587.

3. Read the following:

(1)	.1	.01	.001	.0101	1.1	10.1
(2)	.6	6.06	66.6	.0606	600.06	6000.006
(3)	.60	7.070	8.50	.0600	87.087	10.0008

4. What is the ratio of 5 to .5? Of .5 to .05? Of .5 to .005? Of 55 to 5.5? Of 55 to .55?

5. Find the ratio of 875 to 87.5; of 87.5 to 8.75; of 875 to 8.75; of 875 to .875.

6. How is a number affected by moving the decimal point 1 place toward the left? (See problems 4 and 5.) 2 places? 3 places? 6 places?

7. What is a quick way of dividing any number by 10? By 100? By 1000?

8. Express the ratio of 2.358 to 23.58; to 235.8; to 2358; to 23,580; to .2358; to .02358.

Where is the point supposed to be in 2358? When the decimal point is not written, where is it supposed to be?

9. How is a number changed by moving the decimal point 1 place to the right? 2 places? 3 places? 4 places? 5 places? 6 places?

10. Give a quick way to multiply any number by 10; by 100; by 1000; by 1 followed by any number of zeros.

11. What effect is produced in such a number as .683 or .492 by writing a zero between the decimal point and its first digit? 2 zeros? Any number of zeros?

12. Compare .5 with each of these numbers: .50; .500; .50000. What is the effect of writing any number of zeros to the right of the last digit of a decimal?

A DECIMAL FRACTION, or DECIMAL, is a fraction whose denominator is 10, 100, 1000, or some power of ten, in which the denominator is not written but is indicated by the position of the decimal point.

A POWER of 10 is a number obtained by using 10 as a factor any number of times. Thus, $10 \times 10 = 10^2 = 100$; $10 \times 10 \times 10 = 10^3 = 1000$, and so on, are powers of 10.

§92. Reducing a Decimal to a Common Fraction.

The fractions we have studied whose denominators are actually written, are called *common fractions.*

1. Express the following decimals as common fractions:

.5	.50	.500	.4	.40	.400	.6
.60	.25	.250	.125	.375	.625	.875

2. Reduce the results of problem 1 to their lowest terms.

3. Write these mixed decimals as improper fractions:

1.5	2.50	2.75	10.4	12.5	6.75	18.25

4. Reduce the results of problem 3 to their lowest terms.

5. After dropping the decimal point from the following decimals, what numbers must be written beneath them to express them as common fractions?

.6	.67	.625	.875	.1275	12.75	25.768	33.333

6. Make a rule for expressing any decimal as a common fraction in its lowest terms.

7. Compare the rule you have made with the following rule:

Write the denominator beneath the decimal, omit the point, and reduce the fraction to its lowest terms.

Pure decimals: .25; .038; .508; .0173.
Mixed decimals: $0.3\frac{1}{3}$; $4.67\frac{1}{2}$; $6.66\frac{2}{3}$; $4.444\frac{4}{9}$.

8. Read and give the meaning of the following mixed decimals:

$.12\frac{1}{2}$	$3.33\frac{1}{3}$	$18.66\frac{2}{3}$	$2.1\frac{1}{2}$
$.0\frac{3}{4}$	$.04\frac{2}{9}$	$.00\frac{2}{5}$	$36.000\frac{5}{6}$

To reduce such an expression as $3.44\frac{5}{7}$ to a mixed number proceed thus:

$$3.44\frac{5}{7} = \frac{344\frac{5}{7}}{100} = \frac{\frac{2413}{7}}{100} = \frac{2413}{700} = 3\frac{313}{700}.$$

Or thus: $$3.44\frac{5}{7} = 3\frac{44\frac{5}{7}}{100} = 3\frac{\frac{313}{7}}{100} = 3\frac{\frac{313}{7}\times 7}{100\times 7} = 3\frac{313}{700}.$$

9. Reduce the decimals of problem 5 to mixed numbers, or to common fractions in their lowest terms.

10. Make a rule for expressing any mixed decimal as a common fraction, or a mixed number.

11. Compare your rule with the following:

Any decimal may be expressed as a common fraction in its lowest terms, or as a mixed number, by dropping the decimal point, writing the denominator, and reducing the resulting common fraction to its lowest terms.

12. Express as common fractions in their lowest terms the following:

.1 .24 1.75 $2.33\frac{1}{3}$ 3.175 1.875 5.218 $8.88\frac{8}{9}$

ADDITION OF DECIMALS

§93. Rainfall, Industrial Problems, etc.

See definition of rainfall, page 109.

1. The following quantities of rain fell from week to week during May, 1902, in Chicago: First week, 1.09 in.; second week, 1.11 in.; third week, .45 in., and fourth week, 2.43 inches. Find the total rainfall for the month.

CONVENIENT FORM

1.09 in.
1.11 in.
.45 in.
2.43 in.
Ans. 5.08 in.

It is convenient to write the addends in a column so that the unit digits are all in the same vertical column. Then begin on the right and add as with whole numbers. In the sum the point should stand directly under the points in the addends.

2. A dealer received 8 carloads of coal of the following tonnages: 24.6, 28.785, 31.25, 24.95, 31.8, 25.125, 28, and 29.25. What was the total tonnage (number of tons)?

3. A farm was divided by its owner into lots of the following acreages: $32.87\frac{1}{2}$, $7.12\frac{1}{2}$, $68.33\frac{1}{3}$ $11.66\frac{2}{3}$, $16.28\frac{3}{8}$, and $21.13\frac{7}{8}$. What was the total area of the farm?

4. The following numbers represent in thousands of feet a lumber dealer's sales in 1 da.: 6.865, 24.245, 16.398, 12.28, 18.2, 6.395, and 18.967. Give the total for the day.

5. A quantity of soil contained .125 lb. gravel; .268 lb. coarse sand; .175 lb. fine sand; $.037\frac{1}{2}$ lb. organic matter; $.214\frac{3}{4}$ lb. clay, and .275 lb. water. Give total weight.

6. During 8 hr. a freight train made the following mileages: 32.15, 28.375, 15.687, 20.2, 15.63, 17.5, 8.95, and 21.3. How far did the train run during the 8 hours?

7. Following are the weights in grains of the U. S. coins: 1¢-piece, 48; 5¢-piece, $73.166\frac{2}{3}$; dime, $38.583\frac{1}{3}$; quarter-dollar, 96.45; half-dollar, 192.9; dollar, $412\frac{1}{2}$; quarter-eagle, $64\frac{1}{2}$; half-eagle, 129; eagle, 258; double eagle, 516. Find the weight of all together.

8. The following numbers denote the monthly rain or snowfall in a certain city for 1910: .66, 1.53, 4.16, 2.26, 5.08, 6.45, 4.25, 1.44, 4.83, 1.45, 2.03, 1.90.

What was the total rain or snowfall for the year?

9. Without rewriting the numbers, find the total yearly rain or snowfall for each year from 1901 to 1911 inclusive, from these recorded data:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1.99	1.95	2.13	2.14	2.09	2.42	2.47	4.52	.32	.36	3.83	1.32
1.99	1.57	2.21	2.17	6.77	10.58	2.23	1.85	1.34	1.54	2.68	1.63
2.08	2.44	1.69	4.16	1.93	3.59	3.08	.18	1.98	1.75	2.45	2.14
1.55	2.13	2.66	2.65	3.35	1.96	.60	.60	8.28	.85	1.18	1.66
2.15	1.60	1.32	.36	1.99	1.79	2.43	6.49	1.89	.51	5.60	6.76
1.12	3.48	1.26	2.79	4.16	2.82	3.61	3.52	6.70	1.36	2.16	.16
4.53	2.22	3.56	2.23	.84	3.60	1.47	1.70	.84	.18	3.06	1.62
3.54	2.59	4.60	.76	2.23	5.30	1.94	3.03	3.16	3.26	2.25	1.11
.58	1.60	2.11	.14	4.35	2.71	6.66	.91	2.39	2.09	1.14	6.81
1.21	3.52	1.58	1.02	3.59	2.06	4.64	4.24	1.56	1.35	3.30	.58
1.15	2.05	3.38	.33	2.18	2.42	4.25	2.00	2.92	1.29	.85	1.70

10. Foot and average the vertical columns and tell what the footings and averages mean.

11. Compare the total of the footings of problem 10 with the total of problem 9.

SUBTRACTION OF DECIMALS

§94. Nature Study.

1. 54 cu. in. of soil in its natural state weighed 1.94 pounds. After being thoroughly dried it weighed 1.459 pounds. How much moisture passed off in drying?

Convenient Form

1.940 lb.
1.459 lb.
Ans. .481 lb.

We have seen that 1.94 may be written 1.940. For convenience write the numbers so that units' digits stand in the same column. Beginning on the right subtract as though the numbers were whole numbers. In the result the point should stand directly under the points in the minuend and subtrahend.

2. After drying, the same soil occupied only 37.125 cu. in. How much did it shrink in bulk in drying?

3. 100 green oak leaves weighed .22 pounds. After thorough drying they weighed .087 pounds. How much did they lose in drying?

4. The organic matter was then driven off by burning the dry leaves. The ash weighed .0053 pounds. How much organic matter did the 100 leaves contain?

5. The corresponding numbers for 100 green elm leaves were: Weight of green leaves, .132 pounds; weight of dry leaves, .0345 pounds; weight of ash, .0035 pounds. Answer questions like 3 and 4 for these leaves.

6. 1.135 lb. of dry beans, soaked for 24 hr., weighed 2.212 pounds. What was the weight of water taken up by the beans?

7. The dry beans occupied 34.875 cu. in. and the soaked beans 85.5 cubic inches. Find the increase in bulk.

8. In the table of rainfall and snowfall, page 144, how much greater was the rainfall in May than in Apr., 1902? In June than in Apr., 1902? In June than in May? In June than in July?

9. In 1903 how much greater was the April than the May rainfall? The June than the May? The June than the July? The July than the Aug.? The July than the September?

10. In 10 lb. of roasted peanuts 3.26 lb. is waste, and the rest is food. How much food is there in 10 lb. of roasted peanuts?

11. In 10 lb. of cocoanuts there are 5.12 lb. of food and the rest is waste. How much waste in 10 lb. of cocoanuts?

12. In 20 lb. of Brazil nuts there are 10.08 lb. of food and the rest is waste. How much waste in the 20 lb.?

13. In a pound of filberts .479 lb. is food; in a pound of almonds .352 lb. is food. The rest is waste. How much more food is there in 10 lb. of filberts than in 10 lb. of almonds? How much more waste in the almonds?

14. Ten pounds of pecans contain 4.68 lb. of food, and the same weight of walnuts contains 4.2 lb. of food. The rest is waste. How much more waste is there in 10 lb. of walnuts than in 10 lb. of pecans?

15. The food in 10 lb. of hickory nuts is 3.78 lb.; of chestnuts is 8.4 lb.; of butternuts is 1.36 lb. The rest is waste. How much more food is there in 10 lb. of hickory nuts than butternuts? Of chestnuts than hickory nuts? Of chestnuts than butternuts? How much waste is there in 10 lb. of each kind of nut?

MULTIPLYING DECIMALS

§95. Exercises and Problems.

1. A pound of butter contains 15.6 oz. of fat. How many ounces of fat are there in 3 pounds of butter?

2. In one pound of eggs there are 1.79 oz. of shell. How many ounces of shell are there in 5 pounds of eggs?

3. How many minutes in .05 hour?

4. A lot is 125 ft. deep and 12.5 ft. wide. What is its area?

5. A square-cornered box is .7 of a foot long, .5 of a foot wide and .3 of a foot deep. What is the area of the top? Of a side? Of an end? Of the entire surface?

6. Solve:

(1) $326\times10=?$	(4) $326\times100=?$	(7) $326\times1000=?$
(2) $32.6\times10=?$	(5) $32.6\times100=?$	(8) $32.6\times1000=?$
(3) $3.26\times10=?$	(6) $3.26\times100=?$	(9) $3.26\times1000=?$

7. How may a number be multiplied quickly by 10? By 100? By 1000?

8. Multiply:

(1)	(2)	(3)	(4)	(5)	(6)
68	68	68	6.8	6.8	6.8
27	2.7	.27	27	2.7	.27
476	476	476	476	476	476
136	136	136	136	136	136
1836	183.6	18.36	183.6	18.36	1.836

By the number of decimal places of a number is meant the number of digits (zero included) on the right of the decimal point.

In 2.005 there are 3 decimal places.

9. In problem 8, how many decimal figures are there in the product of (2)? How many in the product of (3)? Of (4)? Of (5)? Of (6)?

10. In problem 8, how many decimal figures are there in both the multiplicand and multiplier of (2)? Of (3)? Of (4)? Of (5)? Of (6)?

11. How may one quickly tell how many decimal places there will be in the product of two decimals?

12. Find the product of 1235×321, and write the answers to the following without multiplying:

(1) $1235 \times 32.1 = ?$
(2) $1235 \times 3.21 = ?$
(3) $123.5 \times 32.1 = ?$
(4) $12.35 \times 3.21 = ?$
(5) $1.235 \times 321 = ?$
(6) $12.35 \times .321 = ?$

13. Solve:

(1) $32 \times 2.5 = ?$
(2) $45 \times 3.8 = ?$
(3) $63 \times .28 = ?$
(4) $3.9 \times 4.5 = ?$
(5) $8.5 \times .48 = ?$
(6) $28.6 \times 32.5 = ?$
(7) $2.86 \times 3.58 = ?$
(8) $28.6 \times 3.56 = ?$

§96. Pointing the Product.

Use pencil and paper only when needed

1. Find the product 4862×784, and from it write the following products:

486.2×784	48.62×784	48.62×78.4
4.862×78.4	4.862×7.84	$4862 \times .784$

2. How many decimal places are there in 486.2? In 48.62? 4.862? 10.03? 3.0060? .0600? 20.0806?

3. How many decimal places are there in each of the products of problem 1?

4. Compare the number of decimal places in each of the products of problem 1 with the sum of the numbers of decimal places in both the multiplicand and the multiplier. What do you find?

5. Make a rule for finding how many decimal places there must be in the product of two decimals.

6. Compare your rule with the following:

The number of decimal places in the product equals the sum of the numbers of decimal places in the factors.

7. From the product $436 \times 328 = 143{,}008$, write the following products:

(1) $43.6 \times 328 = ?$ (3) $4.36 \times 3.28 = ?$ (5) $.436 \times 328 = ?$
(2) $4.36 \times 32.8 = ?$ (4) $436 \times 32.8 = ?$ (6) $.0436 \times .0328 = ?$

8. Make a rule for multiplying decimal numbers.

9. Compare your rule with the following:

With decimal numbers multiply as with whole numbers, and point off as many decimal places in the product as there are in both the multiplicand and the multiplier.

10. A field containing 38.75 A. yielded 23.9 bu. of wheat per acre. What was the total yield?

Convenient Form	
With Integers	With Decimals
3875	38.75
239	23.9
34875	34875
11625	11625
7750	7750
926125	926.125

Ans. 926.125 bu.

11. Find the products for the following:

(1) $426 \times 32.8 = ?$ (4) $654.1 \times 23.6 = ?$

(2) $42.6 \times .328 = ?$ (5) $6.541 \times 236 = ?$

(3) $4.26 \times .0328 = ?$ (6) $.6541 \times .236 = ?$

12. Multiply each of the following numbers by .075:

(1) 3236 (2) 3377.8 (3) 5256.35

13. Multiply each of the following by .125:

(1) 2240 (2) 1709.58 (3) 5054.85 (4) 5735.75

14. A steer weighing 16.22 cwt. sold at $8.85 per hundredweight. What price did it bring?

15. A passenger train ran for 12.27 hr. at the rate of 55.75 mi. per hour. How far did it run during the time?

DIVIDING DECIMALS

§97. Moving the Decimal Point.

1. Solve and express the quotients in decimal form:

(1) $253 \div 10 = ?$

$10)\overline{253}$

25.3

(2) $29 \div 10 = ?$

(3) $907 \div 10 = ?$

(4) $83 \div 10 = ?$

(5) $253 \div 100 = ?$

$100)\overline{253}$

2.53

(6) $843 \div 100 = ?$

(7) $2601 \div 100 = ?$

(8) $277 \div 100 = ?$

To divide a number by 10, 100, 1000, etc., move the decimal point 1, 2, 3, etc., places to the left.

Notice that moving the decimal point to the left means dividing; moving it to the right means multiplying.

§98. Dividing by a Whole Number; Pointing the Quotient.

1. I paid \$941.25 for 15 A. of land; what was the price per acre?

Convenient Form	
With Integers	With Decimals
6275	62.75
15)94125	15)941.25
90	90
41	41.
30	30.
112	11.2
105	10.5
75	.75
75	.75

Compare the steps in the work with decimals with the corresponding steps in the work with whole numbers.

How do the decimal points stand through the problem? How does the point stand in the quotient?

One way to point the quotient is to place the point of the quotient directly above the point in the dividend.

Check: $15 \times 62.75 = 941.25$.

Ans. \$62.75.

2. The 36 members of a society were assessed (made to pay) equally to meet a debt of \$1341 against the society. What was the amount each member was assessed?

Solution:

37.25
36)1341.00
108
261
252
9.0
7.2
1.80
1.80

By writing zeros after the decimal point the quotient may be carried out in a decimal form.

What effect does writing zeros after the point have on a number?

Another way to point the quotient is: Imagine $36\overline{)1341}$ changed to $3.6\overline{)134.1}$. Disregarding the fraction in divisor and dividend, we have $\frac{1}{3}$ of 134 is about 44. Point 3725 to get as nearly 44 as possible, thus: \$37.25.

3. I paid \$2823.75 for 45 A. of land. What was the price paid per acre?

§99. Dividing by a Decimal; Pointing the Quotient.

Answer all you can orally

1. How do the following numbers compare in size: 4320, 432, 43.2, 4.32, and .432?

2. Where is the decimal point supposed to be in 36? (See query p. 140.) In 125? In 6? In 12? In 6829?

3. Read the following horizontally:

(1) $366 \div 6 = 61$	$36.6 \div 6 = 6.1$	$.366 \div 6 = .061$
(2) $496 \div 8 = 62$	$49.6 \div 8 = 6.2$	$.496 \div 6 = .062$
(3) $242 \div 11 = 22$	$24.2 \div 11 = 2.2$	$.242 \div 11 = .022$

How is the quotient changed in value by keeping a fixed divisor and moving the decimal point in the dividend toward the left 1 place? 2 places? 3 places? 4 places?

How is the quotient affected by keeping a fixed divisor and changing the decimal point in the dividend toward the right 1 place? 2 places? 3 places? 4 places?

4. Notice the dividend, the divisor, and the quotient in the following:

$\begin{array}{r} 10 \\ 1000\overline{)10000} \end{array}$	$\begin{array}{r} 100 \\ 100\overline{)10000} \end{array}$	$\begin{array}{r} 1000 \\ 10\overline{)10000} \end{array}$	$\begin{array}{r} 10000 \\ 1\overline{)10000} \end{array}$
$\begin{array}{r} 4 \\ 1111\overline{)4444} \end{array}$	$\begin{array}{r} 40 \\ 111.1\overline{)4444} \end{array}$	$\begin{array}{r} 400 \\ 11.11\overline{)4444} \end{array}$	$\begin{array}{r} 4000 \\ 1.111\overline{)4444} \end{array}$
$\begin{array}{r} 2 \\ 303\overline{)606} \end{array}$	$\begin{array}{r} 20 \\ 30.3\overline{)606} \end{array}$	$\begin{array}{r} 200 \\ 3.03\overline{)606} \end{array}$	$\begin{array}{r} 2000 \\ .303\overline{)606} \end{array}$

How does keeping the same dividend and moving the decimal point in the divisor toward the left 1 place change the quotient? 2 places? 3 places? 4 places?

With the same dividend what would be the effect on the quotient by moving the decimal point in the divisor toward the right 1 place? 2 places? 3 places? 4 places?

5. Answer the following questions:

$\frac{360}{120} = ?$ $\frac{36}{12} = ?$ $\frac{3600}{1200} = ?$ $\frac{3 \cdot 6}{1 \cdot 2} = ?$

$\frac{360}{120}$ is another way of writing "Divide 360 by 120."

6. How, then, does it affect the quotient to move the decimal point toward the left, or right in both dividend and divisor 1 place? 2 places? 3 places? 4 places?

Hence, to point the quotient in division of decimals: (1) *divide through as though both dividend and divisor were whole numbers;* (2) *move the point in both dividend and divisor each the same number of places toward the right, or the left, until there is but one figure to the left of the point of the divisor. Then disregard the decimal parts and so point the quotient as to get about* $\frac{1}{2}$, $\frac{1}{3}$, *etc., to* $\frac{1}{9}$ *of the modified dividend.*

7. Divide 10.1388 by .426.

```
   23.8
.426)10.1388
     852
    1618
    1278
     3408
     3408
```

Dividing as with whole numbers, the digits of the quotient are 238. Imagine the point moved in both 10.1388 and .426, to make 101.388 ÷ 4.26. Then $\frac{1}{4}$ of 101 is about 25. Point 238 to get as near 25 as possible. Thus: 23.8. Ans. 23.8.

Notice that this way of pointing amounts to making the number of decimal figures in the quotient the difference between the number of decimal figures in the dividend and the divisor.

8. Find the following quotients, pointing as described:

(1) 1503.36 ÷ 46.4 = ?
(2) 150.66 ÷ .324 = ?
(3) 118.9026 ÷ .873 = ?
(4) 8.3955 ÷ .087 = ?
(5) 2545.03 ÷ 829 = ?
(6) 16.333 ÷ .0035 = ?

9. Find the following quotients:

(1) 2152.96 ÷ 23.2 = ?
(2) 4596.84 ÷ 226 = ?
(3) 1422.45 ÷ 32.7 = ?
(4) 1295.538 ÷ 52.6 = ?
(5) 40195.6 ÷ 3170 = ?
(6) 692.224 ÷ 41.6 = ?
(7) 33.4236 ÷ .865 = ?
(8) 2.62272 ÷ .0384 = ?

10. The number of states and the total areas in square miles of the land surface of the principal geographic divisions of continental United States are given in the following table. Find the average land surface of a

state to the nearest .001 for each division and for the whole United States.

Division	Number	Land Surface	Average
North Atlantic.....	9	168,665	
South Atlantic.....	9	282,535	
North Central.....	12	765,855	
South Central.....	8	620,290	
Western.....	11	1,187,535	
Continental U. S.....			

§100. Exercises and Problems.

1. Divide 78.975 by 32.5.

Check by comparing the product of the quotient and divisor with 78.975.

2. How long will it take a train running at an average speed of 42.3 mi. an hour to run 2707.2 miles.

3. The total yield of a wheat field was 285.36 bu. and the field averaged 23.2 bu. per acre. How many acres were there in the field?

4. Solve the following:

(1) $89.76 \div 26.4 = ?$
(2) $89.6002 \div 24.82 = ?$
(3) $3.234 \div 24.5 = ?$
(4) $135.88 \div 21.5 = ?$
(5) $3.234 \div .132 = ?$
(6) $9.456 \div .24 = ?$
(7) $524.660 \div 28.36 = ?$
(8) $135.88 \div 6.32 = ?$
(9) $9.456 \div 39.4 = ?$
(10) $409.625 \div 56.5 = ?$

5. Check the results of problem 4 by multiplying the quotients by the divisors and comparing the products with the dividends.

If the next decimal place after the last one required is less than 5, write the given quotient as the required decimal. If it is greater than 5, add 1 to the last figure of the decimal required. If it is exactly 5, increase the last required figure by 1, or leave it unchanged according as is necessary to make the last required decimal figure *even*. This rule applies to all cases of division of decimals.

No.	Weight	Price
1	1503	$9.00
2	1622	8.85
3	1606	8.60
4	1524	8.50
5	1524	8.50
6	1504	8.10
7	936	8.70
8	1327	6.85
9	1273	8.05
10	1130	8.75
11	1326	7.65
12	1141	8.50
Total. Aver...		

6. On Dec. 6, 1909, a herd of 12 prize cattle of weights given in the table sold at the indicated prices per cwt. What was the total weight of the herd? The average weight of the animals?

7. For how much did the entire herd sell?

8. What was the average price per hundredweight?

9. What was the average price per head which the owner received for the herd?

10. The previous year, a carload of 26 prize-winning Angus cattle, averaging 1492 lb., sold at $15.50 per hundredweight. What average price did the cattle bring the owner?

11. How much did the owner receive for the carload of problem 10?

The following table gives, for men, of ages from 18 to 26 years, the average lung capacity in cubic inches, the height in inches, and the weight in pounds.

Age	Lung Capacity	Height	Weight
18	251.4	68.2	134.25
19	251.8	68.2	135.40
20	258.2	68.1	138.65
21	260.4	68.1	140.60
22	264.8	68.2	141.15
23	263.7	68.1	138.60
24	267.1	68.2	143.90
25	267.2	68.2	143.15
26	267.1	68.9	142.30

12. Find the number of cubic inches of lung capacity per inch of height, for each age, by computing (finding) the ratio to 2 decimal places of each number of column 2 to the corresponding number of column 3. Is this ratio the same for all ages?

13. Similarly find the number of cubic inches, to 2 decimal places, of lung capacity per pound of weight, for each age.

14. For each age find the number of pounds of weight per inch of height to 2 decimal places. Are these numbers the same for all ages?

§101. Original Problems.

Make and solve problems based on the following facts:

1. 1 cu. ft. is about .8 of a bushel of small grain. A grain bin is $8' \times 12' \times 22'$.

2. 1 bu. of ear corn is about 2.25 cubic feet. A wagon box is $2.67' \times 2.85' \times 9.6'$.

3. Well settled timothy hay runs about 355.25 cu. ft. to the ton. A hay shed is $24' \times 40'$ and is filled with settled hay to a height of 18.5′.

4. Loose timothy hay runs about 460 cu. ft. to the ton. A load of loose hay is $8' \times 18' \times 22.7'$.

5. Stove coal runs about 35.1 cu. ft. to the ton. A coal bin is $6' \times 12.5' \times 15.35'$.

6. 1 perch of stone is 24.75 cubic feet. A stone wall is $2.75' \times 4.385' \times 126.8'$.

7. 30.25 sq. yd. make 1 sq. rd. A town lot is 60.5 yd. × 42.36 yards.

8. 272.25 sq. ft make 1 sq. rd. A field is 408.375 ft. × 1361.25 feet.

9. A man walks about 3.5 mi. per hour. It is 85 mi. from Chicago to Milwaukee.

10. A horse trots about 7.5 mi. per hour.

11. A horse runs about 18 mi. per hour for short distances.

12. A steamboat runs 18 mi. per hour.

13. A slow river flows 3 mi. per hour.

14. A rapid river flows 7 mi. per hour.

15. A crow flies 25 mi. per hour; a falcon, 75 mi.; a wild duck, 90 mi.; a sparrow, 92 mi.; a hawk, 150 mi., and a carrier pigeon, 80 mi. per hour.

16. Sound travels through air 1134 ft. per second; through water, 5000 ft. per second; and through iron or steel, 17,000 ft. per second.

17. A rifle ball travels 1460 ft. per second at starting; and a 20-lb. cannon ball, 1600 ft. per second at starting.

18. Light travels 186,650 mi. per second; electricity, 288,000 mi. per second. The sun is about 93,000,000 mi. from the earth; the moon, about 240,000 miles.

19. One horse-power raises 33,000 lb. through a height of 1 ft. in 1 minute.

20. The equatorial diameter of the earth is 7925.6 miles, the polar diameter, 7899.1 miles.

§102. Reducing a Common Fraction to a Decimal.

1. Express $\frac{5}{16}$ decimally.

Annex zeros to the right of the decimal point after the numerator, and then divide by the denominator.

$$\begin{array}{r} .3125 \\ 16)\overline{5.0000} \\ 4.8 \\ \hline .20 \\ .16 \\ \hline 40 \\ 32 \\ \hline 80 \\ 80 \\ \hline \end{array}$$

Ans. $\frac{5}{16} = .3125$

2. Express the following common fractions decimally (to 3 decimal places):

$\frac{3}{4}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{7}{8}$ $\frac{5}{8}$ $\frac{3}{16}$ $\frac{5}{32}$ $\frac{14}{64}$

3. Express the following mixed numbers decimally (to 3 places):

$1\frac{1}{2}$ $3\frac{1}{5}$ $2\frac{3}{4}$ $6\frac{5}{8}$ $13\frac{5}{16}$ $18\frac{13}{32}$

4. Express the following decimally to 5 decimal places:

$\frac{1}{7}$ $\frac{3}{13}$ $\frac{8}{17}$ $\frac{9}{23}$ $\frac{16}{37}$ $\frac{5}{19}$

Such decimals as these fractions give rise to, that do not terminate, are called non-terminating decimals.

5. Express the following numbers decimally to 6 decimal places:

$\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{9}$ $\frac{2}{9}$ $\frac{7}{9}$ $\frac{4}{11}$ $\frac{6}{11}$ $\frac{9}{11}$ $\frac{10}{27}$ $\frac{11}{27}$ $\frac{20}{27}$

Such non-terminating decimals as these that repeat the same digit or group of digits indefinitely, are called repetends, or circulating decimals, or circulates.

6. Express the values of these numbers by the use of integers and decimals only:

$1.3\frac{1}{2}$	$16.87\frac{1}{2}$	$5.28\frac{3}{5}$	$17.9\frac{5}{8}$	$20.0\frac{2}{5}$
$1.00\frac{4}{5}$	$.00\frac{7}{8}$	$.0\frac{7}{8}$	$.000\frac{1}{16}$	$30.060\frac{3}{5}$

GENERAL REVIEW OF DECIMALS

§103. Exercises for Practice.

1. Add the following:

(1)	(2)	(3)	(4)	(5)	(6)
37.8	249.001	.96	7.68	862.01	328.
8.65	16.25	3.004	86.043	2.8106	.032
12.08	9.018	10.	190.608	843.7	$7.001\frac{1}{2}$
.75	24.	7.	9.	1200.	$29.080\frac{3}{4}$
16.	3.	.206	.9	.16	109.
14.01	.03	200.	8.3	9.	9000.
.6	.4	860.03	60.5	.0068	.08
10.	17.003	.2	3.7	.0309	.0007

2. Subtract the following:

(1)	(2)	(3)	(4)	(5)	(6)
84.96	98.01	100.01	106.038	67.083	800.00
6.38	1.97	87.08	69.879	29.187	98.68

(7)	(8)	(9)	(10)	(11)	(12)
810.02	3.786	26.	3.	68.	1.0101
86.983	.989	.748	2.983	9.8789	.9098

3. Find the following products:

(1) 89.3×42.1
(2) 68.01×9.82
(3) 86.3×8.761
(4) 100.001×86.8
(5) $76.9 \times .93$
(6) $7.69 \times .093$
(7) $8.08 \times .008$
(8) $16.8 \times .072$
(9) 9.001×1.801
(10) 1.68×7.2
(11) $168 \times .072$
(12) 3.1416×25.18
(13) 31.416×2.518
(14) 314.16×251.8
(15) 785.4×2.781
(16) $.7854 \times .2781$
(17) 3.1416×16.84
(18) $314.16 \times .1684$

4. Find the following quotients:

(1) $120.716 \div 26.82$
(2) $123.1498 \div 26.82$
(3) $276.496 \div 3.142$
(4) $268.928 \div 8.4$
(5) $109.624 \div 3.86$
(6) $548.12 \div 14.2$
(7) $1774.08 \div .384$
(8) $8.8704 \div 192$
(9) $443.52 \div 96$
(10) $14.784 \div .032$
(11) $.3696 \div .0008$
(12) $9.24 \div .00004$

5. Find the following quotients to 3 decimal places:

(1) $8 \div .3$
(2) $631 \div .072$
(3) $9 \div .077$
(4) $11 \div .0009$
(5) $12 \div 36$
(6) $1 \div 3$
(7) $2 \div 3$
(8) $4 \div 9$
(9) $44 \div .09$
(10) $13 \div 3$
(11) $7.7 \div 9$
(12) $123 \div 9.99$
(13) $4 \div .011$
(14) $2.5 \times .033$
(15) $260 \div 3.3$
(16) $2.6 \div 33$
(17) $11.5 \div 3.3$
(18) $10.4 \div .909$
(19) $1.74 \div 9.09$
(20) $.77 \div 1.21$
(21) $1.111 \div .099$
(22) $60.6 \div 1.818$
(23) $.625 \div 3.36$
(24) $.1 \div .09$

6. Express the following fractions decimally to 4 places:

$\frac{11}{12}$ $\frac{10}{13}$ $\frac{7}{19}$ $\frac{3}{11}$ $4\frac{2}{9}$ $\frac{3\frac{1}{2}}{7\frac{1}{2}}$ $\frac{6.7}{7.8}$ $\frac{8.65}{12.17}$ $\frac{7\frac{3}{5}}{12\frac{7}{9}}$

§104. Applications to Area.

1. The distance around the earth at the equator is about 24,900 miles. The area of Indiana is 36,354 square miles. If the area of the state were formed into strips 1 mile wide, and these strips laid end to end, how many times around the earth would they reach?

2. If the state were formed into strips 6 miles wide, how far would they reach if laid end to end? What part of the distance around the earth at the equator? (Answer to two decimal places.)

3. If the state were formed into strips 100 miles wide, how far in miles (to 2 places) would they reach, laid end to end?

4. How long a strip 166 miles wide would have an area equal to that of Indiana?

5. How long a strip would have an area equal to that of Indiana, if the strip was (1) 73 miles wide? (2) 83 miles wide? (3) 219 miles wide?

6. What is the area of a parallelogram, or rectangle, 240 miles long and $151\frac{19}{40}$ miles wide?

7. What is the area of a square that is $190\frac{2}{3}$ miles long and wide?

8. Prove by multiplying that 6 squares of dimensions 77.839 miles, are nearly equal to the area of Indiana.

9. Prove by multiplying that the area of Nebraska, 77,520 sq. mi., equals that of each of the following rectangles:

(1)	2 mi. × 38,760 mi.	(6)	200 mi. × $387\frac{3}{5}$ mi.
(2)	20 mi. × 3876 mi.	(7)	240 mi. × 323 mi.
(3)	50 mi. × $1550\frac{2}{5}$ mi.	(8)	250 mi. × $310\frac{2}{2\,5}$ mi.
(4)	100 mi. × $775\frac{1}{5}$ mi.	(9)	$237\frac{1\,7}{4\,0}$ mi. × $237\frac{1\,7}{4\,0}$ mi.
(5)	150 mi. × $516\frac{4}{5}$ mi	(10)	300 mi. × $258\frac{2}{5}$ mi.

10. What sort of rectangle is the (9)th of the 9th? Find its perimeter.

11. Prove that the area of Oklahoma, 70,057 sq. mi., equals rectangles of the following dimensions:

(1) 3 mi. $\times$ 23,352$\frac{1}{3}$ mi. (5) 150 mi. $\times$ 467$\frac{7}{150}$ mi.
(2) 10 mi. $\times$ 7005$\frac{7}{10}$ mi. (6) 200 mi. $\times$ 350$\frac{57}{200}$ mi.
(3) 50 mi. $\times$ 1401$\frac{7}{50}$ mi. (7) 250 mi. $\times$ 280$\frac{57}{250}$ mi.
(4) 100 mi. $\times$ 700$\frac{57}{100}$ mi. (8) 264.683 mi. $\times$ 264.683 mi.

12. What sort of rectangle is the 8th? Find its perimeter.

13. Prove that the area of Colorado, 103,948 sq. mi., equals in area the following rectangles:

(1) 4 mi. $\times$ 25,987 mi. (4) 240 mi. $\times$ 433$\frac{7}{60}$ mi.
(2) 50 mi. $\times$ 2078$\frac{24}{25}$ mi. (5) 300 mi. $\times$ 346$\frac{37}{75}$ mi.
(3) 100 mi. $\times$ 1039$\frac{12}{25}$ mi. (6) 322.41 mi. $\times$ 322.41 mi.

Find the perimeter of the 6th rectangle.

14. Prove that the area of Missouri, 69,420 sq. mi., equals in area, nearly or exactly, any one of the following rectangles:

(1) 10 mi. $\times$ 6942 mi. (4) 250 mi. $\times$ 277$\frac{17}{25}$ mi.
(2) 100 mi. $\times$ 694$\frac{1}{5}$ mi. (5) 300 mi. $\times$ 231$\frac{2}{5}$ mi.
(3) 200 mi. $\times$ 347$\frac{1}{10}$ mi. (6) 263.47$\frac{27}{40}$ mi. $\times$ 263.47$\frac{27}{40}$ mi.

Find the perimeter of the 6th rectangle.

§105. Applications to Farming.

1. A man's time is worth 20¢ per hour and each horse's time is worth 10¢ per hour. What does it cost per day (10 hr.) for the labor of one man and 4 horses?

2. If a man and 4 horses harrow 35 acres per day, what does it cost the farmer per acre to harrow his land?

3. A man's time is worth 20¢ per hour and each horse's time is worth 10¢ per hour. What is it worth per acre to cut the weeds out of a 60-acre pasture if it takes a man and a two-horse team 1 hour to cut them from 1.2 acres?

4. There were 3200 hills of corn on an acre. Morning glories twined about and destroyed 100 of the hills. What part of the crop was thus destroyed?

5. If this acre of corn yielded 62 bu., how much would it have yielded, if the 100 hills had not been destroyed?

6. A farmer raises 160 acres of corn. 100 hills out of each acre of 3200 hills are destroyed by weeds. How many acres of corn are destroyed by weeds? What is his loss if his yield is 64 bu. per acre at present price of corn?

7. A farmer decides to prepare a field by plowing it 4 times and disking it 15 times. What will it cost per acre to do this, if each plowing costs $1.65 per acre and each disking costs 40¢ per acre?

8. If soil is cultivated to the depth of 3.5 in., how many cubic feet of soil per acre are cultivated? How many, if cultivated to the depth of 7 in.? If cultivated to the depth of 8.5 in.?

9. How much more plant food is made available with cultivation to the depth of 7 in. than with a 3.5 in. depth of cultivation?

10. If a man and a team can plow $1\frac{1}{2}$ acres 7 in. deep, or 2 acres 3.5 in. deep, in a day, how much more does it cost per acre to plow land 7 in. deep than to plow it only 3.5 in. deep, labor being worth $4.00 per day?

11. If a man and team can till thoroughly 4 acres in a day, or 6 acres in a careless manner, how much more per acre does good work cost, labor being worth $4.00 per day?

12. How much more per acre does it cost to plow deeply and to till well, problems 10 and 11? How many additional bushels of wheat worth 90¢ per bu. will it take to pay for the additional labor per acre?

13. How much will be the gain if but 16 bu. of wheat per acre can be raised with shallow plowing and careless seeding, and 31 bu. with more careful work? How much will the increased wheat yield be worth at present price of wheat?

14. A field yields 25 bu. of corn per acre. By careful cultivation the farmer increases this yield to 45 bushels per acre. With corn worth 40¢ per bu. how many additional days' labor at $4.00 per day will the extra yield pay for?

15. If he spends but 10 days' extra time on his 30-acre field of corn to produce the increase in crop, problem 14, how much does he get per day for his extra time?

16. Suppose a farmer is able to double the average yield of 120 bu. of potatoes from an acre of land by putting 10 days' extra time on it. What wages does he get with potatoes at the present price?

In a certain community, the cost of drainage tile is as follows:

3 in. tile cost about 3¢ each.
4 in. tile cost about 4¢ each.
5 in. tile cost about 5¢ each.
6 in. tile cost about 6¢ each.
All sizes are 12 inches in length.

17. A farmer owns a piece of low ground 80 rods long and 40 rods wide. How many acres in this piece?

18. If he puts the tiles crosswise of the field, 4 rods apart, so that they open into the creek that runs lengthwise

of the land, how many rods of tiling will it take? How many tiles?

19. What will be the cost of these tiles according to the table if the 5 in. size is used?

20. What will it cost to dig the ditches and lay the tiles at 40 cents per rod?

21. What will be the entire cost if 5 in. tiles are used? 6 in. tiles?

22. What will be the cost per acre for each of the four sizes of tile?

23. If the farmer can grow only $1\frac{1}{2}$ tons per acre, of marsh hay worth \$5.00 per ton on this land before draining and can grow 50 bu. of corn per acre, worth 40¢ per bu., after draining, what is the increase in the value of the crop due to drainage?

24. In how many years will this increase alone pay for the tiling, if 5 in. tile are used?

25. If a clean field produces 50 bu. of corn per acre and a weedy field only 30 bu. per acre, what is the loss per acre caused by weeds when corn is selling at 40 cents per bushel?

26. What would be the loss on a 40-acre field at the same rate?

27. How many days' labor, at \$1.50 per day, will equal this loss?

28. A farmer raised 300 bushels of potatoes per acre. The land was well cultivated and kept free from weeds to produce this crop. If because of lack of labor but 120 bushels were grown instead, what would be the money loss from lack of labor at present price per bushel?

COMPOUND DENOMINATE NUMBERS

§106. Definitions.

A DENOMINATE NUMBER is a number whose unit is concrete; as, 13 mi., 8 hr., 40A., $125, etc.

A concrete unit is a unit having a specific name; as, 1 yd., 1 lb., $1, 1 hat, 1 horse, etc.

A compound denominate number is a number expressed in two or more units of the same kind; as, 11 hr., 25 min., 15 sec.; 12 gal. 3 qt. 1 pt. 3 gills.

TABLES OF MEASURES

Read carefully the tables that follow and fix them in mind by solving the problems beginning on page 168.

§107. Measures of Value.

The *standards of value* of the United States are here given.

TABLE OF U. S. MONEY

10	mills (m.)	= 1 cent (¢)	5	dollars	= ½ eagle
10	cents	= 1 dime (d.)	2½	dollars	= ¼ eagle
10	dimes	= 1 dollar	20	dollars	= 1 double eagle
10	dollars	= 1 eagle			

United States coins are bronze, nickel, silver, and gold.

§108. Measures of Weight.

Three systems of weight units are used in the United States, viz.: troy, avoirdupois, and apothecaries'.

Troy weight is used in weighing gold, silver, and jewels. Avoirdupois weight is used for ordinary articles. Apothecaries' weight is used by druggists in mixing medicines.

The troy pound is $\frac{12}{16}$ of the avoirdupois pound. The grain is the same in all three systems.

1 lb. (av.) = 7000 grains
1 lb. (troy) = 5760 grains

TABLE OF AVOIRDUPOIS WEIGHT

7000 grains (gr.)	= 1 pound (lb.)
16 ounces	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 long ton (L. T.)

§109. Measures of Length, or Distance (Linear Measure).

The standard unit for common measures of length, or distance, is the *yard.*

TABLE OF COMMON LINEAR MEASURE

12 inches (in.)	= 1 foot (ft.)	
3 feet	= 1 yard (yd.)	
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ ft.	= 1 rod (rd.)	
320 rods	= 1 statute mile (mi.)	

TABLE OF EQUIVALENTS

63,360 in.	= 1 mi.
5280 ft.	
1760 yd.	
320 rd.	

1.15 (about) common, or statute, miles = 1 geographical, or nautical, mile.

§110. Measures of Surface.

The unit upon which surface measure is based is the *square yard,* which is a square each of whose sides equals 1 yard.

TABLE OF COMMON SURFACE MEASURE

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$	square yards	= 1 square rod (sq. rd.)
160	square rods	= 1 acre (A.)

§111. Measures of Volume.

TABLE OF CUBIC MEASURE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
$24\frac{3}{4}$ cubic feet	= 1 perch

Fire wood is measured by the *cord.* A cord of wood is a straight pile, $4' \times 4' \times 8'$, or 128 cubic feet. A *cord foot* is a straight pile of wood, $4' \times 4' \times 1'$. How many cubic feet are there in a cord foot?

§112. Measures of Capacity.

Liquid measure is used in measuring liquids, the capacity of cisterns, tanks, etc.

Table of Liquid Measure		Table of Equivalents	
4 gills (gi.)	= 1 pint (pt.)	32 gi.	= 1 gal.
2 pints	= 1 quart (qt.)	8 pt.	
4 quarts	= 1 gallon (gal.)	4 qt.	

A liquid gallon contains 231 cubic inches.

Dry measure is used in measuring grain, fruit, vegetables, etc. The standard of dry measure is the *Winchester bushel*, which is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep, containing 2150.42 cubic inches.

Table of Dry Measure		Table of Equivalents	
2 pints (pt.)	= 1 quart (qt.)	64 pt.	= 1 bu.
8 quarts	= 1 peck (pk.)	32 qt.	
4 pecks	= 1 bushel (bu.)	4 pk.	

A dry gallon = 4 qt. or $\frac{1}{2}$ pk., or 268.8 cubic inches.

§113. Measures of Time.

The standard unit for measuring time is the *mean solar day*. The mean solar day is the average time interval from the instant when the sun crosses the meridian of a place (noon) to the next instant of crossing the meridian (the next noon).

Table of Time Measure

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
30 days	= 1 calendar month (mo.)
12 calendar months	= 1 calendar year (yr.)
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

One mean solar year = 365 da. 5 hr. 48 min. 46 sec. = $365\frac{1}{4}$ da. (nearly).

According to the *Gregorian calendar*, which is now used by nearly all civilized nations, every year whose date number is divisible by 4 is a leap year, *unless the date number ends in two zeros* (as 1600), *in which case the date number must be divisible by* 400 *to be a leap year.*

The extra day of the leap year is added to February, giving this month 29 da. in leap years.

§114. Measurement by Counting.

TABLE FOR COUNTING		TABLE FOR MEASURING PAPER	
12 things	= 1 dozen (doz.)	24 sheets	= 1 quire
20 things	= 1 score	20 quires	= 1 ream
12 dozen	= 1 gross	2 reams	= 1 bundle
12 gross	= 1 great gross	5 bundles	= 1 bale

§115. Adding and Subtracting Compound Denominate Numbers.

1. What date is 2 yr. 9 mo. 21 da. after Mar. 15, 1909.

Mar. 15, 1909, may be written 1909-3-15, Mar. being the 3d month of the year.

CONVENIENT FORM

yr.	mo.	da.
1909	– 3	– 15
2	– 9	– 21
1912	– 1	– 6

15 da. + 21 da. = 36 da. = 1 mo. and 6 da. Write 6 in day column.

1 mo. + 9 mo. + 3 mo. = 13 mo. = 1 yr. and 1 mo. Write 1 in mo. col.

1 yr. + 2 yr. + 1909 yr. = 1912.

1912-1-6 may be written Jan. 6, 1912. *Ans.*

2. How long is it from Oct. 15, 1908, to May 12, 1911?

Oct. 15, 1908, may be written 1908-10-15, and May 12, 1911, thus: 1911-5-12.

CONVENIENT FORM

yr.	mo.	da.
1911	– 5	– 12
1908	–10	– 15
2	– 6	– 27

Ans. 2 yr. 6 mo. 27 da.

15 da. cannot be taken from 12 da.; but 5 mo. 12 da. = 4 mo. 42 da. 42 da. – 15 da. = 27 da. Write 27 in da. col.; 1 yr. 4 mo. = 16 mo.; 16 mo. – 10 mo. = 6 mo. Write 6 in mo. col. Then 8 from 10 leaves 2. Write 2 in yr. col.

§116. Exercises and Problems on Denominate Number Tables.

STANDARDS OF VALUE

1. How many cents are there in 23 dimes? In $23.00? In 23 eagles?

2. How many dollars are there in 248 cents? In 248 dimes? In 248 eagles?

3. How many half-eagles are there in $245.00? In 250 dimes?

WEIGHT

4. The avoirdupois table is used for weighing iron, coal, and all coarser (less costly) articles. How many grains heavier is a pound of coal than a pound of silver?

5. Express 8 pounds as ounces. Express 8 lb. as grains.

6. How many ounces are there in 24 lb. of iron? In 48 lb. 6 oz. of iron?

7. How many ounces are there in 3 cwt.? In 1 cwt. 40 lb.? In 5 cwt. 8 oz.?

8. Express as avoirdupois ounces the following avoirdupois weights:

(1) 3 lb. 8 oz. (3) 86 lb. 15 oz. (5) 8 cwt. 13 lb.
(2) 18 lb. 12 oz. (4) 1 cwt. 8 lb. (6) $9\frac{1}{2}$ cwt.

9. Express as tons the following:

(1) 40 cwt. (3) 16,000 lb. (5) 12,500 lb.
(2) 30 cwt. (4) 9000 lb. (6) 13,500 lb.

10. Express the following as long tons:

(1) 6720 lb. (3) 112 cwt. (5) 56 tons.
(2) 19,040 lb. (4) 952 cwt. (6) 39 cwt. 20 lb.

11. How many pounds are there in $\frac{1}{4}$ of a ton of coal? In $\frac{1}{2}$ of a long ton of pig iron?

LINEAR MEASURE

12. How many inches are there in a rod? How many inches are there in 17 rods?

13. Express 3 rd. $2\frac{1}{3}$ yd. in inches. Express 2 rd. 1 yd. 2 ft. in inches.

14. Express 6408 inches in rods and yards.

15. A toy railroad track is 10 yd. long. How many feet of toy rails are needed for it?

16. A 12-rod fence is made of 10-inch boards set vertically with no cracks between them. How many boards are needed?

17. The four sides of a garden are 10 rods, 1020 ft., $162\frac{1}{2}$ ft., and 1019 ft. 4 in. How many 10-inch boards will be needed to enclose it with a fence like that described in problem 16?

18. The tire of a bicycle is 88 inches long. How many times will the wheel turn in going 40 rods?

19. The tire of a wagon wheel is 15 ft. 9 in. long How many turns will the wheel make in going 5 miles?

SURFACE MEASURE

20. Express as square inches:

(1) 3 sq. ft.
(2) 3 sq. ft. 8 sq. in.
(3) 1 sq. yd.
(4) 2 sq. yd. 1 sq. ft.
(5) 6 sq. yd. 48 sq. in.
(6) $3\frac{1}{2}$ sq. yd.
(7) $45\frac{1}{4}$ sq. ft.
(8) 1 sq. rd.

21. Express:

(1) 121 sq. yd. as sq. rd.
(2) 640 sq. rd. as A.
(3) 40 A. as sq. rd.
(4) 80 sq. rd. as A.
(5) 1 sq. rd. as sq. ft.
(6) 1 A. as sq. yd.

22. How wide must a rectangle 80 rd. long be to contain 1 A.? 5 A.? 20 A.? 80 A.?

23. How wide must a rectangle 16 rd. long be to contain 1 A.? How many square rods more than 1 A. are there in a 14-rd. square?

24. How many square yards more than 1 A. are there in a 70-yd. square?

25. How many square yards less than 1 A. are there in a 69-yd. square?

26. How many square feet are there in 1 A.?

27. How many square yards more than 1 A. are there in a 213-ft. square?

CAPACITY AND VOLUME

28. Express:

(1) 2 qt. 1 pt. as pt.
(2) 1 qt. $\frac{1}{2}$ pt. as gi.
(3) 3 qt. 1 pt. 2 gi. as gi.
(4) 2 gal. as pt.
(5) 12 gal. as qt.
(6) 11 gal. 3 qt. as pt.

29. Express:

(1) 24 pt. as qt.
(2) 48 gi. as qt.
(3) 76 pt. as gal.
(4) 64 gi. as gal.
(5) 17 pt. as qt. and pt.
(6) 84 pt. as gal. and qt.

30. There are 231 cu. in. in a liquid gallon. How many cubic inches in a quart? A pint?

31. There are 537.6 cu. in. in a peck. How many more cubic inches are there in a dry than in a liquid quart?

32. Express:

(1) 3 bu. as pk.
(2) $2\frac{1}{4}$ bu. as pk.
(3) $2\frac{1}{2}$ bu. as pk.
(4) 1 bu. as dry gallons.
(5) 18 dry gallons as bu.
(6) 20 pk. as pt.

AVOIRDUPOIS POUNDS IN A BUSHEL PRESCRIBED BY VARIOUS STATES

COMMODITIES	California	Connecticut	Illinois	Indiana	Iowa	Kansas	Kentucky	Louisiana	Maine	Massachusetts	Michigan	Minnesota	Missouri	New Hampshire	New Jersey	New York	Ohio	Pennsylvania	Tennessee	Texas	Vermont	Virginia	Washington	Wisconsin
Barley	50	48	48	48	48	48	47	48	48	48	48	48	48	..	48	48	48	47	48	48	48	48	48	48
Beans	..	60	60	60	60	60	60	60	62	..	60	..	60	60	60	62	60	..	60	60	60	60	60	60
Blue Grass Seed	..	..	14	14	14	14	14	14	..	..	14	..	14	..	..	..	..	..	14	..	..	14	..	..
Buckwheat	40	48	52	50	52	50	56	..	48	48	48	50	52	..	50	48	50	48	50	42	48	52	42	48
Castor Beans	..	..	46	46	46	40	..	..	..	..	..	..	46	..	..	..	..	..	..	..	..	..	..	..
Clover Seed	..	..	60	60	60	60	60	60	..	..	60	60	60	..	64	60	60	62	..	60	60	60	60	60
Coal (Anthracite)	..	..	80	..	80	80	76	..	..	..	..	..	..	..	..	..	80	..	..	80	..	80	..	..
Corn on the Cob	..	..	70	68	70	70	70	..	..	..	70	..	..	..	..	..	70	..	70	70	..	70	..	70
Corn, Shelled	52	56	56	56	56	56	56	56	56	56	56	56	56	56	56	58	56	56	56	56	56	56	56	56
Cornmeal	..	50	48	50	..	50	50	..	50	50	50	..	..	50	..	..	..	..	50	..	..	50	..	..
Dried Apples	..	..	24	25	24	24	24	..	..	..	22	28	24	..	25	..	24	..	24	28	..	28	28	25
Dried Peaches	..	..	33	33	33	33	39	..	..	..	28	28	33	..	33	..	33	..	..	28	..	32	28	28
Flax Seed	..	..	56	..	56	56	56	..	..	..	56	..	56	..	55	55	56	..	56	56	..	56	..	56
Hemp Seed	..	..	44	44	44	44	44	..	..	..	..	..	44	..	..	..	..	..	..	44	..	..	..	..
Millet	..	..	50	..	48	50	50	..	..	..	..	..	..	..	..	..	50	..	..	50	..	50	..	55
Oats	32	32	32	32	32	32	32	32	32	32	32	32	32	32	30	32	33	30	32	32	32	32	32	32
Onions	..	50	57	48	57	57	57	..	52	52	54	..	57	..	57	..	55	..	56	57	52	57	50	57
Peas	..	60	..	..	..	..	60	60	60	..	60	..	..	60	60	60	60	..	60	..	60	60	60	..
Potatoes	..	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
Rye	54	56	56	56	56	56	56	56	50	55	56	56	56	56	56	56	56	56	56	56	56	56	56	56
Sweet Potatoes	..	..	50	..	46	50	55	..	..	54	56	..	..	..	54	..	50	..	50	55	..	56	..	..
Timotay Seed	..	..	45	45	45	45	45	45	..	..	45	..	45	..	..	44	45	..	45	45	45	45	40	45
Turnips	..	50	55	..	..	55	60	..	60	..	58	..	..	..	..	..	..	..	..	55	60	55	50	42
Wheat	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60

33. How many bushels are there in a 200-pound bag of beans in Maine? How many in New York? In other states of the above table?

34. How many bushels are there in a $1\frac{1}{2}$-ton load of shelled corn in California? In New York? In other states?

35. How many bushels in a 1-ton load of oats in New Jersey and Pennsylvania? In Ohio? In other states?

36. What is the weight of a quart of buckwheat in California? In Connecticut? In Illinois? In the various other states?

37. What is the difference between the weight of a quart of barley in California and in Illinois?

38. How many bushels are there in a long ton (2240 lb.) of anthracite in Illinois? In a short ton?

39. My coal bin is $7' \times 12' \times 12'$. How many bushels of anthracite will it hold? How many tons? (Count $1\frac{1}{4}$ cu. ft. per bushel.)

40. One cu. ft. is about what part of a Winchester bushel?

41. Express 3.5 bu. as pints; 160 pt. as bushels.

42. Find the error in the saying "A pint's a pound" if there are $7\frac{1}{2}$ gal. in a cubic foot of water weighing 1000 ounces.

TIME

43. Express:

(1) 1 hr. as sec.
(2) 1 da. as sec.
(3) 1 wk. as sec.
(4) 30 sec. as hr.
(5) 7200 sec. as hr.
(6) 604,800 sec. as da.
(7) 1,209,600 sec. as wk.
(8) 1,000,000 sec. as wk.; da. and hr.
(9) 1 wk. as minutes.

44. How many weeks in a common year? July 4 fell on Tuesday in 1905. On what day of the week did July 4, 1906, fall? Why?

45. How many months old are you this month?

46. How many days old are you today? How many hours are there in this many days?

47. A clerk earned \$15 a week for 28 calendar months. How much did he earn in all?

48. A man saved \$6 a week for 7 yr. How much did he save in all?

49. Illinois was admitted as a state in December, 1818. How many years and months old is the state of Illinois this month?

50. On the Saturdays of April, 1905, the sun rose and set at Chicago at the times here given:

DAY	DATE	SUN ROSE	SUN SET
Saturday	April 1	5:46 A.M.	6:23 P.M.
Saturday	April 8	5:34 A.M.	6:30 P.M.
Saturday	April 15	5:24 A.M.	6:37 P.M.
Saturday	April 22	5:14 A.M.	6:43 P.M.
Saturday	April 29	5:05 A.M.	6:50 P.M.

Give the length in hours and minutes of each of these 5 days "from sun to sun," and also give in minutes the change per week of the day's length during April, 1905.

51. It is noon at 12: 00. How long in hours and minutes was the forenoon of April 1, 1905? The afternoon? How many minutes longer was the afternoon than the forenoon?

52. Answer similar questions for Apr. 8; 15; 22; 29.

53. From the numbers of a common almanac compare the day's length of April with the same dates for October.

54. From the following table find how long morning twilight lasted on Jan. 1, 1905; Apr. 1; July 1; Oct. 1.

55. Answer similar questions for evening twilight.

DATE	MORNING TWILIGHT		EVENING TWILIGHT	
1905	BEGINS	ENDS	BEGINS	ENDS
Jan. 1st	5:46 A.M.	7:25 A.M.	4:42 P.M.	6:21 P.M.
April 1st	4:10 A.M.	5:45 A.M.	6:24 P.M.	7:58 P.M.
July 1st	2:27 A.M.	4:32 A.M.	7:35 P.M.	9:40 P.M.
Oct. 1st	4:26 A.M.	5:56 A.M.	5:44 P.M.	7:14 P.M.

COUNTING THINGS

56. A stationer pays 25¢ per dozen for lead pencils and sells them at 5¢ apiece. What is his profit on a gross of pencils?

57. How many are "three score and ten"? How many years are "a score of centuries"? How many months?

58. Lincoln's Gettysburg address was spoken in November, 1863. The address begins "Fourscore and seven years ago," etc. To what date did Lincoln refer?

59. What is the value of 1 bale of paper at 12¢ per quire?

60. What is the value of 3 great gross of steel pens at the rate of 2 pens for 5¢?

61. What is the value of 10 great gross boxes of matches at the rate of 2 boxes for 5¢?

§117. General Exercises.

1. Reduce 9 great gross to units.

2. How many great gross are there in 79,636 units?

3. A dealer bought 3600 lead pencils at $2.00 per gross. He sold them at 25 cents per dozen. What was his profit?

4. Bought 2 gross foot-rules at $.12 a doz.; 7 gross steel pens No. 1 at $.07 a doz.; 9 gross lead pencils at $.30 a dozen. Find the amount of my bill.

5. A stationer bought 3 gross boxes of paper, each box containing 6 reams. How many sheets did he buy?

6. A stationer sold 3 quires, 20 sheets of paper to one man; 18 quires to another; 4 reams, 15 quires to another. How many sheets did he sell in all?

7. There are 2 reams, 9 quires of paper in one package; 3 times as much in a second package, and 7 times as much in a third package as in the second package. How much paper is there in the second package? In the third?

8. How many feet are there in 78 yd.? How many inches?

9. How many inches are there in 78 yd. 2 ft. 6 inches?

10. How many yards, feet, and inches in 2838 inches?

11. How many ounces are there in 12 cwt.? In 10 pounds?

12. How many ounces are there in 12 cwt. 10 lb. 11 ounces?

13. How many cwt., lb., and oz. in 19,360 ounces?

14. Reduce 160 yd. 1 ft. 9 in. to inches.

15. Reduce 5781 in. to yards, feet, and inches.

16. Reduce 8 bu. 2 pk. 5 qt. to quarts.

17. Reduce 27 T. 18 cwt. 15 lb. to pounds.

18. Reduce 23 cu. yd. 14 cu. ft. to cubic feet.

19. A silver dollar is .9 (by weight) pure silver. What weight of silver is there in a silver dollar weighing $412\frac{1}{2}$ grains?

20. U. S. gold coins are .9 pure gold and .1 alloy. The U. S. standard five-dollar gold piece weighs 129 gr. What is the number of grains of pure gold in the standard five-dollar gold piece?

21. The 5-cent piece weighs 73.16 gr. Of the weight of the coin, .75 is copper and .25 is nickel. What is the weight of the copper in the 5-cent piece? Of the nickel?

22. The eagle weighs 258 gr. What is the weight of pure gold in the eagle?

23. How many standard 5-dollar gold pieces can be coined from 1 oz. (av.) of pure gold?

24. A boy laid by a certain sum of money each week. At the end of 1 yr, 3 mo. 2 wk. he had saved $88.50. How much did he save each week? (52 wk. = 1 yr.)

25. The mast of a ship was 78 ft. 4 in. high. During a storm .3 of it was broken off. How high was the remaining piece?

26. A man walked $\frac{1}{5}$ of the length of a breakwater, which was 1 mi. 243 rd. 5 yd. long. How far did he walk?

27. A knot, or geographic mile, equals 6080.27 ft. What is the speed in common or statute miles per hour of a vessel that runs 21 knots per hour?

28. A wheel, 12 ft. in circumference, makes how many revolutions in going $1\frac{1}{2}$ miles?

29. A telegraph wire is 14 mi. 140 rd. long, and is supported by 386 poles, which are placed at equal distances apart, a pole being at each end of the wire. How many feet apart are the poles?

30. What is the cost of 18 A. 120 sq. rd. of land at $52 per acre?

31. A man owned 14 A. of land, and sold 1428 sq. rd. How many acres did he have left?

32. If 1000 shingles are needed to cover 100 sq. ft. of roof, how many shingles are required to cover a roof 40 ft. long and 25 ft. wide?

33. A lawn tennis court 78 ft. long and 36 ft. wide is to be surrounded by a strip of sod 15 ft. wide at each end and 8 ft. wide at each side. What will the sodding cost at $.35 per square yard?

34. Find the cost of a half section of land (a section = 640 A.) at $45 per acre.

35. How many cubic inches are there in a tank containing 120 gal.? How many cubic feet?

36. How many cubic feet are there in a block of stone 5 ft. $\times$ 4 ft. $\times$ 6 inches?

37. How much must I pay for a board 16 ft. long and 8 in. wide, and 1 in. thick if lumber costs $35 per thousand board feet? (A board foot means 144 sq. in. of surface, not over 1 in. thick.)

38. I bought 3 boards 12′ long, 6″ wide; 8 boards 10′ long, 9″ wide, and 2 boards 10′ long, 12″ wide. All were 2″ thick. What did the whole cost at $30 per thousand?

39. What is the value of a straight pile of wood 16 ft. long, 8 ft. wide, and 6 ft. high, at $7.50 per cord? (128 cu. ft. = 1 cd.)

40. What is the weight of a pile of oak boards 14 ft. long, 8 ft. 4 in. wide, and 5 ft. high, at an average weight of 54 lb. per cubic foot?

41. In one year the quarries of Minnesota yielded 4,000,000 cu. ft. of sandstone. What was this worth, at $.76 a perch? (1 perch $= 24\frac{3}{4}$ cu. ft.)

42. Find the cost of building a stone wall $150' \times 10' \times 2\frac{1}{2}'$, at $3.50 a perch.

43. A bin $12' \times 8'6'' \times 5'$ is filled with wheat. What is the weight of the wheat at 60 lb. per bushel?

44. What is the value of a straight pile of pine slabs $32' \times 7' \times 4'$, at $3.25 per cord?

45. A grocer paid $4.50 for a barrel of vinegar ($31\frac{1}{2}$ gal.), and sold it at 5¢ per quart. What was his profit?

46. A jug contained 214 cu. in. of molasses. How much did it lack of containing $1\frac{1}{2}$ gallon?

47. What will 1 mi. of right of way for a railroad cost at $60 per acre, the width of the right of way being 100 feet?

48. What will be the cost per mile for railroad ties at $.45 apiece, the ties being laid one every 2 feet?

49. Find the cost of the rails for 1 mi. (single track) of the road, the rails weighing 77 lb. per linear yard and costing $35 per long ton.

50. Find the cost of fencing 1 mi. on both sides of the track, placing posts costing $.25 apiece 16 ft. from center to center, and using wire weighing 270 lb. per mile @ 4¢, and 30 lb. of staples @ 4¢. The fence is to be 4 wires high, and labor costs 21¢ per rod of fence. (Count the odd post.)

51. Reduce .865 gal. to smaller units.

.865 × 4 qt. = 3.46 qt., .46 × 2 pt. = .92 pt.; .92 × 4 gi. = 3.68 gills. *Ans.* 3 qt. 3.68 gills.

52. Reduce .168 gal. to smaller units.

53. If oil is $.11 per gallon, what will be the cost of three 42-gallon barrels of kerosene?

54. If a certain spring regularly yields 25 gal. daily, how many barrels of the capacity of $31\frac{1}{2}$ gal. would it fill in 20 days?

55. A merchant bought 16 gal. 3 qt. of syrup at 38¢ a qt. and sold 27 qt. for $12; 18 qt. for $9, and the remainder at 35¢ per quart. Did he gain or lose, and how much?

56. Reduce 2 pk. 7 qt. 1 pt. to the decimal of a bushel.

1 pt. = .5 qt. 7.5 qt. = $\frac{7.5}{8}$ pk. = .9375 pk. 2.9375 pk. = $\frac{2.9375}{4}$ bu. = .734375 bushels.

57. Reduce 3 pk. 5 qt. 1 pt. to bushels.

58. How many bushels will a bin hold that is 14 ft. long, 7 ft. 6 in. wide, and 5 ft. 8 in. high?

59. A farmer sold 6 loads of corn, each load averaging 36 bu. 2 pk. at 35 cents per bushel. What did he receive for all the corn?

60. A boy had a bushel of hickory nuts, and sold 3 pk. 7 qt. 1 pt. What fraction of a bushel had he left?

61. If 9 bu. of potatoes cost \$4.80. what is the average cost per peck?

62. A bin contained 5376.25 cu. in. of rye. How much did it lack of containing $3\frac{1}{2}$ bushels?

63. A coal dealer buys two car-loads of coal each weighing 67,200 lb., at \$4.50 a long ton, and sells it at \$5.75 a short ton. What is his gain?

64. How many bushels do 2240 lb. of buckwheat make in Kentucky? In California? (See table p. 171.)

65. How many bushels are there in 500 lb. of dried apples in Ohio? In Indiana? In Michigan? In Texas? (p. 171.)

66. How many bushels of anthracite coal make a long ton (2240 lb.) in Kentucky? In other states?

67. How many bushels of corn on the ear are there in 2 tons of 2000 lb. each in Indiana? In other states?

68. Find the difference between the number of cubic inches in the volume of a cube each edge of which is 12.9 inches long and in a Winchester bushel (p. 166).

THE METRIC SYSTEM

§118. Historical Sketch of the Metric System.

The metric system is a decimal system of weights and measures adopted by the French Government soon after the French Revolution of 1789. The aim of the system is to base all measures upon an invariable standard, and to secure the simplest possible relations between the different units of the system.

The unit of length, which is fundamental to the whole system, is called the *meter*. It was attempted to make the meter 1 ten-millionth of the length of the part of a meridian of the earth, which reaches from the equator to the pole, called a *quadrant* of the earth's meridian. The meridian of the earth was measured, and $\frac{1}{10000000}$ of the quadrant was obtained. A platinum bar equal to this length was very accurately cut and kept by the Government as the official standard of reference.

Later measurements of the earth's meridian showed the former measurement of the meridian to be incorrect. The length of the meter as obtained from the erroneous measures was, however, retained, and the length of this bar is the standard meter. From it all the other units of the system are derived.

The metric system is used for all purposes in France, and for most scientific purposes in Germany, England, and the United States. It is legalized in nearly all civilized countries.

The length of the meter is 39.37 in., or about 1.1 yards.

§119. Tables of Metric Measures.

Fig. 1 shows a scale graduated along the upper edge to 16ths of an inch, and along the lower edge to 1000ths of a meter, called *millimeters*.

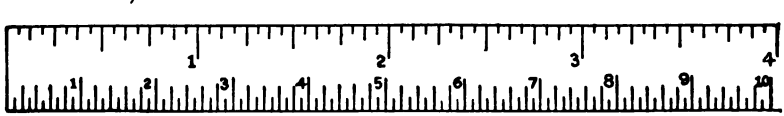


FIGURE 1

Decimal parts of the standard units are denoted by the Latin prefixes, abbreviated in each case to a single small letter:

milli, meaning 1000th, written m.
centi, meaning 100th, written c.
deci, meaning 10th, written d.

Multiples of the standard units are denoted by the Greek prefixes, abbreviated in each case to a single capital letter:

deka, meaning 10 times, written D.
hekto, meaning 100 times, written H.
kilo, meaning 1000 times, written K.
myria, meaning 10,000 times, written M.

Table of Measures of Length

10 *millimeters* (mm.)	=1 *centimeter* (cm.)	=about	.4 in.
10 centimeters	=1 decimeter (dm.)	=about	4.0 in.
10 decimeters	=1 METER (m.)	=about	1.1 yd.
10 meters	=1 dekameter (Dm.)	=about	32.8 ft.
10 dekameters	=1 hektometer (Hm.)	=about	328 ft.
10 hektometers	=1 *kilometer* (Km.)	=about	.62 mi.
10 kilometers	=1 myriameter (Mm.)	=about	6.21 mi.

Table of Surface Measure

100 *sq. millimeters* (mm².)	=1 *sq. centimeter* (cm².)
100 sq. centimeters	=1 sq. decimeter (dm².)
100 sq. decimeters	=1 SQ. METER (m².)
100 sq. meters	=1 sq. dekameter (Dm².)
100 sq. dekameters	=1 sq. hektometer (Hm².)
100 sq. hektometers	=1 *sq. kilometer* (Km².)

The cm². = .155 sq. in.
The m². = 10.764 sq. ft.
The Km². = 247.107 A.
1 m². = 1 centare (ca.)

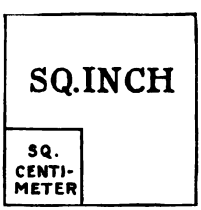


FIGURE 1

Table of Land Measure

100 centares = 1 åre (pronounced âir) (a.)
100 ares =1 *hektare* (Ha.)

Table of Measures of Volume

The standard unit of volume is the cubic meter = 35.314 cu. ft. = about 1.3 cubic yards.

1000 cubic millimeters (mm^3.)	= 1 cu. centimeter (cm^3.)
1000 cubic centimeters	= 1 cu. decimeter (dm^3.)
1000 cubic decimeters	= 1 cu. meter (m^3.)

Table of Measures of Capacity

The standard unit of capacity is the liter (pronounced leeter). It is equal to 1 cu. decimeter, and equivalent to .908 dry quarts.

10 milliliters (ml.)	= 1 *centiliter* (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 *liter* (l.)
10 liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 *hektoliter* (Hl.)
10 hektoliters	= 1 kiloliter (Kl.)

Table of Measures of Weight

The standard unit of weight is the gram, which is the weight of 1 cu. centimeter of distilled water at its temperature of greatest density (39.1° F.).

10 milligrams (mg.)	= 1 *centigram* (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 *gram* (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 *kilogram* (Kg.)
10 kilograms	= 1 myriagram (Mg.)
10 myriagrams	= 1 quintal (Q.)
10 quintals	= 1 *metric ton* (T.)

1 centigram	=	.15432 grain
1 gram	=	15.432 grains
1 kilogram	=	2.20462 lb.
1 metric ton	=	2204.621 lb.

§120. Metric and United States Equivalents.

These equivalents will be of assistance in changing the metric to the common system of measures.

1 m.	= 39.37 in.	1 mile	= 1.6093 Km.
1 Km.	= .6214 mi.	1 yard	= .9144 m.
1 m^2.	= 1.196 sq. yd.	1 square yard	= .8361 m^2.
1 Km^2.	= .3861 sq. mi.	1 square mile	= 2.59 Km^2.
1 are	= .0247 A.	1 acre	= 40.47 a.
1 m^3.	= 1.308 cu. yd.	1 cubic yard	= .7645 m^3
1 stere	= .2759 cord	1 cord	= 3.624 st.
1 liter	= 1.0567 liquid qt. .9081 dry qt.	61.022 cu. in. 1 liquid qt.	= 1 liter. = .9436 l.
1 Hl.	= 2.8376 bu. 26.417 liq. gal.	1 dry qt. 1 bushel	= 1.101 l. = .3524 Hl.
1 g.	= 15.432 grains	1 grain	= .0648 g.
1 Kg	= 2.2046 lb. av.	1 lb. avoir.	= .4536 Kg.
1 metric ton	= 1.1023 ton	1 ton	= .9072 metric ton.

§121. Problems with the Metric Units.

1. Take a smooth lath, lay a straight strip of paper beside the lower edge of the metric rule, Fig. 1, page 180, and with a sharp pencil mark the graduations from the rule to the strip of paper, and then from the strip of paper to the lath. Continue the *centimeter* graduation marks along your lath until you get a meter stick (= 100 centimeters). How does this meter stick compare in length with a yard stick?

2. Explain which digit in 6.8752 m. stands for m.; which for dm.; which for cm., and which for millimeters.

3. Find by measurement the length and the width of your schoolroom in meters, and write the result of your measurements in meters and decimals of a meter.

4. How many c. m. long, wide, and thick is your reader?

5. Express the length of your pencil in centimeters.

6. A sidewalk is 2112 m. long; how many km. long is it?

7. What is the cost of 158 cm. of ribbon at $.45 a meter?

8. How many steel rods, each 16 cm. long, can be cut from a rod 7.68 m. long?

9. From a piece of wire, 98.52 m. long, a piece .047 Km. was cut. How long was the remainder?

10. A bicycle track is .807 Km. around. How many meters would one go in riding around the track 5 times?

11. Express 1 Km^2. in Dm^2.; in square meters.

12. Find the number of m^2. in the surface of your schoolroom floor. Express the same in square dekameters.

13. Express the area of your desk-top in square decimeters; in square inches. How do the two compare?

14. Using the decimeter as a measure, find the number of square centimeters in the surface of your geography.

15. Express the same in dm^2.; in square inches.

16. Find the area of the door in square meters. What part of an *are* is this area?

17. A certain plot of ground contains 20 m^2. How many square feet are there in the plot?

18. Lay off a square meter upon the floor. Within this area, lay off a square yard. Note results.

19. With the aid of your decimeter measure, draw a square decimeter. In the corner, draw a square centimeter. How many square centimeters could be drawn within the square decimeter?

20. What is the area of a floor 8.4 m. by 5 meters?

21. A lot is 7.64 m. by .033 Km. Find the area.

22. What is the value of a piece of land 8 Dm. long and 6.5 Dm. wide, at $32 an *are*?

23. How many *ares* in a street 1.5 Dm. wide and 2.48 Km. long?

24. How many square meters are there in a floor 700 cm. long and 500 cm. wide?

25. A man had 3 pieces of land as follows: 8.4 Ha.; 3846 m^2., and 2.5 Km^2. How many Ha. of land did he have?

26. 20 liters equal how many centiliters? How many Dl.? What part of a Kl.? Find the difference between 12 l. and 12 dry quarts.

27. Using your decimeter measure as a basis, model a cubic decimeter or liter.

28. Using your square centimeter as a basis, model a cubic centimeter. What is the relation of a cubic centimeter to a gram? How many cm^3. are there in a liter?

29. How many cm^3. in the volume of your schoolroom?

30. What is the difference between an ordinary ton and a metric ton (tonneau)?

31. A gram is the weight of 1 cm^3. of distilled water. What is the weight in grams of 1 liter of distilled water?

32. A liter of water weighs nearly $2\frac{1}{5}$ lb. What is the weight of 6 l. of water in pounds? In grams?

33. A package of silver weighs 2.58 grams. What is its weight in grains?

34. How many pounds in 1 Q.? In 5 quintals?

35. How many 2 gr. capsules will 5 g. of quinine fill?

36. Find the cost of the quinine of problem 35 at 10¢ a doz. capsules.

37. What is the cost of 5500 Kg. of coal at \$6.50 per ton (2000 lb.)?

38. What is the cost of 2 Q. of sugar at $\$.08\frac{1}{2}$ a kilogram?

39. If a book weighs 3.2 Dg., how many such books will weigh 1.792 kilograms?

40. A box contains 2500 packages of quinine, each package weighing 1.25 g. How many hektograms does the contents of the box weigh?

DECIMALS LEADING TO PERCENTAGE

§122. Measuring by Hundredths.

Answer all you can orally

1. What is $\frac{1}{100}$ of 500 miles? $\frac{3}{100}$ of 500 miles? $\frac{8}{100}$ of 500 miles?

2. What is $\frac{3}{100}$ of 200 A.? $\frac{12}{100}$ of 200 acres?

3. What is $\frac{50}{100}$ of 100 feet? $\frac{37}{100}$ of 100 feet? $\frac{33\frac{1}{3}}{100}$ of 100 feet?

4. $\frac{1}{2}$ of anything equals how many hundredths of it?

One hundredth is often written "1%" and read "1 percent." The sign (%) is a short way of writing "one one-hundredth." Percent is a short way of saying "one one hundredth."

5. What is $\frac{1}{4}$ of 20? What is 25% of 20?

6. What is $\frac{3}{4}$ of 24? What is 75% of 24?

7. What is $\frac{1}{5}$ of 30? 20% of 30? $\frac{2}{5}$ of 30? 40% of 30?

8. What is $\frac{3}{5}$ of 30? 60% of 30? $\frac{4}{5}$ of 30? 80% of 30?

9. What is $\frac{1}{3}$ of 18? $33\frac{1}{3}\%$ of 18? $\frac{2}{3}$ of 18? $66\frac{2}{3}\%$ of 18?

10. What is $\frac{1}{10}$ of 40? 10% of 40? $\frac{3}{10}$ of 40? 30% of 40? $\frac{7}{10}$ of 40? 70% of 40?

11. What is $\frac{1}{20}$ of 80? 5% of 80? $\frac{3}{20}$ of 80? 15% of 80? $\frac{7}{20}$ of 80? 35% of 80?

12. What is $\frac{1}{25}$ of 50? 4% of 50? $\frac{4}{25}$ of 50? 16% of 50? $\frac{8}{25}$ of 50? 32% of 50?

13. What is $\frac{1}{50}$ of 100? 2% of 100? $\frac{3}{50}$ of 100? 6% of 100?

14. What is $\frac{1}{100}$ of 200? 1% of 200? $\frac{3}{100}$ of 200? 3% of 200?

15. How many hundredths of anything are the following fractional parts of it?

$\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$ $\frac{1}{20}$ $\frac{1}{25}$ $\frac{3}{20}$ $\frac{8}{25}$

16. Read and give the meaning of the following:

2% 6% 8% 12% $12\frac{1}{2}\%$ 25% $33\frac{1}{3}\%$ $87\frac{1}{2}\%$ 100%

§123. Decimals as Percents.

Answer all you can orally

1. What is .10 of 20? What is 10% of 20?

2. What is .20 of 20? What is 20% of 20?

3. What is .40 of 10? What is 40% of 10?

4. What is .60 of 40? What is 60% of 40?

5. What is .25 of 100? What is 25% of 100?

6. What is .25 of 200? What is 25% of 200?

7. What is .01 of 100? Of 200?

8. What is 1% of 100? Of 200?

9. What is .02 of 100? Of 200?

10. What is 2% of 100? Of 200?

11. Write in percents the equivalents of the following decimals:

.15 .25 .75 .18 $.12\frac{1}{2}$ $.33\frac{1}{3}$ $.66\frac{2}{3}$ $.37\frac{1}{2}$ 1.13

12. Write in the form of decimal fractions the equivalents of the following percents:

6% 10% 15% 16% 20% 37% $37\frac{1}{2}\%$ $66\frac{2}{3}\%$

13. From the above examples tell how to change decimals to percents; percents to decimals.

14. Compare your statements with:

To change decimals to percents move the decimal point two places to the right and write % after the number. To change percents to decimals, drop the % sign and move the decimal point 2 places to the left.

15. What is .01 of 80? .02 of 80? .03 of 80? .10 of 80?

16. What is 1% of 80? 2% of 80? 3% of 80? 10% of 80?

17. What is .12 of 100? .12 of 200? .12 of 500?

18. What is 12% of 100? 12% of 200? 12% of 500?

§124. Equivalent Parts, Decimals, and Percents.

Answer all you can orally

1. What is $\frac{1}{4}$ of 12? .25 of 12? 25% of 12?

2. What is $\frac{3}{4}$ of 12? .75 of 12? 75% of 12?

3. What is $\frac{1}{2}$ of 10? .50 of 10? 50% of 10?

4. What is $\frac{1}{3}$ of 15? $.33\frac{1}{3}$ of 15? $33\frac{1}{3}\%$ of 15?

5. What is $\frac{2}{3}$ of 15? $.66\frac{2}{3}$ of 15? $66\frac{2}{3}\%$ of 15?

6. What is $\frac{1}{5}$ of 30? .20 of 30? 20% of 30?

7. What is $\frac{2}{5}$ of 30? .40 of 30? 40% of 30?

8. What is $\frac{3}{5}$ of 30? .60 of 30? 60% of 30?

9. What is $\frac{4}{5}$ of 20? .80 of 20? 80% of 20?

10. What is $\frac{1}{8}$ of 16? $.12\frac{1}{2}$ of 16? $12\frac{1}{2}\%$ of 16?

11. Express in three ways: $\frac{1}{4}$ of 12; $\frac{3}{4}$ of 12; $\frac{1}{10}$ of 20; .2 of 25; .5 of 7; 25% of 6; $33\frac{1}{3}\%$ of 21; .125 of 16.

12. What is $\frac{1}{8}$ of 24? .125 of 24? 12.5% of 24?

13. What is $\frac{3}{8}$ of 16? .375 of 16? 37.5% of 16?

14. What is $\frac{5}{8}$ of 32? .625 of 32? 62.5% of 32?

15. What is $\frac{1}{16}$ of 32? $.06\frac{1}{4}$ of 32? $6\frac{1}{4}\%$ of 32?

§125. Finding the Percentage.

Answer all you can orally

1. What is 50% of 4? Of 12? Of 18? Of 24? Of 36?

2. What is 25% of 8? Of 12? Of 20? Of 32? Of 44?

3. What is 20% of 5? Of 15? Of 20? Of 35?

4. What is $33\frac{1}{3}\%$ of 3? Of 15? Of 21? Of 27?

5. What is 10% of 20? Of 60? Of 80? Of 90?

6. What is 4% of 25? Of 50? Of 100? Of 75?

7. What is 5% of 20? Of 40? Of 60? Of 80?

8. How many pounds are 50% of 8 pounds? Of 18 pounds? Of 24 pounds?

9. How many square feet are 25% of 16 square feet? Of 48 square feet? Of 88 square feet? Of 400 square feet?

10. A man set out 150 fruit-trees and 28% of them died. How many died?

Finding the percentage is the same as finding the product in multiplication of decimals. [$150 \times .28 = (?)$]

150 Base (Multiplicand)
.28 Rate (Multiplier)
1200
300
42.00 Percentage (Product)

The number on which the percentage is to be found is the base.
The rate is the percent to be found.
The percentage is a certain number of hundredths of the base.

IN DECIMALS: multiplicand corresponds to *base*; multiplier corresponds to *rate*; product corresponds to *percentage* — IN PERCENTAGE

11. Find the percentage in the following exercises:

(1) 12% of 35
(2) 18% of 65
(3) 30% of 68
(4) 45% of 90
(5) 24% of 45
(6) 36% of 80
(7) 75% of 650
(8) 95% of 1200
(9) 85% of 1000
(10) 85% of 150
(11) $25\frac{1}{2}\%$ of 600
(12) $37\frac{1}{2}\%$ of 800

12. Heating an iron rod 100 in. long increased its length 1%. How many inches was the length increased?

13. One boy threw a stone 100 ft., and another threw it 12% farther. How many feet farther did the second boy throw the stone?

14. Draw a square and divide it by a line so as to show 50% of it; 25% of it; 75% of it; $12\frac{1}{2}$% of it.

15. Show the same percents using a circle.

16. Draw a line of any length and show $33\frac{1}{3}$% of it; $66\frac{2}{3}$% of it; 100% of it; $12\frac{1}{2}$% of it; 10% of it; 20% of it.

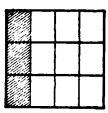


FIGURE 1

17. Draw a square and divide it into small rectangles as shown in the figure. Point out the following percents of it:
25% $33\frac{1}{3}$% 50% $66\frac{2}{3}$% 75% 100%.

18. A pair of shoes costing \$2 was sold at a gain of 50%. What was the amount gained?

19. A boy was 50 in. in height 2 years ago. Since then he has grown 4% taller. How many inches taller is he now than he was two years ago? How tall is he now?

20. A boy goes on an errand a distance of 240 yards and returns the same way. He runs $12\frac{1}{2}$% of the distance. How far does he run?

21. A girl's flower-bed is 80% of the size of her brother's garden bed which is 10 ft. by 12 ft. How many square feet does the flower-bed contain?

22. Two boys ran a 2-minute race. One ran 120 rd. the other 5% farther. How far did the second boy run?

23. A boy on a bicycle raced with a street-car while it ran 350 rd., and was beaten by 4% of this distance. By how many rods was he beaten?

§126. Finding the Percent or Rate.

Answer all you can orally

1. 4 is what percent of 8? Of 16? Of 20? Of 12?

2. 6 is what percent of 12? Of 24? Of 18? Of 30?

3. 10 is what percent of 20? Of 30? Of 40? Of 50?

4. 12 is what percent of 24? Of 36? Of 48? Of 60?

5. The base is 320; the percentage is 48. Find the rate.

Finding the rate in percentage is the same as finding the multiplier in decimals. [$320 \times (?) = 48.00$]

```
          .15   Rate
Base 320)48.00  Percentage
         320
         1600
         1600
```

6. Find the rate in the following exercises:

(1) base 240, percentage, 72.
(2) base 120, percentage, 6
(3) base 65, percentage, 13
(4) base 96, percentage, 24
(5) base, 104, percentage, 26
(6) base, 144, percentage, 48
(7) base, 160, percentage, 80
(8) base, 240, percentage, 80

7. Out of 80 games played by a championship baseball team, 20 were lost. What percent of the games was lost? What percent was won?

8. A man paid \$18 for the use of \$300 for a year. What percent of the sum borrowed was the sum paid?

9. A boy's arithmetic grade in October was 60. In November it was 80. By what percent of his October grade did he increase it in November?

10. The enrollment of a class was 40 and 8 pupils were absent. What percent of the class was absent?

11. On a certain day there were 14 hours from sunrise to sunset? It rained $3\frac{1}{2}$ hours of this time. What percent of the 14 hours was rainy?

12. If school begins at 9 A. M. and closes at 4 P. M. with an hour recess for dinner, what percent of a 24-hour day does school last?

§127. Finding a Number from a Known Fractional Part.

Answer all you can orally

1. $\frac{1}{2}$ of a number is 12. What is the number?
2. $\frac{2}{3}$ of a number is 14. What is the number?
3. $\frac{2}{3}$ of a number is $\frac{2}{7}$. What is the number?
4. $\frac{3}{4}$ of a number is 48. What is the number?
5. $\frac{3}{4}$ of a number is $\frac{9}{16}$. What is the number?
6. $\frac{5}{6}$ of a number is 15. What is the number?
7. $\frac{5}{6}$ of a number is $\frac{15}{22}$. What is the number?
8. $\frac{5}{7}$ of a number is 35. What is the number?
9. $\frac{5}{8}$ of a number is $\frac{25}{32}$. What is the number?
10. $\frac{6}{7}$ of a number is 42. What is the number?
11. $\frac{6}{7}$ of a number is $\frac{54}{77}$. What is the number?
12. $\frac{3}{7}$ of a number is 54. What is the number?
13. $\frac{6}{11}$ of a number is $\frac{24}{55}$. What is the number?
14. $\frac{6}{11}$ of a number is .24. What is the number?
15. $\frac{9}{10}$ of a number is 3.6. What is the number?
16. .8 of a number is 32. What is the number?
17. .7 of a number is 105. What is the number?
18. $\frac{9}{100}$ of a number is 18. What is the number?
19. .09 of a number is 45. What is the number?
20. 9% of a number is 36. What is the number?
21. $\frac{7}{100}$ of a number is 21. What is the number?
22. .07 of a number is 49. What is the number?
23. .05 of a number is 65. What is the number?
24. 5% of a number is 75. What is the number?
25. 10% of a number is 150. What is the number?

26. 10% of a number is 2.5. What is the number?

27. 12% of a number is 84. What is the number?

28. $\frac{5}{8}$ of a number is $27\frac{1}{2}$. What is the number?

29. $\frac{7}{9}$ of a number is $\frac{3}{6}\frac{5}{3}$. What is the number?

30. $\frac{1}{1}\frac{2}{3}$ of a number is 2.4. What is the number?

§128. Finding the Base.

Answer all you can orally

1. 50% of a number is 2. What is the number?

2. 25% of a number is 3. What is the number?

3. 20% of a number is 6. What is the number?

4. 75% of a number is 9. What is the number?

5. 40% of a number is 10. What is the number?

6. 18% of the number of apple-trees in an orchard died. 36 trees died. How many trees were in the orchard at first?

Finding the base in percentage is the same as finding the multiplicand in decimals. [(?)×.18=36.00]

Rate .18)36.00 Percentage
200 Base

7. Find the base in the following exercises:

(1) rate, .20, percentage, 44
(2) rate, .25, percentage, 64
(3) rate, .75, percentage, 120
(4) rate, .64, percentage, 128
(5) rate, .27, percentage, 81
(6) rate, .16, percentage, 48
(7) rate, .12, percentage, 84
(8) rate, .28, percentage, 140
(9) rate, .45, percentage, 180
(10) rate, .06, percentage, 126

8. Fifteen percent of the distance a boy can jump is 12 inches. How far can the boy jump?

9. A flock of pigeons increased 24, which was an increase of 60%. How many were in the flock at first?

10. Twenty percent of the cost of raising a potato crop was $12. What was the whole cost?

11. A girl spent 25 cents which was $12\frac{1}{2}\%$ of her monthly allowance. How much was her allowance?

12. A boy spends $33\frac{1}{3}\%$ of his study-time on his arithmetic lesson. He spends $\frac{3}{4}$ of an hour on his arithmetic lesson. How many hours is his entire study-time?

13. A certain school has 180 school days in a year, which is $12\frac{1}{2}\%$ of the number of school days of the elementary course. How many school days has the elementary course?

14. A boy went to school 16 years which was 20% of the age of his grandfather. How old was his grandfather?

§129. Easy Applications of Percentage.

1. The height of a 14 year old boy is 60 in., and a girl of the same age is 2% taller. How tall is the girl?

2. A dress cost the dealer $2.00 and was sold for $33\frac{1}{3}\%$ more than it cost. For how much did it sell?

3. The price of an article was reduced $1.00, which was 20% of the marked price. Find the marked price.

4. Eighty percent of the cost of a girl's suit is $4.00. What is 1% of the cost? What is 100%, the whole cost?

5. A thermometer reading was 120°, which was 20% higher than the reading previously taken. What was the previous reading?

In such questions as this, answer first the question, "20% of what?" In this case 20% of the previous reading.

6. This year a flat rented for $55 a month, which is 10% more than it rented for last year. For how much did it rent last year?

7. In 25 lb. of macaroni there are 2.5 lb. of water. What percent of the macaroni is water?

8. In 1902 there were 6500 acres sown to sugar beets in the state of New York. In 1903 there were 7150 acres in the same state. What was the percent of increase from 1902 to 1903?

9. There were 3400 acres of sugar beets in Wisconsin in 1902. In 1903, there were 70% more acres in Wisconsin. How many acres of sugar beets were there in Wisconsin in 1903?

10. Michigan raised 150 acres of cranberries in 1900 and Maine raised 60% of this number of acres. How many acres of cranberries did Maine raise?

11. Rhode Island had 300 acres of cranberries in 1900 and Connecticut had 90% of this number of acres. How many acres had Connecticut?

12. A young poplar tree increased in height by 120% during 1908. At the beginning of the year it was 10 ft. tall. How tall was it at the end of the year?

13. Joseph bought 21 young chickens and rats killed 7 of them. What percent of the chickens was killed? What percent remained alive?

14. A boy paid 15 cents apiece for 24 young Plymouth Rock chickens. He spent 40 cents for feed, raised 8 of them and sold the 8 at 60 cents each. What was his percent of gain, or loss?

15. A girl earned 90 cents picking berries and spent 40% of her earnings. How many cents did she not spend?

16. The living expenses of a family for a year are \$640. The cost of clothing is \$128. What percent of the living expenses is the cost of clothing?

17. A fruit farm has 5 acres in apples, which is 40% of the whole farm. How many acres are in the farm?

18. A peach orchard occupies 12% of a $12\frac{1}{2}$-acre farm. How many acres are there in the peach orchard?

19. Fifty-five percent of the metal of a watch case, weighing 2 oz., is gold. What is the weight of gold in the watch case?

20. Potatoes are 75% water. How many pounds of water are there in 4 bu. of potatoes?

21. A cow was bought for \$36 and sold at a gain of 25%. Find the selling price.

22. A cow was sold for \$50, thus gaining 25%. Find the cost of the cow.

23. A merchant paid \$15 for an article, and marked it to sell at a gain of 40%. A few weeks later he marked the price down 10%. What was the "marked down" price?

24. What is the percent of gain on an article that costs \$15 and sells for \$18.90?

25. A man paid \$6 for the use of \$100 for one year. What percent was what he paid of what he borrowed?

26. At the same rate as in problem 25 what should a man pay for the use of \$200 for one year?

27. At the rate in problem 25 what should the man pay for the use of \$100 for 2 yr.? For 3 yr.? For 5 years?

28. How much must a man pay, at the rate of \$4 on the \$100, for the use of \$3200 for 1 yr.? For 2 years?

PERCENTAGE LEADING TO INTEREST

§130. Simple Interest.

INTEREST is money to be paid for the use of money.

1. Two percent interest means that 2¢ is to be paid every year for the use of 100¢ or \$1; \$2 for the use of \$100, and so on at this rate. What then would 6% interest mean? 4% interest? 10% interest?

The rate of interest is the number of dollars to be paid for every \$100 borrowed for 1 year.

In problem 1 above, the rates are 2 percent, 6 percent, 4 percent, and 10 percent, and they are written 2%, 6%, 4%, and 10%.

2. A man pays \$10 for the use of \$200 for 1 year. What is the rate of interest?

3. What is the rate of interest if \$30 is paid for the use of \$500 for 1 year?

4. What are the rates of interest in the following?

(1) \$15 paid for the use of \$300 for 1 year
(2) \$20 for the use of \$400 for 1 year
(3) \$24 for the use of \$400 for 1 year
(4) \$36 for the use of \$600 for 1 year
(5) \$32 for the use of \$400 for 1 year
(6) \$42 for the use of \$700 for 1 year

The interest for one year is the sum of money to be paid each year for the whole sum borrowed.

5. What is the interest for 1 yr. on \$200 at 6%? At 5%?

6. What is the interest for 1 yr. on \$400 at 4%? At 7%?

7. What is the interest for 1 year on the following sums of money?

(1) \$500 at 6%	(5) \$800 at 7%
(2) \$400 at 5%	(6) \$600 at 5%
(3) \$700 at 6%	(7) \$350 at 6%
(4) \$800 at 4%	(8) \$560 at 4%

Reckoning interest for 1 year is the same thing as reckoning percentage; thus:

PERCENTAGE	INTEREST FOR ONE YEAR
\$800 Base	\$800 Principal
.05 Rate	.05 Rate
\$40.00 Percentage	\$40.00 Interest for 1 year

The only things to be carefully noted are:

(1) that two of the numbers are named differently in interest problems; and

(2) that the interest for any number of years is the interest for 1 year multiplied by the number of years.

Thus, if the interest for 1 year is \$40, as above, the interest for 4 years is 4 times \$40, and for $5\frac{1}{2}$ years is $5\frac{1}{2} \times \$40$, etc.

8. Find the interest on \$200 at 4% for 1 yr. 6 years.

9. What is the interest at 6% on \$1 for 1 year?

In computing interest a year means 12 months of 30 days each.

10. What is the interest at 6% on \$1 for $\frac{1}{2}$ year? For 6 mo.? For 2 mo.? For 1 month?

11. What is the interest at 6% on \$1 for 3 months? For 5 months? For 7 months? For 14 months? For 16 months? For 34 months? For any number of months?

12. From the answer to problem 9, find the interest at 6% for 1 yr. on \$8; on \$25; on \$48; on \$85; on \$124.

13. If you know the interest at 6% for 1 yr. on \$1, how can you find the interest at 6% for 1 yr. on any number of dollars?

14. From the last answer to problem 12, find the interest at 6% on \$124 for 2 years; for 3 years; for $4\frac{1}{2}$ years; for $12\frac{1}{2}$ years; for any number of years.

INTEREST, WHEN THE TIME IS IN YEARS AND MONTHS

15. If the interest on a sum of money at 4% for 1 year is known, how would you find the interest on the same sum at 4% for $\frac{1}{12}$ of a year, or 1 month? For 2 months? For 4 months? For 3 months? For 5 months? For 7 months?

16. The interest on \$40 at 6% for 1 year is \$2.40. Find the interest at 6% for $\frac{1}{12}$ of a year, or 1 month; for 2 months; for 4 months; for 3 months; for 7 months.

17. Tell how to find the interest at 6% on any number of dollars for any number of months.

18. Find the interest on \$60 at 6% for 1 year and 1 month; for 1 yr. and 2 months; for 1 yr. and 3 months; for 1 yr. 5 mo.; for 1 yr. 7 months.

19. Find the interest on \$80 at 5% for 2 years; for 2 yr. 1 mo.; for 2 yr. 3 mo.; for 3 yr. 6 mo.; for 2 yr. 7 mo.; for 3 yr. 5 months.

20. Find the interest on \$1200 at 4% for 2 yr. and 6 mo.; for 2 yr. 9 mo.; for 2 yr. 3 mo.; for 3 yr. 3 mo.; for 3 yr. 8 months.

INTEREST RECKONED ON THE 6% BASIS

21. What part of the interest on any sum at 6% is the interest on the same sum for the same time at 1%? At 3%? At 2%? At 4%?

22. The interest on a sum of money for 1 yr. at 6% is \$24. What is the interest on the same sum for the same time at 3%? At 2%? At 12%? At 18%? At 1%?

23. The interest on a sum of money for a certain time at 6% is \$24, and at 1% it is \$4. What is the interest on same sum for same time at 7%? At 5%? At 8%? At 4%?

24. The interest at 6% on a certain sum for a certain time is \$36. Find the interest on the same sum for the same time at the following rates: 1%, 7%, 5%, $\frac{1}{2}$%, $6\frac{1}{2}$%, $5\frac{1}{2}$%, 3%, $3\frac{1}{2}$%, $7\frac{1}{2}$%, $\frac{1}{4}$%, $6\frac{1}{4}$%, $5\frac{3}{4}$%.

25. By finding the interest first at 6%, then taking $\frac{1}{6}$ of it, find the interest at 7% on \$800 for 3 years.

26. By finding the interest first at 6%, find the interest for $4\frac{1}{2}$ years at 5% on \$560.

27. Find the interest at 9% on \$720 for 3 yr. on the 6% basis.

CALCULATING FIRST THE INTEREST ON \$1

Some prefer to calculate interest by finding first the interest on \$1 for the given time and at the given rate.

28. What is the interest on \$1 for 1 yr. at 6%? At 4%? At 7%? At 8%? At 3%? At 1%? At 9%? At 10%?

29. What is the interest on \$1 for $\frac{1}{2}$ year at 6%? At 4%? At 7%? At 8%? 3%? 1%? 9%? 10%?

30. What is the interest for 3 months on \$1 at 6%? At 4%? At 8%? At 7%? At 5%?

31. Find the interest on \$1 at 6% for 2 yr.; for 6 mo.; for 1 yr. 6 mo.; 2 mo.; 2 yr. 2 mo.; 2 yr. 6 months.

32. Find the interest on \$1 at 8% for 6 mo.; for 3 mo.; for 9 mo.; for 1 yr. 3 mo.; for 1 yr. 9 mo.; for 2 yr. 6 mo.

33. From the results of problem 32 find the interest for 9 mo. at 8% on \$5; on \$10; on \$20; on \$50; on \$100.

34. Tell how to find the interest on any sum by finding first the interest on \$1 for the given time and rate.

35. The interest on \$1 for a given time and rate is \$.15. Find the interest for the same time and rate on \$50; on \$150; on \$450; on \$865; on \$1260.

To find the interest by the \$1*-method on any sum of money multiply the interest on* \$1 *for the given time and at the given rate, by the number of dollars.*

36. Find by the \$1-method the interest at 6% on \$320 for 2 yr. 6 months.

37. Find the interest on \$350 at 4% for 2 yr. 6 mo.; for 6 yr. 9 mo.; for 8 yr. 3 months by the \$1-method.

38. Find by the \$1-method the interest on:

(1) \$500 at 7% for 1 yr. 3 mo.
(2) \$460 at 8% for 1 yr. 9 mo.
(3) \$620 at 5% for 4 yr. 4 mo.
(4) \$150 at $8\frac{1}{2}$% for 3 yr. 6 mo.
(5) \$810 at $4\frac{1}{2}$% for 4 yr. 4 mo.

39. Find by the \$1-method and check by using the 6%-basis the interest on:

(1) \$650 at 7% for 2 yr. 9 mo.
(2) \$800 at 5% for 5 yr. 6 mo.
(3) \$1250 at $4\frac{1}{2}$% for 4 yr. 3 mo.
(4) \$1560 at 4% for 6 yr. 6 mo.

40. Find by any method the interest on the following:

(1) \$210 at 8% for 1 yr. 6 mo.
(2) \$360 at 3% for 2 yr. 3 mo.
(3) \$450 at 4% for 3 yr. 9 mo.
(4) \$600 at 7% for 1 yr. 5 mo.
(5) \$720 at 5% for 3 yr. 7 mo.
(6) \$1224 at 4% for 2 yr. 11 mo.

GENERAL REVIEW PROBLEMS

§131. Problems on Factories and Farming.

1. In 1910 Evansville, Ind., had about 10,166 persons engaged in the work of 299 factories. Find the average number of persons engaged in a factory.

2. If each of these 10,166 persons supported 4 people, how many persons in this city got their living from factory work?

3. If half of the 10,166 persons supported 5 persons each and the rest supported 2 persons each, how many got their living from factories?

4. If there were 69,667 people living in Evansville in 1910, how many people were there to each factory?

5. Taking 40,000 for the number of people supported by factories and the total population of Evansville as 70,000, what fractional part of the population got its support from factories?

6. In 1910 Ft. Wayne, Ind., had about 12,190 persons engaged in her 230 factories. Find the average number of persons engaged in a factory.

7. If each of these 12,190 persons supported 4 people, how many Ft. Wayne people got their living from factories? If half supported 5 persons each, and the rest 2 persons each, how many were supported by factories?

8. If the population of Ft. Wayne in 1910 was 63,940, how many people in Ft. Wayne were there to one of the 230 factories?

9. Calling the number of factory-supported people of problem seven 49,000 and the total population 64,000, what fractional part of the population got support from factories? Taking 43,000 as the number of factory-supported people find the fractional part of the populaton supported by factories. Express the fractions as decimal equivalents to the nearest .01 and tell what they mean here.

10. In 1910 Indianapolis had about 38,000 persons engaged in her 855 factories. Find to the nearest .1 the average number of persons engaged in a factory.

11. If each of these 38,000 persons supported 4 persons how many Indianapolis people were supported by factories? If half supported 5 persons each and the rest 2 persons each, how many were supported by factories?

12. If the population of Indianapolis was 234,270, find the number of people to each of her 855 factories. Calling the population 234,000, what fractional part of the people was supported by factories? Reduce the fractions to their decimal equivalents to the nearest .01 and tell what they mean here.

13. In 1910 South Bend had a population of 53,625, and 13,625 persons engaged in its 218 factories. Terre Haute had a population of 58,140, and 5185 persons engaged in its 170 factories. Find the average population per factory, and the average number of persons engaged by a factory, for each city.

Other problems may be made on the facts of this problem, if they are desired.

14. It takes 18 bu. of corn worth 45¢ per bu. and $4 worth of other feed to raise a hog until it is 8 months old. What is the cost of the hog to the farmer?

15. A "scrub" will weigh about 180 lb. when 8 months old and will bring $5.50 per cwt. Will the farmer gain or lose, and how much?

16. A Poland-China, or Duroc-Jersey will weigh about 250 lb. when 8 months old, and be worth $6.75 per cwt. What is the hog worth?

17. If it costs the farmer the same to raise the well-bred hog as the "scrub," how much does the farmer gain on the well-bred hog?

18. How much more will the blooded hog bring on the market than the scrub?

19. A growing pig requires daily $\frac{1}{2}$ lb. of protein and $2\frac{1}{2}$ lb. of carbohydrates in its food. A bushel of corn contains $3\frac{1}{2}$ lb. of protein and 40 lb. of carbohydrates. What part of a bushel of corn would supply protein for a day to the pig? How much would the corn be worth at 42¢ a bushel?

20. What part of a bushel of corn would supply the pig with carbohydrates, and what would it be worth @ 42¢ a bushel?

21. A dairy cow or a work horse requires daily 2 lb. of protein and 12 lb. of carbohydrates in their feed. How much would corn enough to supply these nutrients cost @ 42¢ a bushel?

22. A bushel of oats contains 3 lb. of protein and 19 lb. of carbohydrates. With oats at 30¢ a bushel how much would it cost to supply the cow, or horse, with the required protein by feeding oats for 30 days? With carbohydrates for 30 days?

23. How much would it cost to supply the pig (prob. 19) 30 da. with protein by feeding it oats @ 30¢ a bushel?

24. Which is the cheaper protein food, corn at 45¢ a bu., or oats at 30¢; a bushel of corn containing $3\frac{1}{2}$ lb. protein and a bu. of oats containing 3 lb. of protein?

25. A cow eats 3.5 tons of hay worth \$7.00 per ton, 1200 lb. of ground feed worth 90 cents per cwt., and pasture amounting to \$8 in a year. What does it cost a farmer to keep a cow?

26. A "scrub" cow will give 13 lb. of milk, testing 4.2% butter-fat, worth 28 cents a lb. daily for 300 days in the year and raise a calf worth \$5. What is the profit on her?

27. A Jersey cow will give 24 lb. of milk daily for the same time and raise a calf worth \$8. What is the farmer's profit on her? How much more does he make on the Jersey than on the "scrub"?

28. To raise a colt to a 4-year old horse it takes on an average 2.5 tons of hay, 50 bu. of oats, and \$8 worth of pasture annually, for 4 years. What is the total cost with hay at \$9 per ton and oats at 35 cents per bushel?

29. A "scrub" 4-year old horse will bring about \$85. Has the farmer lost or gained, and how much?

30. A well bred 4-year old horse will bring \$200. What has the farmer gained or lost on this horse?

31. A flock of 72 hens average 90 eggs a year each. With eggs worth 20 cents per dozen, what is the value of these eggs?

32. It takes 15 bushels of corn, 7 bushels of oats, and \$9 worth of other feed to keep this flock for one year, besides what they pick up for themselves. What is the profit over and above the cost of feed at present price of eggs?

33. What would have been the profit if they had laid 140 eggs each?

34. In 1905-6, Louisiana produced about $\frac{3}{10}$, Porto Rico about $\frac{1}{5}$, Hawaii about $\frac{7}{20}$, and the Philippines about $\frac{1}{7}$ of the total cane sugar production of the United States What fractional part did they produce altogether?

35. The rest of the total production was furnished by all the other southern states? What fractional part did these states produce?

36. The total production of cane sugar of the U. S. in 1905-6 was about 1080 thousand long tons (2240 lb.). Express this in thousands of tons of 2000 lb. each.

37. The total cane sugar production of the U. S. in 1906-7 was about 1038 thousand long tons. Louisiana produced 230 thousand long tons. Express to 3 decimal places Louisiana's part of the total production that year.

38. Porto Rico's production in 1906-7 was 255 thousand long tons. Express to three decimal places Porto Rico's part of the total production that year.

39. The cane sugar production in 1906-7 of Porto Rico was 255,060 long tons, which was .654 of Hawaii's production. Find Hawaii's production in long tons.

40. In 1905-6 the cane sugar production (1080 thousand tons) was .796 of the total production of both beet and cane sugar. Find to the nearest thousand tons, the beet sugar production of that year.

41. In 1906-7 the cane sugar production (1038 thousand tons) was .706 of the total beet and cane sugar production. Find to the nearest thousand tons, the beet sugar production of that year.

PART III

GENERAL REVIEW

§132. Areas.

1. If you hold a board 1′ wide and 10′ long flatwise against the wind, how hard does the board push against you if the wind pushes against each square foot of the board with a force of 2 pounds? If it pushes with a force of $5\frac{3}{4}$ pounds?

The wind is a stream of air moving along with a certain speed, or *velocity*. The greater the speed, or *velocity*, the stronger it pushes against objects it strikes.

The velocity of wind in miles per hour and the pressure per square foot are given below:

	Velocity in Miles per Hour	Pressure per Square Foot	
		lb.	oz.
Light breeze	$3\frac{1}{4}$	..	.75
Moderate breeze	$6\frac{1}{2}$	..	3.33
Fresh breeze	$16\frac{1}{4}$	1,	5
Stiff breeze	$32\frac{1}{2}$	5,	3
Strong gale	$56\frac{1}{4}$	15,	9
Hurricane	$79\frac{1}{2}$	31,	4
Violent hurricane	$97\frac{1}{2}$	46,	12

Make other problems using measures of your schoolhouse or other buildings in your neighborhood.

2. What is the total pressure in pounds on the west side of a house 30 ft. by 25 ft., due to a light breeze from the west? To a stiff breeze? To a strong gale?

3. What is the pressure on the side of a tall building 265 ft. by 80 ft., due to a hurricane blowing squarely against it?

4. Find the wind pressure against the side of a load of hay 25 ft. long and 12 ft. high, due to a strong gale blowing squarely against it.

5. Find the number of pounds pressure against a sign-board 100 ft. by 14 ft., due to a strong gale blowing squarely against it.

6. The side of a passenger coach against which the wind blows is 70 ft. long and 12 ft. high. (a) Find the wind pressure due to a stiff breeze blowing squarely against the coach.

(b) Find the wind pressure due to a strong gale blowing squarely against it.

7. Find the wind pressure on the side of a train of 10 such cars as that of problem 6, due to a hurricane, blowing squarely against it.

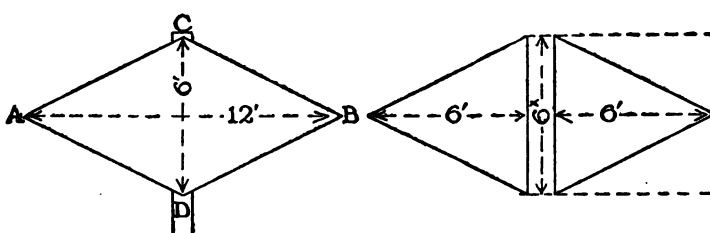


FIGURE 1

8. Find the wind pressure on a diamond-shaped sign-board, due to a violent hurricane blowing squarely against it, if the diamond is 12′ long (AB = 12′) and 6′ broad at the middle (CD = 6′). (See Fig. 1.)

9. Find the wind pressure due to a strong gale blowing squarely against the flat side of a dense tree-top of the shape of a triangle, 30 feet wide and 40 feet high.

§133. Scale Drawing.

One inch in Fig. 1 represents 80 rods.

1. How many acres are in the tract *ABCDE*? What is the farm worth at \$95 per acre?

2. The field is entirely enclosed, except along the river, by a barbed wire fence having 4 lines of wire. Barbed wire weighs about 1 lb. per rod of single wire. Find the cost of the wire at \$3.50 per bale of 100 pounds.

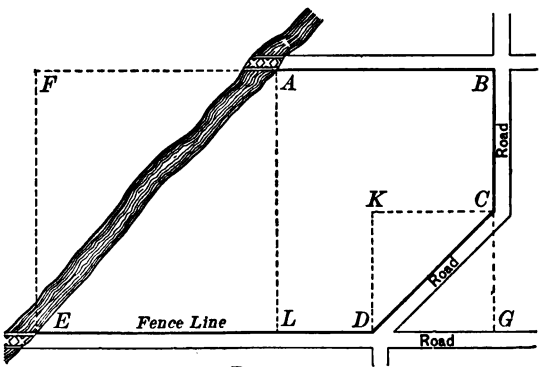


FIGURE 1

3. If posts cost 35¢ each and are set 2 rd. apart, find the cost of the posts.

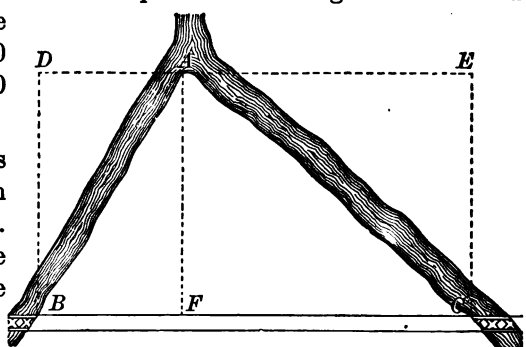


FIGURE 2

4. In Fig. 2, 1 in. represents 80 rd., or, *the scale is 1″ to 80 rd.* How many acres are in the field *ABF*? What is the field worth at \$75 per acre?

5. Find how many acres there are in the field *AFC*. What is its value at \$87$\frac{1}{2}$ per acre?

6. Find the value of the field *ABF* at \$80 per acre.

7. How many rods of fence would be needed to enclose the field *ABF*? *AFC*? The entire field *DBCE* (deduct 65 rd. for river)?

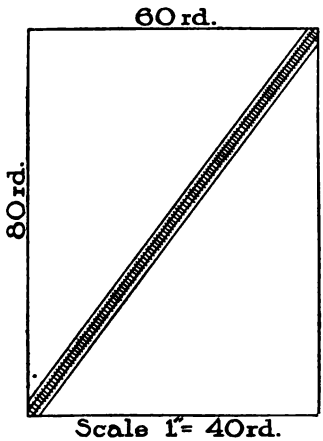


FIGURE 1

8. A railroad runs diagonally across a rectangular field, dividing it into two triangles as shown in Fig. 1. How much is each triangular part worth at \$120 per acre, supposing the road takes off equally from each part a strip containing $2\frac{1}{2}$ acres?

9. The railroad company fences both sides of the strip with 4-wire fences, each 96 rd. long. Posts are set a rod apart, and cost 25¢ each. The wire used costs \$3.25 per bale of 100 rd., and 2 men at \$1.50 per day work 3 days to build the fence. What does it cost the company to fence the road?

10. If 1 square inch of the rectangle of Figure 2 represents $\frac{5}{8}$ of an acre, how many acres are represented by the whole rectangle?

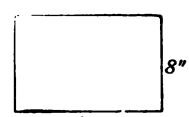


FIGURE 2

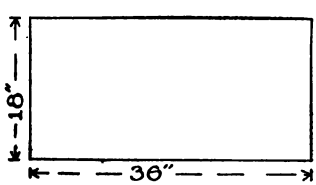


FIGURE 3

11. If one square foot of a sheet of zinc weighs $2\frac{1}{2}$ ounces, how much will a piece of the same sheet of the size shown in Figure 3 weigh?

§134. Fencing a Farm.

Answer all you can orally

A farm was divided into fields and seeded as shown in the drawing (Fig. 1). The scale of the drawing is 1 in to 80 rd.

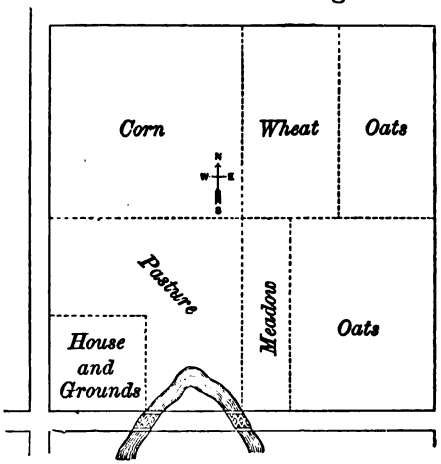


FIGURE 1

1. How long is the cornfield north and south? How wide?

2. What is the length and width of the wheatfield? The north oatfield? The meadow? The south oatfield?

3. The drawing of the road is $\frac{1}{10}$ in. wide. How wide is the road?

4. How long and how wide is the farm?

5. How long must a wire be to reach entirely around the farm?

6. How many rods of barbed wire will be needed to inclose the farm with a 3-wire fence? How many miles (320 rd. = 1 mile)?

Use pencil only when needed

7. How many rods long is the partition (division) fence across the farm from east to west? What part of a mile is its length? How many rods of wire will be required for a 4-wire fence?

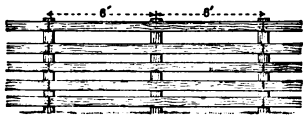
8. How many rods of wire will run the 4-wire fence across the farm from the middle of the north side to the middle of the south side?

9. How many rods of wire will run a 4-wire fence between the wheatfield and the north oatfield? Between the meadow and the south oatfield?

10. How many bales of wire of 100 rd. each will be needed for these cross fences and division fences?

11. How much will the wire cost for the 3-wire inclosing fence @ $3.50 a bale of 100 rods?

12. The house and grounds lot is separated from the pasture by a 5-board fence. The boards used are 1 in. thick, 6 in. wide, and 12 ft. long. How many boards will reach once along the north side of this lot? Once along the east side?



TWO FENCE PANELS
FIGURE 1

13. How many boards will be needed for the 5-board fence along both sides?

14. The posts are set 6 ft. between centers, that is, a post is set every 6 ft. How many posts will be needed for the north side? For the east side? For both?

15. How much will these posts cost @ 23¢ apiece?

Lumber is sold by the *board foot*. A board foot is a board 1 ft. long, 1 ft. wide, and 1 in., or less, thick.

16. How many board feet of lumber in a board 1 in. thick, 1 ft. wide, and 5 ft. long? The same thickness and width and 10 ft. long? 12 ft. long?

17. How many board feet in a board 1 in. thick, 10 ft. long, and 2 ft. wide? The same thickness and length and 6 in. wide? 9 in. wide? 18 in. wide? 1 in. wide? 8 in. wide?

18. How many board feet in a board 12 ft. long, 4 in. wide, and 1 in. thick? 2 in. thick?

19. How many board feet in a "two by four" scantling 10 ft. long ($2'' \times 4'' \times 10'$)? 16 feet long?

20. How many board feet in a "four by four" scantling 10 ft. long? 12 ft. long? 18 ft. long?

21. How many board feet are there in a fencing board $1'' \times 6'' \times 12'$? In 5 such boards?

22. How much will the fencing lumber of problem 13 cost @ $18 per M (thousand) board feet?

§135. Areas of Fields.

Answer all you can orally

Problems 1 to 14 refer to Fig. 1, page 211.

1. The whole farm contains 160 acres. How many acres are there in the cornfield? In the wheatfield? In the meadow? In the south oatfield? In the pasture? In the house and grounds lot?

2. What is the width of each of these fields in rods?

3. If the meadow were divided by a north and south central line, how many rods wide would each half be? How many acres would there be in each half?

4. How many square rods in a strip 1 rd. wide and 80 rd. long?

5. How many square rods in an acre?

6. How wide must a strip of land be to contain an acre, if it is 80 rd. long? 40 rd. long? $\frac{1}{2}$ mile long?

7. Compare the sizes of the wheatfield and the north oatfield; of the cornfield and the wheatfield; of the meadow and the wheatfield; of the meadow and the south oatfield; of the meadow and the house and grounds lot; of the lot and the pasture; of the pasture and the south oatfield.

Use pencil and paper only when needed

8. What would be the income per acre from the farm planted in corn, if the average yield were 47 bu. per acre, sold at 28¢ per bu.? The gross (total) income from the farm (except the house and grounds lot) for this year?

9. If the farm were rented @ $5 per acre, how much would remain for the tenant* from each acre that is planted in corn (conditions as in prob. 8)? From the entire farm?

10. What would be the owner's income at $5 per acre from the whole farm, not allowing for expenses?

11. Which is the more profitable way for the owner to rent his farm, @ $6 for every acre cash, or for ½ of all the crop delivered to market, supposing that the farm, except lot around house, is planted to corn, yielding 48 bu. per acre, worth 30¢ per bushel?

Rent of the first sort is called "cash rent," of the second sort, "grain rent."

12. If this same farm will produce 20 bu. of wheat per acre and a price of 60¢ per bushel can be obtained for it, which brings the more money to the owner—the wheat crop or the corn crop? How much more? (Conditions as in problem 8.)

13. Compare the owner's gross profits of a grain-rented farm from an oats crop of 40 bu. per A. sold at 22¢ per bushel with the gross profits of the wheat crop of problem 12.

14. In problem 12 which would bring the larger income to the owner, and how much, cash rent @ $5 per A., or grain rent @ ½ of crop delivered to market?

15. A cornfield of 68 acres produced an average yield of 48 bu. per acre. How many bushels did the field yield?

*The tenant is the farmer who raises the crop on another man's farm.

16. It cost \$5.85 per acre to raise the crop, problem 15. At 46¢ per bu. what was the net value of the crop?

17. If the tenant paid cash rent @ \$5.50 per acre, what was his net profit from the crop? How much did the owner receive?

§136. Cost and Value of Country Roads.

1. If 14 teams pass over a road each day, how many trips are made over the road in a year? There would be a saving of 2¢ per mile each trip, if the roads were improved. How much would be saved per mile per year?

2. If \$25 per mile annually were invested in good roads, and \$102.20 per mile were saved annually by the improvement, how much would the farmers of a community save in ten years on 50 miles of road?

3. It costs a farmer about 35 cents per ton to haul farm products one mile on the average dirt road as now built. What does it cost him per ton to market his products on such a road, at an average distance of four miles? What is the total cost per year, if he markets 40 tons of produce?

4. It costs about 20¢ per ton to haul farm produce 1 mile on well built and well kept dirt roads. What will it cost per ton for a distance of 4 miles? What will be the total cost per year, if 40 tons of produce are hauled to market? What does the farmer save on account of having good roads? How much would 20 such farmers in a community save?

5. A road is 4 rods wide. How many sq. ft. of surface are there on a mile of such road?

6. There are 34 inches of rainfall in a year. How many tons of water fall on a mile of road 4 rods wide in a year? (A cu. ft. of water weighs 62.42 lb.)

7. How many cubic yards of gravel are required to cover a mile of road 14 ft. wide with a layer 8 in. deep?

8. How many board feet of lumber in two timbers 3 inches thick, 12 inches wide, and 8 ft. long? What is it worth at $36 per M (thousand)?

9. What will two strips of steel $\frac{3}{8}$ inches thick, 3 inches wide and 8 feet long, weighing 3.5 pounds per foot of length, cost at 3¢ per pound?

10. A farmer can make a road drag with the material mentioned in problems 8 and 9 in 3 hours. His time is worth 20¢ per hour. What is the cost of the drag, including steel, timber, and labor?

11. A farmer's boy with three horses can drag a mile of road in 1 hour. How much will it cost if the boy's time is worth 20¢ per hour and the time for each horse is worth 10¢ per hour? How much will it cost if he drags the mile of road 10 times during the year? If he drags it 15 times?

12. On bad roads a farmer can haul $\frac{7}{8}$ of a ton at each load; on good roads he can haul 2 tons. If he can haul 5 loads a day on good roads and 3 loads a day on poor roads, how much time does he lose in hauling 2400 bushels of wheat and 1500 bushels of potatoes? (See table, page 171.)

§137. Accounts.

A man has on hand at the beginning of February, 1909, $250. He receives during the month $200 for salary and $60 for the rent of two houses. During the month he pays $72.50 on an insurance policy; $110 for household expenses; $25.50 for incidentals, and $5 for books and stationery. His account book shows the following entries:

CASH

1909	(*Receipts*)	*Dr.*	1909	(*Expenditures*)	*Cr.*
Feb. 1	On Hand.....	$250 00	Feb. 1	By Insurance....	$ 72 50
" 15	To Salary.....	200 00	" 28	" Household Ex.	110 00
" 27	" Rent.....	60 00		" Incidental "	25 50
				" Books and Sta.	5 00
				" Balance.....	297 00
		510 00			510 00
" 28	On Hand.....	297 00			

1. With what items is cash the man's debtor? With what items is cash his creditor?

2. During the month, how much money did the man receive? How much money did he actually possess on February 28 after paying his expenditures?

3. How much was spent? What is the difference between the receipts and the expenditures? How much more was spent than was received as salary?

Draw forms and prepare cash accounts for each of the following:

4. March, 1908. On hand first of the month $1000. March 5, sold land for $1500; received on the 15th, $150 for rent, and sold a team for $225 on the 25th. Bought real estate on the 8th for $1000; paid $65.25 for repairs on the 15th; taxes on the 16th amounted to $17.50; household expenses for the month amounted to $100, and personal expenses to $50.

5. June, 1908. A boy's receipts and expenditures were as follows: Received 25¢ for mowing the lawn on the 1st; 50¢ a day for running errands on the 4th and 5th; 25¢ for mowing the lawn again on the 15th. On the 17th he paid $1.25 for a hat. The same day he earned 50¢ for delivering packages for the grocer.

6. July, 1908. A farmer sold on the 13th two loads of hay at $20 a load; 4 hogs weighing in all 1000 lb. at 6¢; 15 lb. butter at 20¢. On the 13th he bought 26 lb. granulated sugar at 6¢; 3 lb. coffee at 35¢; and dry goods to the amount of $15. On the 26th he sold vegetables to the amount of $1.25, and 9 doz. eggs at 14¢. The same day he bought clothing to the amount of $10.

7. Find cost of each purchase, also the amount due, on the bill below:

E. S. WICKWIRE,

1910 *In account with* MANNHEIMER BROS., *Dr.*

Jan. 4	To 1 Rug.....at $75.50				
" 4	" 1 Overcoat. " 35.00				
" 12	" 7 yd. Dress Gds " 2.00				
" 14	" $3\frac{1}{2}$ yd. Silk..... " 1.50				
	Amount due.....	.....	.....		

Received payment,

MANNHEIMER BROS.

Jan. 31, 1910.

Draw forms and prepare bills for the following accounts:

8. Edward Morris in account with J. W. Bowlby, Dr., St. Paul, Minn., May 30, 1908. 1 doz. collars at $12\frac{1}{2}$¢ each; $\frac{1}{2}$ doz. pr. cuffs at 25¢ a pair; 2 neckties at 50¢; 3 shirts at $1.25; 1 hat at $3.50; 1 pr. shoes at $3.50; 2 pr. gloves at $1.25. Paid, May 30.

9. A. H. Simons bought of Yerxa Bros., Chicago, June 1908, 1 sack appleblossom flour at $1.30; 8 lb. ham at 17¢; 15 lb. granulated sugar at 6¢; $\frac{1}{2}$ lb. Oolong tea at $1.60; 3 lb. Mocha and Java coffee at 40¢; 1 bunch of celery at 30¢. Paid in full the same day.

10. Edward Ryan bought of M. J. Doran, February 7, 1908, 2 cords of hard wood at $8.75; 3 T. of hard coal at $8.25; 2 cords of pine slabs at $3.25.

11. May 4, 1908, Alvin Johnson bought of the John Martindale Lumber Company, cash payment, 1000 cedar shingles at $3 per thousand; 1 barrel of lime at 68¢; 1000 feet of pine flooring at $35 per thousand; 3000 laths at $1.50 per thousand; 15 pounds shingle nails at 7¢ per pound.

Accounts are not always settled in full at the close of the month. When a portion of the amount due is carried over to the next month, this item appears as the first item on the bill, under the title "account rendered."

12. Find amount of this bill:

J. FIRESTONE, *Dr.*

1908 *To* ALLEN, MOON & Co.,

Feb. 1	To Account rendered.....	.....	.....	$24	85
" 10	" 2 Sacks of Flour @.	$2.60			
" 10	" 25 lb. Ham @.....	.18			
" 10	" 4 doz. Eggs @.....	.22			
" 10	" 6 lb. Butter @.....	.25			
" 15	" 25 lb. Gran. Sugar... @	.06			
" 15	" Vegetables.....	5.00			
	By Cash.....				

13. A. L. Parker, in account with Franklin, Smith, and Company, lacks $12.75 of paying the entire bill of February, 1909. His March purchases were as follows: March 5, ½ dozen towels, @ 50¢ apiece; 2 table cloths, @ $6.50; 1 dozen napkins, @ $5.50 a dozen; 2 bed spreads @ $2.50; ½ dozen sheets @ 75¢ apiece; ½ dozen pillow cases @ 20¢ apiece; 7 yards dress goods @ $1.50; findings, $5.25. Bill paid in full March 28, 1909.

14. Edward D. Young in account with F. W. Salisbury, wood and coal dealer, has left over from a previous account \$8.25. On the 3d of January, 1911, he bought 2 T. of hard coal @ \$8.25; 1 cord of hard maple wood, \$8.50; 1 cord of pine slabs, \$3.50. Paid the whole amount January 29.

15. On Nov. 15, 1911, Mrs. F. M. Jones purchased 1 can lima beans @ 12¢; 1 can peas @ 12¢; 1 qt. cranberries @ 12¢; 4 lb. pork chops @ 12¢; $4\frac{1}{2}$ lb. chicken @ 12¢; 2 lb. butter @ 32¢, and paid the account with a \$5.00 bill. What change was due her?

§138. Farm Account Keeping.

The following forms show how a certain farmer keeps a systematic account of receipts and expenditures with each of the fields shown in Fig. 1, page 211.

The items for which money was paid out or received were put down with the dates of payment or of receipt in a day-book thus:

Day-Book for Forty-Acre Cornfield

1. Paid for 4 da. work removing stalks @ \$1.15, Mar. 18.
2. Paid for 9 da. plowing @ \$1.15, Apr. 30.
3. Paid for 2 da. harrowing @ \$1.00, May 1.
4. Paid for $3\frac{1}{2}$ da. planting corn @ \$1.20, May 18.
5. Paid for 6 bu. seed corn @ 90¢, May 18.
6. Paid for 2 da. replanting corn @ \$1.15, May 30.
7. Paid for 5 da. harrowing corn @ \$1.15, May 30.
8. Paid for 7 da. cultivating @ \$1.15, June 15.
9. Paid for 6 da. cultivating @ \$1.15, June 30.
10. Paid for 6 da. cultivating @ \$1.15, July 15.
11. Paid for cutting 240 shocks corn @ $8\frac{1}{3}$¢, Sept. 15.
12. Paid for husking 1392 bu. corn @ 3¢, Nov. 20.
13. Paid for shelling 1336 bu. corn @ $\frac{3}{4}$¢, Jan. 10.
14. Sold 1336 bu. corn @ 38¢, Jan. 10.
15. Received pay for 3 mo. pasture @ \$1.50, Feb. 28.

1. These items are here arranged in the form of a receipt and expenditure account. Fill out on a separate sheet an account-form like the one below, and find the totals and the net profit for this forty-acre cornfield:

In Account with Forty-Acre Cornfield

Expenditures				Receipts			
Mar. 18	Removing stalks, 4 da., @ $1.15			Jan. 10	1336 bu. corn @ 38¢		
Apr. 30	Plowing, 9 da., @ $1.15			Feb. 28	3 mo. pasture @ $1.50		
May 1	Harrowing, 2 da., @ $1.00				Total		
" 18	Planting corn, 3½ da., @ $1.20				Expenditures to be deducted		
" 18	Seed corn, 6 bu., @ 90c				Net profit from 40 acres		
" 30	Replanting, 2 da., @ 1.15				Net profit per acre		
" 30	Harrowing corn, 5 da., @ $1.15				Account closed March 1, 1901		
June 15	Cultivating, 7 da., @ $1.15						
" 30	Cultivating, 6 da., @ $1.15						
July 15	Cultivating, 6 da., @ $1.15						
Sept. 15	Cutting 240 shocks corn @ 8½¢						
Nov. 20	Husking 1392 bu. corn @ 3¢						
Jan. 10	Shelling 1336 bu. corn @ ¼¢						
	Total						

2. Make a statement from these items of general expense, and find the total expenditure:

1. Apr. 20. Bought 3 sets of harness @ $28
2. " 25. Paid tax on 160 acres @ 25¢
3. " 30. Bought 1 wagon @ $60
4. " 30. Bought 2 plows @ $35
5. May 10. Bought 3 cultivators @ $25
6. Aug. 20. Paid 84 rods tile ditching @ 50¢
7. Aug. 30. Built corn crib, cost $112.00
8. Sept. 10. Paid for 120 mo. pasturing stock @ $1.50
9. Feb. 28. Paid for 45 mo. pasturing stock @ $1.25
10. " 28. Paid 30 da. fertilizing @ $1.25
11. " 28. Paid 8 da. mending fence @ $1.25

§139. Exercises for Practice.

1. Add until you can get correctly the sum for any one of these exercises in one minute:

(1)	(2)	(3)	(4)	(5)
3786	803	2863	30642	12
309	26099	18	96	864216
9167	1807	5	2875	7
43	391	680	758	3864
168	8786	85316	6	8
9	67	445	6412	4
28	732	78	7	7
887	6	8029	45369	629
21	42183	66	801	139621
4	27	3	94	5

2. Subtract. and prove by adding the difference to the subtrahend:

(1)	(2)	(3)	(4)	(5)
60821	164021	286002	780004	800103
5908	96479	190884	89746	73846

3. Two 3's are 6. How many 3's in 3 sixes? In 4 sixes? In 7 sixes? In 20 sixes?

4. Two 6's are 12. How many sixes in 4 twelves? In 6 twelves? In 10 twelves? In 18 twelves?

5. $2x=y$. How many x's make $4y$? $6y$? $5y$? $9y$? $15y$? $20y$? $30y$?

6. Three 8's are 24. How many 8's in 3 24's? In 5 24's? In 12 24's? In 20 24's?

7. Three 9's are 27? How many 9's in 4 27's? In 8 27's? In 7 27's? In 20 27's?

8. Multiply the following numbers mentally, writing only the products:

(1) 244×25
(2) 628×50
(3) $408 \times 12\frac{1}{2}$
(4) 372×11
(5) 146×250
(6) $108 \times 33\frac{1}{3}$
(7) $303 \times 66\frac{2}{3}$
(8) 176×125
(9) 212×99

9. Solve the following exercises mentally, writing only the quotients:

(1) $1200 \div 25$
(2) $5700 \div 300$
(3) $4500 \div 150$
(4) $15000 \div 500$
(5) $4200 \div 130$
(6) $6300 \div 90$
(7) $2460 \div 82$
(8) $2940 \div 98$
(9) $3270 \div 109$

10. Divide:

(1) 8450 by 26
(2) 10914 by 34
(3) 15281 by 37
(4) 12958 by 38
(5) 17864 by 44
(6) 24384 by 48
(7) 16686 by 54
(8) 23199 by 57
(9) 26752 by 64
(10) 32708 by 68
(11) 59422 by 73
(12) 79083 by 87

11. Divide:

(1) 9690 by 102
(2) 10769 by 121
(3) 13668 by 134
(4) 17150 by 175
(5) 19474 by 182
(6) 20304 by 216
(7) 66795 by 365
(8) 102910 by 502
(9) 190035 by 615
(10) 361398 by 899
(11) 1083903 by 901
(12) 1078032 by 1214

12. Divide:

(1) 6015356 by 1763
(2) 13036697 by 2683
(3) 13799478 by 3234
(4) 17713971 by 3807
(5) 24534861 by 4007
(6) 21443457 by 4237
(7) 32398582 by 5141
(8) 50537881 by 7109
(9) 133764048 by 13648
(10) 401292144 by 29403

§140. Rainfall and Crops.

1. About how many barrels of water fall on an acre in a 2-in. rainfall?

2. How many tons does this much water weigh?

3. The total rainfall for a year at a certain city is about 32 inches. What does the water weigh that falls here during the year on a square yard of ground? On a square rod? How many tons on an acre?

4. If the plants use $\frac{1}{8}$ of this, what is the weight of water used by an acre of vegetation?

5. How many pounds of water are there in the potatoes grown on an acre of ground, yielding 160 bu. per acre, if a bushel of potatoes contains 45 lb. of water?

IMPORTANT FERTILIZING SUBSTANCES IN FARM CROPS

Crop	Ounces per Bushel			Crop	Pounds per Ton		
	Nitrogen	Phosphoric Acid	Potash		Nitrogen	Phosphoric Acid	Potash
Wheat	20 oz.	8 oz.	5 oz.	Timothy or Red Top Hay	20 lb.	9 lb.	30 lb.
Rye	17 "	9 "	5 "	Clover Hay	40 "	10 "	40 "
Corn, shelled	14 "	5 "	3 "	Tobacco (leaves)	60 "	13 "	80 "
Barley	12 "	6 "	4 "	Straw (average)	10 "	4 "	20 "
Buckwheat	12 "	4 "	2 "	Sugar Beets	3 "	$\frac{1}{5}$ "	4 "
Oats	10 "	3 "	2 "				
Potatoes	3 "	1 "	4 "				
Root Crops Av'ge	3 "	1 "	2 "				

6. How many pounds of each of the 3 important fertilizers are there in a wheat crop that yields 13 bu. per acre? 26 bu. per acre? 45 bu. per acre?

7. Answer a like question for a corn crop of 60 bu. per acre. 70 bu. 228 bushels.

8. Answer a similar question for oats yielding 45 bu. per acre. 60 bu. 80 bushels.

9. Answer a similar question for a potato crop of 100 bu. per acre. 120 bu. 200 bushels.

10. Answer a similar question for a timothy hay crop of $1\frac{1}{2}$ tons per acre. 2 tons. $2\frac{1}{2}$ tons.

11. Corn is planted in hills 4 feet apart each way. How many hills to the acre? With 3 good ears to the hill, how many good ears to the acre?

12. Which is the better crop, 4 stalks to the hill that bear ears requiring 150 ears to make a bushel, or 3 stalks to the hill that bear ears requiring 100 to make a bushel?

13. How many more bushels per acre in the first crop of problem 12 than in the second?

14. If a 40-acre field produces 70 bu. of corn per acre the first year, but falls off 6 bu. per acre yearly when corn is grown continually, what will be the yield the 5th year?

15. What will be the total loss in the 5 years due to the decrease in yield? With corn at 40 cents a bushel, what is the money loss?

COMMON FRACTIONS

§141. Problems and Exercises.

1. Find the cost of $9\frac{1}{2}$ T. hard coal at $\$7\frac{3}{4}$.

CONVENIENT FORMS

(1)
$$\begin{array}{r} 7\frac{3}{4} \\ 9\frac{1}{2} \\ \hline \frac{3}{8}=\frac{1}{2}\times\frac{3}{4} \\ 3\frac{1}{2}=\frac{1}{2}\times 7 \\ 6\frac{3}{4}=9\times\frac{3}{4} \\ 63\ =9\times 7 \\ \hline 72\frac{13}{8}=73\frac{5}{8} \end{array}$$

(2)
$$7\tfrac{3}{4}=\tfrac{31}{4}$$
$$9\tfrac{1}{2}=\tfrac{19}{2}$$
$$\tfrac{31}{4}\times\tfrac{19}{2}=\tfrac{589}{8}=73\tfrac{5}{8}.$$

Ans. $\$73\frac{5}{8}$.

Either way may be used to check the other.

Find areas of surfaces having the following dimensions:

2. $44\frac{3}{7}$ feet long and $28\frac{3}{4}$ feet wide.

3. $36\frac{3}{8}$ feet long and $27\frac{2}{7}$ feet wide.

4. $24\frac{3}{4}$ feet long and $18\frac{2}{3}$ feet wide.

5. $45\frac{5}{6}$ feet long and $30\frac{1}{3}$ feet wide.

Find the cost of the following items:

6. $24\frac{3}{4}$ doz. eggs @ $\$.16\frac{2}{3}$.

7. $46\frac{3}{4}$ gal. of oil @ $\$.12\frac{1}{2}$.

8. $15\frac{1}{3}$ yd. of cloth @ $\$.66\frac{2}{3}$.

9. $52\frac{3}{4}$ lb. of sugar @ $\$.05\frac{5}{9}$.

10. $265\frac{3}{8}$ M. of pine flooring @ $\$35\frac{1}{4}$.

11. Make original problems from the following items:

(1) A cubic inch of water weighs $252\frac{2}{2}\frac{4}{5}$ grains.

(2) A water tower is $2\frac{4}{6}\frac{7}{3}$ ft. higher than the mound upon which it is built. The mound is $81\frac{1}{2}$ ft. high.

(3) $69\frac{1}{5}$ statute miles = 1 degree of longitude at the equator.

(4) $\frac{3}{8}\frac{1}{9}$ of a library of 55,447 volumes was burned.

(5) Flaxseed cost $\$\frac{2}{5}$ per pound when a certain linseed oil factory bought supplies.

(6) A spring furnishes $28\frac{7}{16}$ bbl. of water daily.

(7) A steam-boat makes $11\frac{3}{4}$ miles per hour, on an average.

(8) A room is 32 ft. long and $24\frac{5}{8}$ ft. wide. Painting costs $\$\frac{1}{5}\frac{1}{0}$ per square foot.

(9) A cubic foot of water weighs 62.5 pounds.

(10) Gold is $19\frac{1}{4}$ times as heavy as water.

(11) $30\frac{1}{4}$ sq. yd. = 1 square rod.

12. $\frac{7}{8} \div 5 = \frac{7}{x}$. What number does x stand for?

13. What number should stand in place of x in the following?

(1) $\frac{16}{17} \div 4 = \frac{x}{17}$ (7) $\frac{15}{3} \div 14 = \frac{15}{x}$

(2) $\frac{80}{37} \div 40 = \frac{x}{37}$ (8) $\frac{240}{173} \div 3 = \frac{240}{x}$

(3) $\frac{40}{47} \div 8 = \frac{x}{47}$ (9) $\frac{40}{45} \div 5 = \frac{40}{x}$

(4) $\frac{144}{167} \div 24 = \frac{x}{167}$ (10) $\frac{40}{45} \div 5 = \frac{x}{45}$

(5) $\frac{7}{9} \div 4 = \frac{7}{x}$ (11) $\frac{180}{211} \div 10 = \frac{x}{211}$

(6) $\frac{9}{10} \div 7 = \frac{9}{x}$ (12) $\frac{180}{211} \div 10 = \frac{180}{x}$

A fraction is divided by a whole number (1) *by dividing its numerator by the whole number, or* (2) *by multiplying its denominator by the whole number.*

14. A strip of tin-foil $\frac{8}{9}$ ft. long is to be cut across into 4 equal pieces. What is the length of one strip?

15. 4 children have a garden containing $\frac{2}{5}$ A. If the children have equal portions, what part of an acre does each have?

16. If $1\frac{1}{8}\frac{5}{}$ acres of farmland is divided equally among 7 men, how many acres will there be in each man's share?

17. Five measurements of last year's growth of twigs from the lower branches on the north side of an oak tree are: $5\frac{1}{2}''$, $6\frac{1}{4}''$, $4\frac{3}{4}''$, $5\frac{1}{2}''$, and $6\frac{1}{4}''$. What is the average growth of twigs on the north side?

18. From the lower branches on the east and west sides of the same tree the measurements are: $6\frac{3}{4}''$, $4\frac{3}{4}''$, $6\frac{1}{4}''$, $6\frac{1}{4}''$, and $6\frac{1}{2}''$. What is the average growth of twigs on the east and west sides?

19. For twigs on the south side of the same tree the measurements are: $6\frac{1}{4}''$, $6\frac{3}{4}''$, $6\frac{1}{4}''$, $8\frac{3}{4}''$, and $4\frac{1}{2}''$. What is the average growth of twigs on the south side of the tree?

20. From the middle branches of the oak tree 5 measurements of branches from the north side are: $5\frac{3}{4}''$, $4\frac{3}{4}''$, $7\frac{1}{2}''$, $3\frac{3}{4}''$, and $5\frac{1}{8}''$; of branches from the east and west sides are: $4\frac{3}{4}''$, $7\frac{1}{2}''$, $5\frac{1}{2}''$, $7\frac{1}{4}''$, and $6\frac{3}{4}''$; of branches from the south side: $5\frac{1}{2}''$, $6\frac{1}{2}''$, $7\frac{3}{4}''$, $6\frac{5}{8}''$, and $7\frac{1}{2}''$. How much greater is the average growth of twigs on the south side than the average growth of those on the north side of the tree? Than twigs from the east and west sides?

21. Make and solve problems based on these items:

(1) A ship sails $246\frac{3}{10}$ mi. in 36 hours.

(2) A man paid $\$195\frac{1}{4}$ for board for 72 days.

(3) A lot contains $2172\frac{1}{6}$ sq. rd. One side is $40\frac{3}{4}$ rd. long.

(4) 15 lb. of fresh salmon cost $\$2.73\frac{3}{4}$.

§142. Dividing by Inverting the Divisor.

A general method of dividing *any number* by a fraction is given here..

1 divided by $\frac{5}{6}$ equals $\frac{6}{5}$, and $1 \div \frac{6}{5} = \frac{5}{6}$; $1 \div 8 = \frac{1}{8}$, and $1 \div \frac{1}{8} = 8$..

The RECIPROCAL of any number, whole or fractional, is the number inverted.

Inverting a number, or what is the same thing, taking its reciprocal, is a short way of showing how many times the number is contained in 1.

1. If, then, the reciprocal of a fraction is multiplied by 6, what operation is performed?

2. Make a rule for quickly dividing any whole number by a fraction.

The reciprocal of any fraction shows how many times the fraction is contained in 1.

Any number, integral, fractional, or mixed, may be divided by a fraction by multiplying the number by the reciprocal of the divisor.

3. Why is the divisor inverted? Why is the inverted divisor multiplied by the dividend?

§143. Problems and Material for Problems.

1. $\frac{2}{9}$ yd. of silk costs $\$\frac{3}{4}$. Find the cost of 1 yard.

2. How many feedings will $5\frac{1}{4}$ bu. of oats supply, if each feeding is $\frac{3}{16}$ bushel?

3. How many pieces of wire $\frac{3}{4}$ ft. long can be cut from a piece $49\frac{1}{2}$ ft. long?

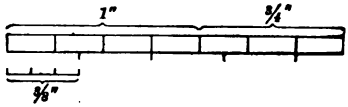


FIGURE 1

4. Measure $1\frac{3}{4}$ in. by $\frac{3}{8}$ in. (See Fig. 1.)

5. Find the number of strips of floor covering running the long way of the room:

WIDTH OF ROOMS	FLOOR COVERING
$22\frac{1}{2}$ ft.	$2\frac{1}{2}$ ft. wide
$7\frac{2}{3}$ yd.	$\frac{5}{6}$ yd. "
$18\frac{3}{4}$ ft.	$2\frac{1}{2}$ ft. "
$8\frac{1}{4}$ yd.	$\frac{5}{6}$ yd. "

6. Make and solve problems based upon these items:

(1) Portland cement $\$2\frac{3}{4}$ per bbl.; $9 purchasing fund.

(2) Sidewalk having an area of $607\frac{3}{4}$ sq. ft.; flagstones $5\frac{1}{4}$ ft. square.

(3) $15\frac{3}{8}$ gal. maple syrup; $\frac{3}{4}$ gal. measures.

(4) $29\frac{5}{6}$ doz. oranges; $\frac{9}{12}$ doz. oranges.

(5) The ratio of two numbers is $\frac{3}{5}$; one of them is $\frac{6}{7}$.

7. A man bought a house for $1860 which was $\frac{3}{5}$ as much as he paid for a mill. What was the cost of the mill?

8. Of a number of cars of grain inspected on a certain day 36 were rejected. This number was $\frac{1}{3}\frac{2}{5}$ of the entire number inspected. Find the number.

§144. Complex Fractions.

Any problem in division of fractions may be written in fractional form, the dividend being written above and the divisor below the dividing line.

Thus $\frac{6}{7} \div \frac{5}{11}$ may be written $\dfrac{\frac{6}{7}}{\frac{5}{11}}$

Or, $18\frac{2}{3} \div 6\frac{2}{5}$, may be written $\dfrac{18\frac{2}{3}}{6\frac{2}{5}}$

Fractions like $\dfrac{18\frac{2}{3}}{6\frac{2}{5}}$, $\dfrac{4}{\frac{2}{3}}$, $\dfrac{\frac{6}{7}}{\frac{5}{11}}$ that contain fractions in one or both terms, are called complex fractions.

The second principle of page 228 enables us to solve all such problems. If either the divisor or the dividend is a mixed number, it should first be reduced to an improper fraction.

Following this principle:

(1) $\dfrac{\frac{6}{7}}{\frac{5}{11}} = \frac{6}{7} \times \frac{11}{5} = \frac{66}{35} = 1\frac{31}{35}$

(2) $\dfrac{3\frac{2}{3}}{5\frac{3}{8}} = \dfrac{\frac{11}{3}}{\frac{43}{8}} = \frac{11}{3} \times \frac{8}{43} = \frac{88}{129}$

The second step in (1) becomes unnecessary if we notice that the product of the outside terms (6 and 11), called *the extremes*, gives the numerator, and that the product of the inside terms (7 and 5), called *the means*, gives the denominator of the result. Show that this is the same as dividing the product of *the extremes* by the product of *the means* in the given fractions.

A complex fraction is simplified when its terms are freed of fractions, and the result is expressed in its simplest form. Factors may often be cancelled from both numerator and denominator before multiplying, as follows:

(1) $\dfrac{\frac{1}{\cancel{6}}}{7} \times \dfrac{5}{\cancel{2}_{4}} = \frac{5}{7 \times 4} = \frac{5}{28}$ (2) $\dfrac{\frac{1}{\cancel{7}}}{\cancel{8}_{1}} \times \dfrac{\cancel{2}}{\frac{\cancel{2}}{\cancel{5}_{5}}} = \frac{1 \times 2}{1 \times 5} = \frac{2}{5}$

Simplify these complex fractions, cancelling when possible:

1. $\dfrac{\frac{3}{4}}{\frac{1}{2}}$ 4. $\dfrac{\frac{5}{16}}{\frac{4}{3}}$ 7. $\dfrac{\frac{2}{7}}{3}$ 10. $\dfrac{20\frac{2}{5}}{12\frac{3}{4}}$ 13. $\dfrac{84\frac{2}{7}}{62\frac{4}{5}}$

2. $\dfrac{\frac{5}{6}}{23}$ 5. $\dfrac{\frac{7}{12}}{\frac{3}{8}}$ 8. $\dfrac{15\frac{2}{3}}{12\frac{1}{2}}$ 11. $\dfrac{25\frac{2}{3}}{6\frac{3}{10}}$ 14. $\dfrac{123\frac{5}{9}}{116\frac{2}{3}}$

3. $\dfrac{\frac{7}{9}}{\frac{4}{15}}$ 6. $\dfrac{4}{6\frac{2}{3}}$ 9. $\dfrac{1\frac{5}{9}}{\frac{3}{7}}$ 12. $\dfrac{44\frac{4}{9}}{32\frac{2}{5}}$ 15. $\dfrac{\frac{a}{b}}{\frac{c}{d}}$

§145. Practical Applications for Review of Fractions.

1. The number of acres of corn in Indiana in 1911 was $\frac{18}{85}$, and the number of acres of wheat was $\frac{2}{7}$ of the entire acreage of all grains. What part of the state's entire grain acreage did both wheat and corn include?

2. Find the difference of the parts in wheat and in corn.

3. In 1911 about $\frac{4}{21}$ of the state's grain area was in oats. What part of the entire state's grain area did the corn, wheat, and oats acreages make up?

4. Find the difference between the fractions that represent the corn and the oats acreages, and tell what the result means.

5. Find the difference between the fractions representing the wheat and the oats acreages, and tell what the result means.

6. Of the total acreage in wheat of the five leading counties in 1911, the leading county had $\frac{1}{4}$, the second county $\frac{1}{5}$. What fraction represents the part of the acreage of the five counties that is in the remaining three counties?

7. These three remaining counties all had the same acreage. What fraction represents the acreage of one of these three counties?

8. If $\frac{11}{50}$ of the entire wheat acreage of Indiana in 1911 was in the leading ten counties, and $\frac{8}{25}$ was in the next 20 counties, what part of the state's wheat acreage was in all the rest of the counties?

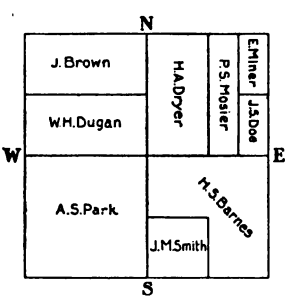
9. Find how many times the wheat acreage of the leading 10 counties is contained in the wheat acreage of the next 20 counties, using the numbers of problem 8.

10. If the average yield of an 18-acre wheat-field was $14\frac{3}{8}$ bushels per acre, what was the total yield?

11. If the average yield of Posey County's 75,876 acres of wheat in 1911 was $19\frac{1}{8}$ bushels per acre, what was the county's total yield?

MEASUREMENT

§146. Measuring Land.



SECTION OF LAND—FIGURE 1

A *section* of land is a tract 320 rd., or 1 mi., square. It is usually divided into halves, quarters, eighths, and sixteenths, as shown.

1. How many square rods make a section of land?

2. An acre = 160 square rods. How many acres are there in a section of land? In a quarter section?

3. A section of land is divided up into farms, as shown in Fig. 1. How many acres are there in the farm belonging to A. S. Park? To H. S. Barnes? To J. Brown? H. A. Dryer? J. M. Smith? P. S. Mosier? J. S. Doe?

4. Point out the south half of the section; the east half; the N $\frac{1}{2}$; the W $\frac{1}{2}$.

5. Point out the SE $\frac{1}{4}$; the NW $\frac{1}{4}$; the SW $\frac{1}{4}$; the NE $\frac{1}{4}$.

6. Point out the N $\frac{1}{2}$ of the SE $\frac{1}{4}$; the E $\frac{1}{2}$ of the SE $\frac{1}{4}$; the W $\frac{1}{2}$ of the NE $\frac{1}{4}$; the S $\frac{1}{2}$ NW $\frac{1}{4}$; the W $\frac{1}{2}$ E $\frac{1}{2}$.

7. Point out the SW $\frac{1}{4}$ of the SE $\frac{1}{4}$; the NE $\frac{1}{4}$ SW $\frac{1}{4}$; the NW $\frac{1}{4}$ NW $\frac{1}{4}$; the NE $\frac{1}{4}$ SW $\frac{1}{4}$.

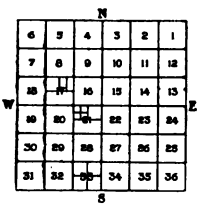
8. The farm of E. Miner would be described as the N $\frac{1}{2}$ E $\frac{1}{2}$ E $\frac{1}{2}$ NE $\frac{1}{4}$. Describe the farms of the other 8 owners of this section.

9. How many rods of fence would be required to enclose the section and separate it into the farms shown in the cut on page 232?

10. How many rods long is a ditch starting from the NE corner of H. A. Dryer's farm, thence running due south to H. S. Barnes's N line, thence due W to Barnes's W line, thence S to J. M. Smith's N line, thence due W across Park's farm to his W line?

11. Each small square in Fig. 1 represents a section. The large square represents a *township*. How long is a township? How wide?

12. How many square miles are there in 1 township? How many acres are there in 1 township?



TOWNSHIP—FIGURE 1

13. Point out in Fig. 1 the following:

(1) NW $\frac{1}{4}$ Sec. 33.

(2) S $\frac{1}{2}$ Sec. 17; NE $\frac{1}{4}$ Sec. 17; E $\frac{1}{2}$ NE $\frac{1}{4}$ Sec. 17.

(3) NE $\frac{1}{4}$ NW $\frac{1}{4}$ Sec. 21; SE $\frac{1}{4}$ NW $\frac{1}{4}$ Sec. 21.

14. How many farms of 160 A. each could be made of a township?

15. How long and how wide is a square farm which contains 160 acres?

§147. Plotting Observations and Measurements.

1. The hourly temperatures in a certain city from 6 A. M. to 6 P. M. December 26, 1902, were:

6 A. M.	7 A. M.	8 A. M.	9 A. M.	10 A. M.	11 A. M.
7°	7°	10°	10°	12°	14°

12 M.	1 P. M.	2 P. M.	3 P. M.	4 P. M.	5 P. M.	6 P. M.
14½°	14½°	15°	16°	15°	14½°	14½°

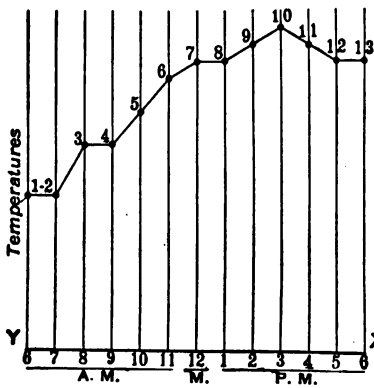


FIGURE 1

Draw 13 equally spaced vertical parallels, one for each hour. Draw a horizontal line as YX across the parallels. Using $\frac{1}{16}''$ to represent 1°, measure off on the first vertical the distance Y1 to represent 7°; on the second vertical the distance 7-2 to represent the second 7°, and so on. Mark the top of each distance with a dot. Draw a line connecting these dots as in Fig. 1. This line shows the temperature change during the 12 hours. *This is called plotting the readings.*

This plotting is helped by the use of paper ruled into small squares. A horizontal side of a small square may represent 1 hr. and a vertical side 1°.

Pupils should have notebooks of squared paper for this work.

2. Read the out-door temperatures from hour to hour at your school and plot the readings as in problem 1.

3. The average hourly temperatures from 6 A. M. to 6 P. M., December 26 to January 2, were:

6 A. M.	7 A. M.	8 A. M.	9 A. M.	10 A. M.	11 A. M.
21.5°	22.1°	22.2°	22.2°	22.5°	23.2°

12 M.	1 P. M.	2 P. M.	3 P. M.	4 P. M.	5 P. M.	6 P. M.
23.9°	24°	24.5°	25.2°	25°	24.7°	24.3°

Plot these readings on squared paper, or by measurement, as in Figure 1. Plot the tenths by estimate.

4. Does the line for the averages for a week agree in a general way with the line for December 26? What does a comparison of the two lines show?

This table gives heights and weights of boys and girls.

Age	Height in Feet		Weight in Pounds	
	boys	girls	boys	girls
2	2.6	2.6	25.0	23.5
4	3.0	3.0	31.4	28.7
6	3.4	3.4	38.8	35.3
9	4.0	3.9	50.0	47.1
11	4.4	4.3	59.8	56.6
13	4.7	4.6	75.8	72.7
15	5.1	4.9	96.4	89.0
17	5.4	5.1	116.6	104.4
18	5.4	5.1	127.6	112.6
20	5.5	5.2	132.5	115.3

5. Using squared paper, or a drawing like Fig. 1, plot the numbers in the first column horizontally and those in the second column vertically. Draw a line through all the points and obtain the curve for growth of boys. At what age do boys cease growing rapidly in height?

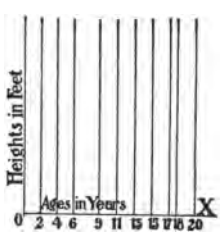


Figure 1

6. Using the numbers in columns 1 and 3 obtain a similar curve for girls. At what age do girls cease growing rapidly in stature?

7. When do boys grow fastest? Girls?

8. Using the numbers in columns 1 and 4 draw the curve for growth of boys in weight.

9. Use numbers in columns 1 and 5 similarly for girls.

10. Compare the curves of problems 8 and 9 and note any likenesses or differences in the two curves.

11. Plot the following 12 lengths of sides and areas in square inches of squares, using any convenient horizontal and vertical scales:

Sides...	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″	12″
Areas..	1	4	9	16	25	36	49	64	81	100	121	144

Do the plotted points lie on a straight line?

Date of Census	Population of U. S. in Millions	Date of Census	Population of U. S. in Millions
1790	3.9	1850	23.2
1800	5.3	1860	31.4
1810	7.2	1870	38.6
1820	9.6	1880	50.2
1830	12.9	1890	62.6
1840	17.1	1900	76.3

12. Plot the above dates on horizontal, and populations on vertical, lines.

Draw a smooth free-hand curve through the plotted points. By continuing the curve, can you estimate approximately the 1910 population?

THE EQUATION

§148. Balance of Values.

1. y lb. of sugar are placed on the left scale pan, and a weight of 10 lb. on the right pan balances it (see Fig. 1). How heavy is y?

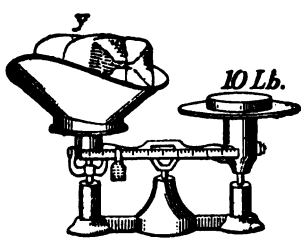


Figure 1

The balance of these two weights is expressed thus:

$$y = 10. \qquad \text{(I)}$$

This expression is called an equation, and is read "y equals 10."

2. If a 4-lb. weight is now added to the right pan, how many additional pounds of sugar must be placed upon the left pan to balance the

scales? Write an equation to express the relation between the weights now in the pans.

3. If x lb. on the right balance y lb. on the left, how would you state the fact in an equation?

4. If 20 lb. are added to the x lb. already on the right, how many pounds must be added on the left to balance the equation?

5. If 35 is added on the right side of equation (I), page 236, what change must be made on the left side to balance the equation? Write the equation thus changed.

Just as the horizontal position of the scale beam shows that there is a balance of the weights on the pans, so the equality sign (=), shows that there is a *balance of value* of the numbers between which it stands. It must never be used between two numbers, or combinations of numbers, that are not equal in value.

The number on the left of the sign of equality is called the first member, or the left side of the equation. The number on the right is called the second member, or the right side of the equation.

§149. Exercises.

1. Four equal weights and a 5-lb. weight just balance 21 pounds (Fig. 1, p. 236.) How heavy is one of the equal weights?

$4x+5=21$. Subtract 5 from both sides.

$\quad 5 \quad 5$

$4x \quad =16$. If $4x=16$, what is x? What is the answer to the problem?

2. Find the value of the letter:

(1) $3y+2=14$ (3) $7x+8=29$ (5) $15a+3=48$

(2) $5x+12=32$ (4) $9z+3=75$ (6) $12x-9=27$

$12x-9=27$. Add 9 to both sides.

$\quad 9 \quad 9$

$12x \quad =36$. If $12x=36$, one x, or x, equals what?

3. Solve for y:

(1) $22y-6=38$	(5) $25y+12=87$
(2) $18y-9=45$	(6) $17+27y=98$
(3) $28y-6=50$	(7) $36y-20=124$
(4) $30y-10=50$	(8) $45y-42=93$

4. Solve for t:

(1) $3t+7=28$	(5) $32t-4=60$
(2) $5t-8=17$	(6) $48t+6=102$
(3) $9t+5=41$	(7) $42t-5=79$
(4) $12t-8=40$	(8) $39t+7=124$

5. Express by an equation that 3 times an unknown number, n, increased by 7 is 28.

$$3n+7=28$$

6. Find from the equation, $3n+7=28$, the number that n stands for.

7. Write an equation showing that 5 times an unknown number, x, decreased by 8 is 17, and find the number x stands for.

8. In a pile of 12 books a certain number, n, are histories, and the remainder, twice as many, $2n$, are arithmetics. Show that $n+2n$, or $3n=12$. Find the number n.

9. In a pile of 13 readers and story books, x are readers and there are 3 more story books than readers. Show that $x+x+3$, or $2x+3=13$. Find the numbers x and $x+3$ stand for and tell what they mean.

10. In a class of 24 pupils there are x boys and twice as many girls. How many boys in the class? How many girls?

11. In a 6th grade of 48 pupils, there are 8 more girls than boys. How many boys in the grade? How many girls?

12. In a month of April there were twice as many cloudy, and 3 times as many partly cloudy, as clear days. How many days of each kind were in this month?

RATIO AND PROPORTION

§150. Ratio.

Relations like 1 to 2, 3 to 1, 12 to 13, $\frac{1}{2}$ to $\frac{2}{3}$, a to x, are called **ratios.** See page 125.

1. Other ways of writing the ratio 1 to 2 are 1:2, and $\frac{1}{2}$. Give other ways of writing ratios, 3 to 1; 12 to 13; $\frac{1}{2}$ to $\frac{2}{3}$.

The two numbers of a ratio are called terms of the ratio. The first term is the antecedent, and the second term is the consequent.

A ratio, like a fraction, is in its simplest form when the terms are the smallest possible whole numbers.

2. Express the following ratios in their simplest forms:

(1) 2:4 (3) 12:15 (5) $\frac{2}{3}:\frac{1}{3}$ (7) 120:12
(2) 8:12 (4) 75:90 (6) $\frac{7}{8}:\frac{4}{5}$ (8) $5a:25a$

The value of a ratio is the quotient of the antecedent divided by the consequent.

3. Compare the values of the following pairs of ratios:

(1) 6:3 and 4:2 (4) $\frac{3}{4}$ and $\frac{12}{16}$ (7) 7:14 and $\frac{14}{28}$
(2) 1:4 and 5:20 (5) $\frac{5}{7}$ and $\frac{25}{35}$ (8) 9:90 and $\frac{2}{20}$
(3) 2:9 and 8:36 (6) $\frac{2}{6}$ and $\frac{8}{36}$ (9) $\frac{3}{7}$ and $\frac{24}{50}$

§151. Proportion.

A PROPORTION is an equation of ratios. Thus 6:12 = 3:6, and $\frac{6}{12} = \frac{3}{6}$ are two different ways of writing the proportion.

The first, second, third, and fourth numbers of the proportion are called the first, second, third, and fourth terms of the proportion.

The first and fourth terms are called the extremes, and the second and third terms, the means.

1. Which pairs of ratios in problem 3 above do not make a proportion?

2. Write two ratios that make a proportion. Make a proportion of them.

3. What are the means of the proportion you made in problem 2? What are the extremes?

4. Compare the product of the means with the product of the extremes in all parts of problem 3, §150, that make proportions. What seems to be true of the products?

In a proportion, the product of the means equals the product of the extremes.

5. Tell which of the following are proportions, by comparing the products of the means and the extremes:

(1) $2:3 = 8:12$ (5) $\frac{1}{2}:8 = 2:32$ (9) $\frac{7}{8} = \frac{14}{16}$

(2) $4:16 = 1:5$ (6) $\frac{1}{3}:\frac{1}{12} = \frac{1}{6}:\frac{1}{18}$ (10) $\frac{5}{7} = \frac{40}{56}$

(3) $7:21 = 9:27$ (7) $3:9 = \frac{1}{2}:\frac{3}{2}$ (11) $\frac{9}{10} = \frac{18}{22}$

(4) $1:40 = 2:81$ (8) $2:\frac{1}{2} = 8:3$ (12) $\frac{15}{25} = \frac{5}{1}$

6. The first expression of problem 5 is read "2 to 3 as (or equals) 8 to 12"; and the ninth is read "7 to 8 equals 14 to 16." Read all the expressions of problem 5.

7. Find the value of a in the proportion $4:6 = 8:a$.

As the product of the means equals the product of the extremes, $4 \times a = 6 \times 8$, or $4a = 48$; $a = \frac{1}{4}$ of $48 = 12$, Ans.

8. What does the letter stand for in each of these proportions?

(1) $\frac{3}{8} = \frac{12}{a}$ (6) $\frac{14}{a} = \frac{7}{12}$ (11) $\frac{5}{x} = \frac{13}{65}$

(2) $\frac{3}{7} = \frac{b}{35}$ (7) $\frac{a}{10} = \frac{16}{40}$ (12) $\frac{11}{44} = \frac{d}{8}$

(3) $\frac{8}{7} = \frac{x}{63}$ (8) $\frac{2a}{16} = \frac{9}{36}$ (13) $\frac{23}{161} = \frac{a}{14}$

(4) $\frac{2}{11} = \frac{16}{m}$ (9) $\frac{x}{18} = \frac{36}{324}$ (14) $\frac{32}{4} = \frac{1}{n}$

(5) $\frac{3}{c} = \frac{21}{30}$ (10) $\frac{3}{y} = \frac{1}{5}$ (15) $\frac{12}{3} = \frac{2}{m}$

§152. Problems.

Use pencil only when needed

1. A boat runs 30 miles in 2 hours. How many miles will it run in 6 hours at the same rate? In 12 hours? In 20 hours?

Solutions for 20 hours:

(1) As $\frac{2}{20} = \frac{30}{x}$; $2x = 600$, or $x = 300$. Ans. 300 mi.

Or (2) As 20 is 10 twos, the boat will run 10×30, or 300 mi. in 20 hours.

2. A steamboat runs 72 sea miles in 3 hours. How many sea miles will it run in 18 hours at same rate? In 15 hours? In 30 hours? In $16\frac{1}{2}$ hours? In $1\frac{1}{2}$ hours?

3. Three sea miles equal 3.45 land miles. How many land miles are there in 6 sea miles? In 24 sea miles? In 72 sea miles? In $\frac{1}{3}$ sea mile? In $33\frac{1}{3}$ sea miles? In 100 sea miles? In 1 sea mile?

4. If 10 acres of land yield 440 bushels of corn, at the same rate of yield how many acres would be needed to yield 220 bushels? 660 bushels? 1100 bushels? 4400 bushels?

5. A tree 65 feet high casts a shadow 60 feet long. What is the height of a tree near it which casts a shadow 30 feet long, at the same moment? 90 ft. long? 120 ft. long? 13 ft. long? 5 ft. long? 25 ft. long?

6. A clock ticks 7 times in 5 seconds. How many times does it tick in 2 da. 6 hours?

7. If 60 A. cost \$3000, how many acres will cost \$2450 at the same rate?

8. The line BC (Fig. 1) represents 20 rods. FE is twice as long, AF four times as long, CD and ED five times as long, and AB six times as long. Find the length of each side.

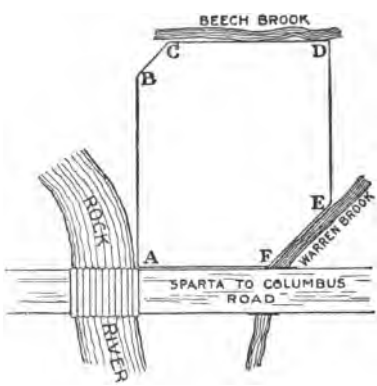


FIGURE 1

9. Find the value of x in the following:

(1) BC : FE = AB : x
(2) AF : FE = x : BC
(3) BC : x = FE : CD
(4) x : BC = FE : AF

10. When a foot-rule is held $2\frac{1}{2}$ ft. (arm's length) in front of the face as shown in Figure 1, 7 in. on the ruler seems just to cover the edge of a door 7 ft. high. If the ruler is held parallel to the edge of the door, how far is the eye from the door?

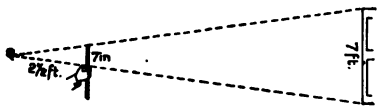


FIGURE 1

11. If the shadow cast by a 4-ft. stake is 5 ft. long, what is the height of a tree near by which casts a shadow 50 ft. long on the same day and hour?

12. The shadow cast by a $3\frac{1}{2}$-ft. stake is $10\frac{1}{2}$ ft. long. What is the height of a flagpole near by which casts a shadow 225 ft. long at the same time?

13. How high is a house which casts a shadow 120 ft. long at the moment when a 6-ft. stake near it casts a shadow 20 ft. long?

14. A road runs 2 miles south, then 1 mile west, then $\frac{3}{4}$ mi. south, then $1\frac{1}{2}$ mi. southeast, and then $2\frac{1}{2}$ mi. south. In a scale drawing of the road the two-mile part is represented by a line 18 in. long. How long a line will represent each of the other parts of the road in the same drawing?

15. What is the scale of the drawing just mentioned?

16. What is the scale of a drawing in which a distance of 20 ft. is represented by a line $2\frac{2}{5}$ in. long? By a line $\frac{4}{5}$ in. long? By a line $2\frac{1}{2}$ ft. long?

17. The food parts of beef are water, fat, and protein. In 1 lb. of loin steak there are $8\frac{2}{5}$ oz. of water, $2\frac{4}{5}$ oz. fat, $2\frac{3}{5}$ oz. protein, and $2\frac{1}{5}$ oz. of waste material. How much of each of these parts are there in $3\frac{1}{2}$ lb. of loin steak? In $5\frac{1}{2}$ lb.? In a $12\frac{3}{4}$-pound loin steak?

18. In 4 lb. sirloin steak there are $34\frac{2}{5}$ oz. water, $10\frac{2}{5}$ oz. fat, $10\frac{2}{3}$ oz. protein, and $8\frac{8}{15}$ oz. waste matter. How much of each is there in an $8\frac{1}{3}$-lb. sirloin steak? In 24 lb.?

19. In a 10-lb. porterhouse steak there are $5\frac{1}{4}$ lb. water, $1\frac{3}{4}$ lb. fat, $1\frac{2}{3}$ lb. protein, and $1\frac{1}{3}$ lb. waste. How much of each in a 28-lb. porterhouse steak? In 7 lb.? In 48 pounds?

DECIMAL FRACTIONS

§153. Exercises.

1. Add the following and note time required:

(1)	(2)	(3)	(4)
16.25	3.875	2068.121	63.2471
206.09	40.161	110.011	2.0604
8.26	325.654	1216.714	1000.
$.75\frac{1}{2}$	.079	618.	2.0095
9.28	14.687	.928	316.
$10.06\frac{1}{2}$	90.896	3.2	.78

(5)	(6)	(7)	(8)
68.35	832.06	8.012	384.129
8.76	15.75	700.07	35.7
98.63	5.4	17.38	8129.15
.89	1268.	1236.5	7.
2.57	86.37	.9	463.0086
.24	200.02	3.875	92015.215
83.64	1683.1	7.9	865.3

2. Subtract the following and check by adding subtrahend and remainder, and note time required:

(1)	(2)	(3)	(4)
8700	58.00	1062.700	9036.0900
7916	19.64	898.807	972.0168

3. Multiply the following:

(1) 487.3×3.63 (4) 29.847×2.48 (7) $64.8 \times 32.8\frac{1}{4}$
(2) 128.75×4.42 (5) $867.29 \times .393$ (8) $78.9 \times .33\frac{1}{3}$
(3) $78.53 \times .639$ (6) $78.430 \times .217$ (9) $48.64\frac{2}{3} \times 15\frac{7}{8}$

§154. Division, and Pointing the Quotient.

We have seen that one way to point a quotient is:

First, divide as with whole numbers, and then move (or imagine moved) the decimal points in dividend and divisor over the same number of places toward the right, or left, until *but one digit stands on the left of the point in the divisor.* Then, neglecting the fractional parts of dividend and divisor, estimate about what the quotient must be, by asking about what is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{9}$, etc., of the integral part of the dividend.

1. Solve the following exercises, pointing by this method:

(1) $211.248 \div 32.6 = ?$
(2) $2112.48 \div 64.8 = ?$
(3) $21.1248 \div 32.6 = ?$
(4) $21124.8 \div 6.48 = ?$
(5) $1768.96044 \div 24.831 = ?$
(6) $176.896044 \div 24.831 = ?$
(7) $17689.6044 \div 248.31 = ?$
(8) $.3312 \div .96 = ?$

2. Divide and point the quotients as explained in problem 1, and check the work by multiplication:

(1) $989.440 \div 25.6 = ?$
(2) $182.428 \div .472 = ?$
(3) $3398.340 \div 83.6 = ?$
(4) $13690.355 \div 416.5 = ?$
(5) $1551.0138 \div 27.51 = ?$
(6) $338.51584 \div .6032 = ?$

Problem 3 shows another way to point the quotient.

3. $933.0188 \div 36.82 = ?$

```
      25.34
36.82)933.01'88
      7364
      19661
      18410
       12518
       11046
        14728
        14728
```

Mark off as many places to the right of the decimal point in the dividend as there are decimal places in the divisor. The quotient is a whole number until this mark is reached, and the rest is a decimal.

Check: $36.82 \times 25.34 = 933.0188$.

4. Divide, pointing the quotient as in problem 3:

(1) $98.944 \div 38.65 = ?$ (4) $1369.0355 \div 4.165 = ?$
(2) $1824.28 \div 386.5 = ?$ (5) $155.10138 \div 5638 = ?$
(3) $339.834 \div 4.065 = ?$ (6) $3385.1584 \div 5612 = ?$

Either of these ways of pointing the quotient amounts to this third way:

Make the number of decimal places in the quotient the difference between the number of decimal places in the dividend and the divisor.

Any one of these three ways may be used to check another.

5. Divide, pointing as you prefer, and check the work:

(1) $4062.824 \div 105.2 = ?$ (4) $13876.778 \div 328.6 = ?$
(2) $792.4824 \div 38.62 = ?$ (5) $138978.93 \div 42.23 = ?$
(3) $8348.970 \div 116.2 = ?$ (6) $2234.3893 \div .5291 = ?$

§155. Ratio of Circumference of Circle to Diameter.

1. The distance around the rung of a chair was measured and found to be 3.625 in.; the diameter was found to be 1.153 in. Divide the distance around the rung by the diameter and find the quotient to the nearest .001.

The distance around a circle is the circumference of a circle.

2. The circumference of a circular rod, 1.875 in. in diameter, was measured and found to be 5.884 inches. Find to the nearest .001, the ratio of the circumference to the diameter.

Object	Circumference	Diameter	Ratio
Ink bottle	5.5 in.	1.75 in.	
Tin box	6.154 "	1.953 "	
Globe of lamp	26.538 "	8.444 "	
Terrestrial globe	57.080 "	18.062 "	
Barrel-head	71.458 "	22.750 "	
Iron ring	18.125 "	5.675 "	
Bicycle wheel	81.177 "	26.025 "	
Front carriage wheel	151.189 "	48.125 "	
Rear carriage wheel	176.333 "	56.125 "	
Average			

3. Measure the diameters and the circumferences of any circles in your schoolroom and find to the nearest .001, the ratio of their circumferences to their diameters. If no circles are at hand use the measures of the table.

4. It is proved in Geometry that the ratio of the circumference of any circle to its diameter is about $3\frac{1}{7}$, or more accurately 3.1416. If, then, the diameter of a circle is known, how may the circumference be found without measurement?

5. Find the average of all the values of the ratios found in problems 1 to 3 and compare this last average with $3\frac{1}{7}$; with 3.1416. What is the difference in each case?

For most purposes this ratio, denoted by the Greek letter π, called *pi*, may be taken as $3\frac{1}{7}$. For greater accuracy use $\pi = 3.1416$.

§156. Area of a Circle.

1. Draw a circle and diameter AB (see Fig. 1).

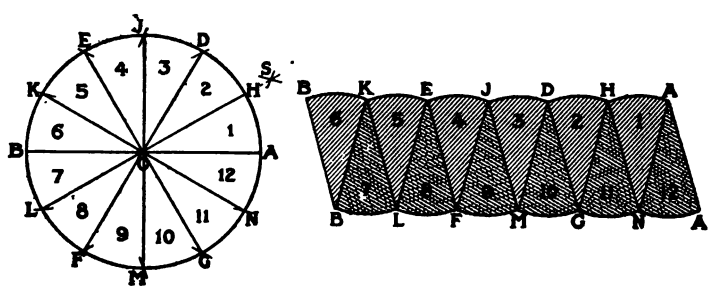


FIGURE 1

With an opening of the compasses equal to the radius of the circle and with A as center draw short arcs at D and G; also, with same radius and with B as center, draw short arcs at E and F. Draw radii OD, OE, OF, and OG.

With compasses opened wider than the distance, DH, and with A as a center, draw one of the arcs at S. With the same opening, draw the other arc with D as a center.

Connect SO. SO divides the angle, AOD, and the arc, AD into two equal parts.

With an opening of the compasses equal to the distance between D and H and with center D draw an arc at J; with center B draw arcs at K and L; also, with center G draw arcs at M and N. Draw radii OJ, OK, OL, OM, and ON.

Cut the circle into the twelve parts (sectors) thus formed and place these sectors as shown on the right-hand part of the figure. How long is the curvilinear base AB of the *approximate* parallelogram thus formed? How wide is the approximate parallelogram?

2. If each of the 12 sectors were split into halves and the 24 sectors fitted together how long and how wide would the new *approximate* parallelogram be? What would be its area?

3. Having the circumference and the radius of a circle how can you find the area of the circle?

4. Having the radius of any circle how can you find the area of the circle (See pp. 245 and 246)?

Let r be the length of the radius, c the circumference, and A, the area of a circle.

Work problems 5 to 7 in two ways: (1) using both r and c, and (2) using r and π $(=3\frac{1}{7})$ and the equation, $A=\pi\times r^2$.

5. Find the area, A, of each of these circles:

(1) $r = 12.5'$, $c = 78.57'$ (3) $r = 20''$, $c = 125.714''$
(2) $r = 6.25'$, $c = 39.29'$ (4) $r = 60''$, $c = 377.143''$

6. The diameter of a drum-head is 2.5′ and the circumference is 7.857′. How many square inches of skin are in the two circular heads?

7. The diameter of a circular window is 18.5″ and its circumference is 58.143″. What is its area?

In the following problems π is taken as 3.1416. Results should be calculated to the nearest .001.

8. The wind is blowing squarely against a circular signboard, whose diameter is 32.75 ft., with a pressure of 25.5 lb. to the square foot. Find the total wind pressure against the board.

9. The circumference of a cylindrical chair rung is 5.5″; find the diameter and area of a right section.

A right section of the rung is the section that would be made by sawing the rung square across.

10. Find the area, A, of a circle whose radius is r ft. long and whose circumference is c ft. long.

11. Find the circumference, c, of a circle whose radius is r rods long. Find the area.

12. A cow is tied to the corner of a corn crib, $18' \times 18'$, by a rope 18′ long. Over how many sq. ft. can she graze?

§157. Problems on Crops.

The table gives the number of acres and the number of bushels of the corn crops of 1910 and 1911 for the ten counties of Indiana that led in corn yield in 1911:

County	Acres in Thousands		Bushels in Millions	
	1911	1910	1911	1910
Allen.....	74.86	69.68	3.233	2.522
Benton.....	104.16	100.20	4.359	3.867
Cass.....	64.46	62.?9	2.797	2.305
Clinton.....	74.64	76.67	3.473	3.622
Knox.....	76.56	81.37	2.817	3.360
Rush.....	80.14	76.07	3.877	3.934
Shelby.....	83.69	80.36	3.201	3.326
Tippecanoe..	95.53	89.72	3.770	3.543
Warren.....	76.59	72.04	2.772	2.781
White.....	95.34	90.29	4.003	3.331
Totals,				
Whole state..	4345.18	4297.88	155.100	156.521

1. Find the total of each of the four columns and tell what it means.

2. Find the difference in corn acreage from 1910 to 1911 for each of the 10 counties.

3. Find the difference in millions of bushels from 1910 to 1911 for each of the 10 counties.

4. What decimal part of the state's entire corn acreage was in these 10 counties in 1910? In 1911? Give results to the nearest .01.

5. What part of the state's entire yield in bushels was in these 10 counties in 1910? In 1911? Results to the nearest .01.

6. Annex whatever zeros are needed to express the numbers of the table in acres and in bushels, and find to the nearest .1 bu. the average yield of corn per acre for Allen County in 1910. In 1911.

7. Similarly find the average yield per acre for each of the other counties.

8. In the same way from the last line of numbers of the table find the average yields per acre for the whole state for 1910; for 1911.

9. In 1911 there were 80.3 thousand acres of oats in Benton County, Ind., that yielded 2674.08 thousand bushels. Find to the nearest .1 bu. the average yield per acre of oats for this county.

10. The same year White County had 65.42 thousand acres of oats, yielding 2131.53 thousand bushels. Find to the nearest .1 bu. the average yield per acre for White County.

11. The same year Warren County had 55.59 thousand acres, yielding 1647.34 thousand bushels. Find to the nearest .1 bu. the average yield per acre.

12. Find similarly the average yields per acre for Allen and Newton Counties, the numbers being: Allen County, 54.34 thousand acres, yielding 2184.86 thousand bushels, and Newton County, 49.18 thousand acres, yielding 1575.27 thousand bushels.

13. Collect the results of problems 9 to 12, and give the difference of yield per acre of each from the largest yield.

14. In 1911 Dearborn County, had 5.92 thousand acres of alfalfa, yielding 10.93 thousand tons. Find to the nearest .01 ton the average yield per acre.

15. The same year Lake County had 34.33 thousand acres of timothy yielding 37.42 thousand tons of hay. Find to the nearest .01 ton the average yield per acre.

16. In 1910 the State of Indiana had 1.003 million acres of clover meadow, yielding 1.108 million tons of hay. Find to the nearest .01 ton the average yield of clover hay per acre for the whole state that year.

17. Find to the nearest .1 bu. the average wheat yield per acre of Indiana for 1911 from the numbers 2.403 million acres, yielding 34.596 million bushels.

18. Find to the nearest .1 bu. the average potato yield per acre of Indiana for 1911 from the numbers 43.33 thousand acres, yielding 2.532 million bushels.

PERCENTAGE

§158. Percent as How Many in 100.

Solve all you can orally

In a city district there are 100 buildings. 50 of them are stores; 40 are dwelling houses; 5 are warehouses; 3 are hotels, and 2 are banks. 10 of them are built of brick; 30 are stone, and 60 are frame houses; 35 of them are 3 stories high; 25 are two stories, and the rest are one story.

1. What part of the buildings are stores? How many hundredths of the buildings are stores?

"How many percent?" "What percent?" means "How many in a hundred?" or "How many hundredths?"

2. How many percent of the buildings are stores?

3. $\frac{1}{2}$ equals how many percent?

4. What part of the buildings are 2 stories high? What percent of them are 2 stories high?

5. What percent are warehouses? Hotels? Banks? Dwellings?

6. What percent are brick? Stone? Frame?

7. What percent are 3-story buildings? 1-story buildings?

8. What part of the buildings are warehouses? $\frac{1}{20}$ equals how many percent?

9. What part of the buildings are dwellings? $\frac{2}{5}$ equals how many percent?

10. What part of the buildings are frame houses? $\frac{3}{5}$ equals how many percent?

A sidewalk 200 rd. long is made up of 10 rd. of stone walk, 25 rd. of cement walk, 40 rd. gravel walk, 50 rd. board walk, and 75 rd. cinder walk.

11. What percent of the entire walk is stone? What percent cement? Gravel? Board? Cinder?

12. What percent of the whole was the stone and the cement together? The cement and the cinder? The gravel and the board? The stone and the board?

In a certain nursery there are 100 oak, 75 ash, 50 maple, 30 elm, 25 hickory, 15 poplar, and 5 walnut trees.

13. What percent of the trees are oak? What part are oak?

14. What percent of the trees are ash? What part are ash?

15. What percent of the trees are maple? Elm? Hickory? Poplar? Walnut?

16. Of 100 kinds of animals of a neighborhood, 27 die, 18 migrate, and 10 hibernate as winter comes on. What percent die? Migrate? Hibernate?

17. 10 kinds store food, 20 remain and feed abroad, and 10 kinds appear only in winter. What percent store food? Feed abroad? Appear only in winter?

§159. Percent as Fractional Parts and as Hundredths.

Answer all you can orally

1. \$2 equals what part of \$8? 9 bu. equals what part of 12 bu.? 8 hr. equals what part of 24 hr.? \$6 of \$9? 18 of 27? 27 of 36?

2. To how many hundredths of a number are the following fractional parts of that number equal?

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{1}{20}$ $\frac{3}{20}$ $\frac{4}{25}$

3. Express the following as hundredths:

$\frac{3}{50}$ $\frac{9}{25}$ $\frac{7}{20}$ $\frac{9}{10}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{12}$ $\frac{5}{12}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{3}{16}$

The sign, "%," is a short way of writing *percent*, or *hundredths:* thus, 2%, 8%, $33\frac{1}{3}\%$, mean $\frac{2}{100}$, $\frac{8}{100}$, $\frac{33\frac{1}{3}}{100}$, and are read, "2 percent," "8 percent," "$33\frac{1}{3}$ percent."

The number (as $2, 8, 33\frac{1}{3}$), written before the sign, "%," is called the *rate percent.*

It is well to recall that we have the following ways of writing such numbers as 6 percent: (1) 6 percent; (2) 6%; (3) 6 hundredths; (4) $\frac{6}{100}$; and (5) .06. The sign, "%," is merely an abbreviation for $\frac{1}{100}$, or .01.

4. In a schoolroom containing 40 pupils, $\frac{3}{5}$ of the pupils were girls. How many girls were there?

5. In problem 4, what percent of the pupils were girls?

6. In the month of September, 12 days were cloudy. What percent of the days were cloudy?

7. A sample stick of timber, weighing 19 lb., contained 2 lb. of water. What percent of the weight of the timber was due to the water it contained?

Annex zeros after the numerator and divide thus:

$$19)\overline{2.00} \qquad \text{But } .10\tfrac{10}{19} = 10\tfrac{10}{19}\%. \quad \text{Why?}$$
$$\overline{.10\tfrac{10}{19}}.$$

8. The first number of the following pairs is what percent of the second?

(1) 8 of 150	(4) $4\frac{5}{8}$ of 60	(7) 32.91 of 263.28
(2) 13 of 700	(5) $27\frac{3}{5}$ of 600	(8) $\frac{1}{4}$ of $\frac{1}{2}$
(3) $7\frac{1}{2}$ of 12	(6) 38.45 of 769	(9) $\frac{3}{5}$ of $12\frac{1}{2}$

9. Change the following percents to their fractional equivalents (fractions in their lowest terms):

5%	8%	$8\frac{1}{3}\%$	$12\frac{1}{2}\%$	12%
16%	$16\frac{2}{3}\%$	$22\frac{1}{2}\%$	$67\frac{1}{2}\%$	$87\frac{1}{2}\%$.

10. Change to their decimal equivalents the following:

7%	$6\frac{1}{4}\%$	$83\frac{1}{3}\%$	$87\frac{1}{2}\%$	$\frac{1}{10}$ of 1%
$2\frac{1}{2}\%$	$2\frac{3}{4}\%$	$\frac{7}{8}\%$	$1\frac{1}{2}\%$	$\frac{1}{100}$ of 1%

§160. Decimals as Percents.

Answer all orally and rapidly.

1. What is .10 of 40? 10% of 40?

2. What is .20 of 30? 20% of 30?

3. What is .05 of 60? 5% of 60?

4. What is .60 of 20? 60% of 20?

5. What is .50 of 64? 50% of 64?

6. What is .25 of 48? 25% of 48?

7. What is $.12\frac{1}{2}$ of 16? $12\frac{1}{2}\%$ of 16?

8. What is $.37\frac{1}{2}$ of 24? $37\frac{1}{2}\%$ of 24?

9. What is $.33\frac{1}{3}$ of 27? $33\frac{1}{3}\%$ of 27?

10. What is .75 of 36? 75% of 36?

11. What is .01 of 200? 1% of 200? .02 of 200? 2% of 200? .03 of 300? 3% of 300?

12. Give in percents the equivalents of:

.12, .29, .35, $.12\frac{1}{2}$, .75, .125, .265, .375, .625, .875.

13. Give as decimal fractions, the equivalents of:

6%	15%	3%	$12\frac{1}{2}\%$	$33\frac{1}{3}\%$
$37\frac{1}{2}\%$	$62\frac{1}{2}\%$	5%	.5%	12.5%.

14. Tell how to change decimal fractions to percent equivalents; to change percents to decimal equivalents.

15. Compare your first answer to 14 with:

To change decimals to percents, move the decimal points two places to the right and write % after the number.

§161. Equivalent Parts, Decimals, and Percents.

Answer all you can orally

1. What is $\frac{1}{2}$ of 4? 50% of 4? .50 of 4?

2. State the question: "What is $\frac{1}{4}$ of 16?" in two other ways, and answer each question.

3. State "What is $\frac{1}{3}$ of 63?" in the language of decimal fractions, and answer your question; in percentage language, and answer your question.

4. State in two other ways each of the following questions and answer your questions:

(1) What is $\frac{1}{5}$ of 25?
(2) What is .60 of 120?
(3) What is $\frac{1}{3}$ of 75?
(4) What is $37\frac{1}{2}\%$ of 40?
(5) What is $33\frac{1}{3}\%$ of 21?
(6) What is .125 of 64?
(7) What is .25 of 12?
(8) What is $\frac{3}{4}$ of 52?
(9) What is $87\frac{1}{2}\%$ of 72?
(10) What is $\frac{1}{8}$ of 32?
(11) What is $\frac{3}{8}$ of 56?
(12) What is $\frac{5}{8}$ of 48?
(13) What is $\frac{1}{16}$ of 80?
(14) What is $6\frac{1}{4}\%$ of 32?
(15) What is $\frac{3}{5}$ of 75?
(16) What is $66\frac{2}{3}\%$ of 42?
(17) What is $\frac{2}{3}$ of 84?
(18) What is $\frac{5}{6}$ of 78?

§162. The Percentage Law.

For all the different kinds of percentage problems and applications, there is only the one law:

$$percentage = base \times rate$$

The rate must be expressed as a decimal fraction.

The PERCENTAGE is the result of finding some percent of an amount, or a number.

The amount or number that the percentage is computed on is often called the BASE.

Letting p stand for the percentage, b for the base, and r for the rate, expressed *as a decimal fraction*, the law is

$$p = b \times r$$

This is the only law of percentage.

For example, to find 12% of 640.

This problem means: *To find the percentage.*

The rate expressed as a decimal fraction is .12, and the law says:

$$p = .12 \times 640 = 76.8$$

Other problems of this nature were solved in §161, on page 255.

The other sort of percentage problem is that in which the product, p, and one of the factors, b, or r, is given, to find the other factor. This requires no other law than the one just given.

When we know that the product of two factors is 20 and that one of the factors is 5, we can get the other factor at once by remembering that $\frac{20}{5} = 4$, *i.e.*, by a simple process of division. But this law $\frac{20}{5} = 4$, is only another way of stating the law $4 \times 5 = 20$.

So if the percentage, p, and base, b, are given to find r, we divide in the same way, thus:

$$r = \frac{p}{b}$$

For example the percentage is 60 and the base is 1200. Find the rate.

$$r = \frac{60}{1200} = \frac{1}{20} = .05, \text{ or } 5\%$$

If the percentage, p, and the rate, r, are given and b is to be found we say:

$$b = \frac{p}{r}$$

For example the percentage is 42 and the rate is 7%. Find the base.

$$b = \frac{42}{.07} = 600$$

Hence, to find either the base or the rate we have only an easy *division* problem to solve.

To solve any percentage problem that you can not solve mentally:

(1) *Decide what numbers are the base, the rate, and the percentage, and which is required.*

(2) *Solve a multiplication problem if the percentage is required, and a division problem if the base, or the rate, is required.*

§163. Finding the Percentage.

The percentage equals the product of the base and rate, expressed decimally, or more briefly,

$$p = b \times r$$

1. What is 6% of 30? Of 50? Of 80? Of 200? Of 700?

2. What is 15% of 10? Of 20? Of 70? Of 90?

3. How many miles is 4% of 200 mi.? Of 450 mi.? Of 75 miles?

4. How many trees is 6% of 60 trees? Of 95 trees? Of 125 trees?

5. How many square yards is 7% of 50 sq. yd.? Of 80 sq. yd.? Of 140 sq. yards?

6. Of 60 chickens, 15% died. How many chickens died?

7. At the beginning of a school year, the lung capacity of a boy was 161 cu. in. By the middle of the year it was 7% greater. How many cubic inches had his lung capacity increased?

8. The lung capacity of a boy at the beginning of the year was 160 cu. in. By the middle of the year it had increased 4%. How many cu. in. had it increased?

9. The world's automobile speed record to May 30, 1911, for $\frac{1}{2}$ mi. was 16.83 seconds. For a mile the record was nearly 210% of that for $\frac{1}{2}$ mile. Find the approximate record for a mile.

10. For 5 miles the world's automobile record (May 30, 1911) was 195.62 seconds. For 2 mi. it was nearly $38\frac{2}{3}\%$ and for 10 miles, nearly $202\frac{1}{4}\%$ of the record for 5 miles. Find the world's records for 2 miles, and for 10 miles.

11. For 25 mi. the world's record was 1100 seconds. For 20 mi. it was $76\frac{9}{40}\%$, and for 50 miles, 199.62% of the record for 25 miles. Find the records for 20 mi. and for 50 miles.

12. For 200 mi. the world's record was 9800.14 seconds. For 100 mi. it was $45\frac{3}{5}\%$, for 250 mi. $126\frac{1}{8}\%$, and for 500 mi. $246\frac{8}{25}\%$ of the record for 200 miles. Find the world's records for 100 mi., for 250 mi., and for 500 miles.

§164. Finding the Rate Percent, or Rate.

The rate equals the percentage divided by the base, or

$$r = \frac{p}{b}$$

Answer all you can orally

1. What percent of 8 is 4? Of 16 is 4? Of 48 is 6?

2. 7 is what percent of 21? Of 28? Of 35? Of 70?

3. 9 is what percent of 18? Of 36? Of 72? Of 45?

4. 11 is what percent of 33? Of 44? Of 55? Of 88? Of 110?

5. 24 is what percent of 160?

$24 = r \times 160$

$\frac{24}{160} = r$

$$\begin{array}{r} .15 \\ 160)\overline{24.00} \\ 160 \\ \hline 800 \\ 800 \\ \hline \end{array}$$

Hence 24 is 15% of 160. 15 is the *rate percent*, or 15% is the *rate*.

6. What percent of \$9000 is \$5400?

7. The standing of the several clubs in the National League during one season was determined from this table:

Club	Games Won	Games Lost	Percent Won
Chicago	104	50	67.5%
New York	91	63	
Pittsburg	86	67	
Philadelphia	78	75	
Cincinnati	75	79	
Brooklyn	64	90	
St. Louis	63	90	
Boston	53	100	

Find for each club what percent of the total number of games played were won. Carry results to the first decimal place, as shown for Chicago.

8. A farmer sold his corn for 55¢ a bushel in 1910. In 1911 he got 44¢ a bushel. What percent of the 1910 price was the 1911 price?

9. A merchant had a suit of clothes marked to sell for $25. He sold it for $20. What percent of the marked price was the selling price of the suit?

10. One boy ran a certain distance in 28 minutes, and a second boy ran the same distance in 35 minutes. What percent was the second boy's time of the first boy's time?

§165. Finding a Number from a Known Part.

Answer all you can orally

1. $\frac{1}{4}$ of a number is 7. What is the number?
2. $\frac{3}{4}$ of a number is 27. Find the number.
3. 25% of a number is 9. Find the number.
4. 75% of a number is 33. What is the number?
5. $\frac{1}{5}$ of a number is 12. What is the number?
6. 20% of a number is 8. What is the number?

7. $\frac{3}{5}$ of a number is 21. What is the number?

8. $\frac{4}{5}$ of a number is 24. Find the number.

9. $\frac{4}{7}$ of a number is 72. Find the number.

10. 8% of a number is 24. Find the number.

11. .6 of a number is 78. Find the number.

$$.6 \times n = 78, \text{ and } n = \frac{78}{.6} = 130$$

12. .09 of a number is 81. Find the number.

13. 7% of a number is 49. Find the number.

$$7\% = .07. \quad .07 \times n = 49. \quad n = \frac{49}{.07} = 700$$

14. 16% of a number is 144. Find the number.

15. .17 of a number is 85. Find the number.

16. $\frac{3}{4}$ of a number is $1\frac{2}{17}$. Find the number.

§166. Finding the Base.

The base equals the percentage divided by the rate expressed decimally, or

$$b = \frac{p}{r}$$

1. $37\frac{1}{2}\%$ of a number is 36. Find the number.

$$b = \frac{36}{.37\frac{1}{2}} = \frac{36}{.375} = 96$$

Check: $96 \times .37\frac{1}{2} = 36$.

2. A wheatfield yielded 840 bushels in 1910, which equals 60% of the yield in 1909. What was the yield in 1909?

3. An automobile was insured for $3750, which was 75% of its estimated value. What was its estimated value?

4. I pay 12% of the value of the house I occupy as rent every year. My rent is \$240 a year; what is the value of the house?

5. A house was damaged by fire, and an insurance company paid the owner \$840 damages, which was 40% of the original cost of the house; what was the original cost?

6. The season's record of games lost for a certain association of baseball clubs was as follows:

Clubs	Games Lost	Percent Lost
First	14	20
Second	17	25
Third	18	30
Fourth	27	$33\frac{1}{3}$
Fifth	28	35
Sixth	28	40
Seventh	32	$45\frac{5}{7}$

The last column of the table contains the percent of all the games played that were lost. Find the number of games played by each club.

§167. Percent Off, Percent Less, and Percent More.

1. 20% *less* than a number means how many percent of the number?

2. 30% *off* a number leaves how many percent of the number?

These are only short ways of saying 20%, or 30% of a a number less than 100% of the number, that is 80%, or 70% of the number.

3. 20% *less* than a distance is 72 ft. What is the distance?

4. 30% *off* a sum of money left \$4.90. What was the sum?

5. 35% less than what number is 26?

Call the number, a, and note that "35% less" means 65% remaining, and we write, or think:

$$.65 \times a = 26$$

Having the product 26, and one of the factors .65, we get the other factor by dividing the product by the given factor.

$$a = 26 \div .65 = 40$$

6. Sold a horse for \$230, which was 15% *more* than it cost. Find the cost of the horse.

15% *more* than the cost means 115% of the cost, c.

$$1.15 \times c = \$230. \quad c = \frac{\$230}{1.15} = \$200.$$

7. Solve the following, as many as you can orally:

(1) 9 is 10% less than what number?
(2) 5 is 25% more than what number?
(3) 7 is $12\frac{1}{2}$% less than what number?
(4) 14 is $16\frac{2}{3}$% more than what number?
(5) 27 is $33\frac{1}{3}$% less than what number?
(6) 55 is 10% more than what number?
(7) 20% off what number leaves 48?
(8) 30% off what number leaves 50?
(9) $33\frac{1}{3}$% off what number leaves 6?
(10) 32 is 15% less than what number?
(11) 7 is 30% less than what number?
(12) 36 is 80% more than what number?
(13) 24 is $12\frac{1}{2}$% less than what number?
(14) 80 is $33\frac{1}{3}$% more than what number?
(15) 98 is 40% more than what number?
(16) $62\frac{1}{2}$ is $87\frac{1}{2}$% less than what number?
(17) $87\frac{1}{2}$ cents is $37\frac{1}{2}$% less than what number?

8. A dealer sold a wagon for \$17.60 losing 12% on it. What did it cost him?

9. A merchant sold a suit for \$18.00, which was 20% less than (20% discount on) the marked price. What was the marked price?

10. $66\frac{2}{3}$ cents is $66\frac{2}{3}\%$ less than what sum of money?

11. $33\frac{1}{3}$ cents is $33\frac{1}{3}\%$ less than what sum of money?

A certain *percent discount*, or *percent loss*, means the same as *percent off*, or *percent less*.

12. A man sold a horse that cost him \$170 at 20% discount, or 20% off. What did he get for the horse?

13. An article marked \$32 sold at 30% discount. What did the article sell for?

14. A man sold a house for \$2400, which was 20% more than the cost. What did the house cost him?

§168. General Problems.

Read the statements in italics at the end of §162.

1. A merchant paid \$16 for a suit of clothes. At what price must he sell the suit to gain 25 percent?

2. A dealer pays \$1.80 a dozen for blank books, and retails them at a profit of $66\frac{2}{3}\%$. Find the selling price.

3. A grocer buys eggs at wholesale at 10c a doz., and retails them at a profit of 30%. Supposing there is no loss due to breakage or spoiling, at what price per dozen does he sell them?

4. Allen paid \$1 for a sled, and sold it at a loss of 20%. How much did he receive for the sled?

5. From a box containing 200 oranges, 80% were sold in one day. How many oranges were sold in the day?

6. What is the percentage on \$640 at 20%? At 28%? At 35%? At $6\frac{1}{4}\%$? At $12\frac{1}{2}\%$? At $87\frac{1}{2}\%$?

7. A farm, costing \$4400, was sold at a gain of 18%; for how much did the farm sell?

8. A farm sold for \$5192, which was 18% more than was paid for it. How much was paid for it?

The 18% here means 18% of what was paid for the farm (the cost price). \$5192 is then equal to what percent of the cost? The statement may be written thus:

1.18 times the cost price = \$5192,

or, more briefly, thus: $1.18x = \$5192$. Hence, $x = \frac{\$5192}{1.18}$. Divide and find the value of x.

9. A center fielder threw a ball 90 ft., which was $12\frac{1}{2}\%$ farther than the third baseman threw it. How far did the third baseman throw it?

First answer the question, "$12\frac{1}{2}\%$ of what?"

10. A boy bought oranges at the rate of 5 for 3¢, and sold them at the rate of 3 for 5¢. What was his rate percent of gain or loss?

11. A merchant in shipping 150 crates of eggs lost 25 crates by freezing. What percent did he thus lose?

12. Hats, costing \$2.75, sold for \$3.50. What was the percent of profit?

13. An automobile, costing \$750, sold for \$1250. What was the percent of profit?

14. A man lost 20% of his money and after losing 10% of the remainder had \$3600 left. How much had he at first?

15. A dealer sells a cask of 30 gal. of oil, agreeing to deliver it, and receives \$30. In delivering it 4.5 gal. are spilled. What percent is spilled, and how much should the dealer pay back?

16. Goods damaged by fire were sold at a loss of 40%. The amount received was \$555. What was the cost?

17. A suit of clothes was marked at $45, which was 50% more than the cost. It sold at a reduction of 20% from the marked price. What was the percent of profit on the first cost of the suit?

GENERAL APPLICATIONS OF PERCENTAGE

§169. Nature Study Data.

The number of clear, cloudy, partly cloudy, and rainy or snowy days for the first 4 mo. of 1903, at Chicago, are here tabulated:

1903	Clear	Cloudy	Partly Cloudy	Rain or Snow
Jan.	9	15	7	9
Feb.	10	11	7	9
Mar.	12	14	5	11
Apr.	10	13	7	13

1. What percent of the number of days of Jan. were clear? Cloudy? Partly cloudy? Rainy or snowy?

2. Answer similar questions for Feb.; for March; for April.

3. What percent of the total number of clear days of 1903 to May 1st fell in Jan.? In Feb.? In March? In April?

4. Answer similar questions for cloudy, partly cloudy, and rainy or snowy days.

5. The highest velocities (speeds) in miles per hour of the wind in Chicago for each of the 12 mo. (beginning with Jan.) of a certain year were: 50; 48; 57; 70; 54; 45; 56; 47; 52; 58; 52; 58. What percent of the highest velocity for April was the highest velocity for Jan.? For Feb.? For each of the remaining months?

6. 100 green oak leaves weighed 100 grams. When thoroughly dried they weighed 39.5 grams. The dried leaves were then burned, and the ash weighed 2.4 grams. What percent of the leaves was dry solid? Mineral matter (ash)?

7. 100 green elm leaves weighed 60 grams. $38\frac{1}{3}\%$, by weight, was dry solid, and $2\frac{2}{3}\%$ was mineral matter. What was the weight of the dry solid? Of the mineral matter?

8. A school garden, $15' \times 40'$, was seeded as follows: 25%, potatoes; $16\frac{2}{3}\%$ of the remainder, peas; 20% of the remainder, beans; 25% of the remainder, lettuce; $33\frac{1}{3}\%$ of the remainder, radishes; 20% of the remainder, parsley, and the rest in beets. How many square feet were seeded to beets?

§170. Almanac Data.

In this section "length of the day" means the length of daylight, or the time interval from sunrise to sunset.

1. On Jan. 1, at Chicago, the sun rose at 7 hr. 29 min., and set at 4 hr. 38 min., and on July 1, it rose at 4 hr. 28 min., and set at 7 hr. 39 min. The length of the day on Jan. 1 equals what percent of the length on July 1?

2. On Jan. 1, the length of the day in St. Louis was 103.8%, and in St. Paul 96.4%, of the length of the same day in Chicago. Find the day's length in each of these cities.

3. On July 1, in the same places, the day's lengths were, respectively, 97.9% and 102.2% of its length in Chicago. Find the day's length in each of these cities on this date.

4. Make and solve similar problems from the calendar pages of a common almanac.

5. On the first of Feb., of March, of April, of May, of June, of July, of Aug., of Sept., of Oct., of Nov., and of Dec., the day's lengths in Chicago were 108%, 122.3%, 137.8%, 152.6%, 163.2%, 163.5%, 154.9%, 144.6%, 129.2%, 115.5%, and 102.6% of the day's length on Jan. 1. Find the day's length on the dates named.

6. On vertical lines, one for each month, plot these rates to scale and draw through the points a smooth curve.

7. From the almanac find what percent the period from "moon rises" to "moon sets" on the first of some month is of the same period on the 15th or 20th.

8. Find what percent the time from full moon to new moon is of the time from new moon to first quarter; from new moon to second quarter, or full moon.

§171. Geography.

Read statements in italics at the top of page 257.

The following table shows the growth in territory of the United States and the dates of the additions.

Acquisition	Date	Sq. Mi.
Original territory	1783	827,844
Louisiana purchase	1803	1,182,752
Florida	1819	59,268
Texas	1845	371,063
Mexican purchase	1848	522,568
Texas purchase	1850	96,707
Gadsden purchase	1853	45,535
Alaska	1867	590,884
Hawaii	1898	6449
Porto Rico	1899	3600
Philippine Islands		114,000
Guam		200
Isle of Pines		882

1. By what percent was the territory of the U. S. increased by the Louisiana purchase? The purchase of Texas?

2. State and solve other similar problems on the table.

3. The area in millions of square miles of the surface of the earth is 148.18; of this surface 108.77 is water, and 39.41 land. Find what percent of the surface of the earth is water. What percent is land? The land surface equals what percent of the water surface?

4. If the coast line of Asia were smoothly curved the length would be 13,860 miles, which is 45% of the actual length. Find the actual length.

5. If the coast line of Australia were smoothly curved the length would be 5852 miles, which is 77% of the actual length. Find the actual length.

6. If the following continents had smoothly curved coast lines the lengths, and the percents that these lengths are of the actual lengths are here given:

CONTINENT	COAST-LINE IF SMOOTH	% EQUIVALENT OF ACTUAL LENGTH
North America	10,387 mi.	43.2%
South America	9034.4 "	66.4 "
Africa	11,771 "	83.6 "
Europe	6622 "	38.5 "

Find the actual lengths of the coast lines of these continents.

7. The Amazon River is 3612 miles long, which is 86% of the length of the Mississippi River. How long is the Mississippi?

8. The Ohio River is 3000 mi. long, which is 77% of the length of the Nile. How long is the Nile?

9. The Congo is 3000 mi. long, which is $90\frac{10}{11}\%$ of the length of the Yangtsekiang. How long is the Yangstekiang?

10. The area of Lake Michigan is 22,450 sq. mi. which is 71.9% of the area of Lake Superior. Find the area of Lake Superior.

11. The area of Victoria Lake is 22,167 sq. mi., which is 93.14% of the area of Lake Huron. Find the area of Lake Huron.

12. The area of Lake Ontario is 7244 sq. mi., which is 72.8% of the area of Lake Erie. Find the area of Lake Erie.

13. The greatest known depth of Lake Superior is 1008 ft., which is 115.2% of the greatest known depth of Lake Michigan. Find the greatest known depth of Lake Michigan.

14. The Mississippi River drains 1,350,500 sq. mi., which is $18\frac{1}{4}$% of the area of North America. Find the area of North America.

15. Other problems may be made and solved from data furnished by a Geography.

§172. Commission.

Commission is a sum of money paid by a person or firm called the principal, to an agent for doing business for the principal.

Commission is usually reckoned as some percent of the amount of money received or expended for the principal.

1. An agent sells 250 bu. of wheat @ 80¢ on a 2% commission. How much money was due the agent?

2. An agent bought for his firm 1000 bbl. of apples @ $1.25, and received a commission of 5%. How much did he receive?

3. What is the commission on 100 baskets of peaches @ 50¢, the rate of commission being 3%?

4. What is the commission on the following sales?

(1) 80 acres of land @ \$75, the rate of commission being 5%.

(2) City property sold at \$8500, rate of commission $4\frac{1}{2}\%$.

(3) 40 bu. tomatoes @ 75¢, rate of commission 10%.

(4) 1000 bu. wheat @ 95¢, rate of commission $1\frac{1}{2}\%$.

(5) 100 doz. eggs @ 25¢, rate of commission 12%.

(6) 100 bu. green vegetables @ 90¢, rate of commission $12\frac{1}{2}\%$.

(7) \$500 collected debts, rate of commission 15%.

(8) \$1200 bad debts collected, rate of commission 20%.

5. Find the commission on \$1850 at $2\frac{1}{2}\%$; the commission on the same amount at $3\frac{1}{2}\%$.

6. A clothing house sent its agent \$1224, which included a sum of money to be invested and 2% commission on this sum for the agent. What sum was to be invested?

7. 5% commission on a certain amount of money was \$684.20. What was the amount? ($.05x = \$684.20$. Find x.)

A shipment of goods sent to an agent to be sold is called a consignment.

8. A consignment of 4560 bu. of wheat was sold by an agent at $78\frac{3}{8}$¢ per bushel. What was the agent's commission at $1\frac{1}{2}$ percent?

9. An agent sold the following consignment of goods:

20 doz. eggs at $14\frac{3}{4}$¢	16 boxes oranges at \$2.25
40 lb. creamery butter at 21¢	4 boxes oranges at \$2.50
36 lb. cheese at $13\frac{1}{2}$¢	12 boxes grape fruit at \$2.75
80 lb. chicken at $12\frac{1}{2}$¢	25 bunches bananas at \$1.25
8 doz. live chickens at \$3.75	12 boxes lemons at \$2.76
4 bbl. apples at \$3.25	16 bbl. potatoes at \$4.25

Find the agent's commission at $6\frac{1}{4}$ percent.

§173. Trade Discount.

A discount is a certain rate percent of reduction (sum thrown off) from the prices at which articles are listed or quoted.

The discount is usually allowed for cash payments or for payment within a specified time.

1. A retail merchant buys silk at \$1.20 a yard. He is allowed a discount of 10% for cash. If he pays cash, how much does the silk cost him?

\$1.20 − 10% of \$1.20 = \$1.08.

2. A publisher sells 50 books at \$1.50 and allows 20% discount. How much does he receive for the books?

Merchants and manufacturers often publish expensive catalogues containing price lists of articles, prices for which change rapidly. When prices fall, instead of publishing new lists, they mark off an additional discount. For example, the price of a certain article may be catalogued thus: \$60, discount, 20%, 10%, 5%. This means a 20% reduction, then a 10% reduction on the reduced price, and then a 5% reduction on the second reduced price. It would be computed thus:

\$60	\$48	\$43.20
.20	.10	.05
\$12.00	\$4.80	\$2.1600

\$60 − \$12 = \$48. \$48 − \$4.80 = \$43.20. \$43.20 − \$2.16 = \$41.04.

Notice this is not the same as a discount of 20% + 10% + 5% (= 35%). Wherein is it different?

3. A New York merchant sells to a customer goods marked thus: Price, \$3500; discount, 10% 60 da. and 5% for cash. What must the customer pay to settle the bill by cash payment?

(The customer gets the benefit of both discounts.)

4. What would the customer have paid if he had been given a single discount of 15 percent?

5. A bill of plumber's supplies was marked thus:

Price: \$7.50, discount 20%, and 7%, and 5%.

How much did the supplies really cost?

6. Compute the amount of money needed to settle the following bills if paid in cash or within the shortest time mentioned:

(1) \$45; discount 25% 60 da. and 10% 5 days.

(2) \$180; discount $16\frac{2}{3}\%$ 30 da., and 10% 10 da., and 5% cash.

(3) \$1800; discount 30% and 10% and 7% cash.

(4) \$54; discount $12\frac{1}{2}\%$, and 8%, and 5% 30 days.

(5) 46 tons coal at \$5.75 per ton; discount 10% and 2% cash.

(6) 50 men's suits at \$18; discount 20% and 5% 10 days.

(7) 4 gross tablets at 50¢ per doz.; discount 20% and 7% cash.

7. To what single rate of discount is a discount of 20% and 5% equivalent?

8. A hardware dealer bought the goods named in the following bill:

(1) 2 doz. braces, @ 45¢ each
(2) 50 lb. $\frac{3}{4}''$ bolts, @ $3\frac{1}{2}$¢
(3) 12 bales barb wire, @ \$2.15
(4) 25 kegs nails, @ \$2.50
(5) 8 cooking stoves, @ \$28.75
(6) 12 baseburners, @ \$32.50
(7) 150 lb. screws, @ \$4.75 per cwt.

The bill was subject to discounts of 10%, 7%, and also 5% 30 da., or 6% 10 da. What was the total cost if the bill was paid in 30 days? What was the total cost if it was paid in 10 days?

9. Find the amount needed to settle the following bill subject to the successive discounts, 10%, 8%, and 5%, for 30 da., or 6% for cash, the bill being paid in cash:

(1) 2 chests carpenter's tools, @ $48.50
(2) 2 doz. augers, @ 25¢ each
(3) 2 doz. carpenter's rules, @ $1.80
(4) 480 lb. hinges, @ $4.75 per C. (Hundred).
(5) 3 doz. tin buckets, @ 22¢ each
(6) 14 scoop shovels, @ 85¢
(7) 6 wash boilers, @ 78¢
(8) 12 washtubs, @ 52¢

10. What amount will be needed to settle the bill of problem 9 in 30 days?

11. What amount will settle the following bill of house furnishings, the bill being subject to the discounts, 20%, 10%, 8%, and 5%, 30 da., or 6% cash, the bill being paid in 30 da.?

(1) 8 dining tables @ $30
(2) 12 sets dining-room chairs @ $18.50 a set
(3) 10 bookcases @ $12.50
(4) 8 rocking chairs @ $12.75
(5) 15 center tables @ $15
(6) 12 Wilton rugs @ $25
(7) 250 yd. ingrain carpet @ 45¢

12. What amount paid in cash will be needed to settle the bill of problem 11?

INTEREST

§174. Simple Interest Defined.

In reckoning simple interest the year is commonly regarded as containing 12 mo. of 30 da. each.

1. A man is charged \$6 for the use of \$100 for 1 yr. What percent of the sum borrowed (\$100) equals the sum (\$6) he is charged for its use?

Interest for one year is the same as percentage.

2. A man is charged \$21 for the use of \$350 for 1 yr. The sum charged equals what percent of the sum borrowed?

INTEREST is money charged for the use of money.

Interest is reckoned at a certain rate percent of the sum borrowed for each year it is borrowed.

When money earns 3, 6, 7, or 10 cents on the dollar *annually* (each year) the *rate* is said to be 3%, 6%, 7%, or 10% *per annum* (by the year) and the *rate percent* is said to be 3, 6, 7, or 10.

The only difference between calculating percentage and calculating simple interest is that after the percentage is calculated it must be multiplied by the *time expressed in years.*

To find the simple interest on \$480 at 5% for $2\frac{1}{4}$ years:

To find percentage		To find simple interest	
\$480	Base	\$480	Principal
.05	Rate	.05	Rate
\$24.00	Percentage	\$24.00	Interest for 1 yr.
		$2\frac{1}{4}$	Time
		\$54.00	Interest for $2\frac{1}{4}$ yr.

The numbers used in percentage are named differently in simple interest, though they are used the same way. It may help to notice that:

IN PERCENTAGE		IN SIMPLE INTEREST
base	corresponds to	*principal*
rate	corresponds to	*rate*
percentage	corresponds to	*interest for 1 yr.*

§175. Six Percent Method.

Answer all you can orally

1. At 6% per annum, how much interest does \$360 earn in 1 yr.? In 3 yr.? In $4\frac{1}{2}$ yr.? In $2\frac{2}{3}$ yr.? In t years?

2. Make a rule for computing the interest on any sum of money at 6% when the time is in years.

3. At 6% per annum, how much interest does \$1 earn in 1 yr.? In 2 mo.? In 1 mo.? In 6 da.? In 12 da.? In 18 days?

4. When the time is in months, how may the interest on any sum of money at 6% per annum be computed?

5. How may the interest at 6% per annum on any sum of money be computed when the time is given in months and days? In years, months, and days?

The sum of money on which the interest is computed is called the PRINCIPAL. The principal plus the interest is called the amount.

Since the borrower must not only pay the interest on the borrowed principal, but must also return the principal, the debt he must discharge (pay) is the *amount.*

6. How long will it require any principal (\$1) to amount to twice its value (\$2), or to double itself, at 6%?

7. Give the reasons for these statements:

Any principal, at 6%, (1) doubles itself in 200 months; (2) earns $\frac{1}{10}$ of itself in 20 months; (3) earns $\frac{1}{100}$ of itself in 2 mo. or 60 days.

§176. General and 6% Methods.

TO FIND THE INTEREST, *I*

1. What is .07 of $450? What is the interest on $450 at 7% for 1 yr.? For 2 yr.? For $3\frac{1}{2}$ yr.? For t years?

2. What is .08 of $1250? What is the interest at 8% on $1250 for 1 yr.? For 3 yr.? For $5\frac{1}{2}$ yr.? For x years?

3. Find the interest at 5% on $640, for 4 years.

4. $1800 was on interest at 7% for 3 yr. 7 mo. 21 da. Find the interest.

CONVENIENT FORMS

General Method

I. Interest computed first at the given rate.

	$1800,	principal
	.07	rate
	$126.00	int. for 1 yr.
	3	whole years
	$378.00	int. for 3 yr.
½ of $126.00 =	63.00	int. for 6 mo.
⅛ of 63.00 =	10.50	int. for 1 mo.
½ of 10.50 =	5.25	int. for 15 da.
⅕ of 10.50 =	2.10	int. for 6 da.
	$458.85	int. for 3 yr. 7 mo. 21 da.

Six Percent Method

II. Interest computed first at 6%.

$.18	int. on $1 for 3 yr. at 6%
.035	int. on $1 for 7 mo. at 6%
.0035	int. on $1 for 21 da. at 6%
.2185	int. on $1 for 3 yr. 7 mo. 21 da. at 6%
1800	
393.3000	int. on $1800 for given time at 6%
65.55	int. on $1800 for given time at 1%
$458.85	int. on $1800 for given time at 7%

SUGGESTIONS for II:

(1) If the interest on $1 for 1 yr. is 6¢, what is the interest for 3 years?

(2) If the interest on $1 for 2 mo. is 1¢, what is the interest for 7 mo.?

(3) If the interest on $1 for 6 da. is 1 mill, what is the interest for 21 da.?

One of these methods may be used to check the other.

5. How much interest must be paid for the use of the following:

(1) \$600 for 1 yr. 5 mo. 24 da. at 7 percent?
(2) \$350 for 2 yr. 4 mo. 21 da. at 8 percent?
(3) \$575 for 2 yr. 8 mo. 18 da. at $6\frac{1}{2}$ percent?
(4) \$860 for 3 yr. 9 mo. 15 da. at $5\frac{1}{2}$ percent?

6. Find the amount of:

(1) \$300 for 2 yr. 4 mo. 25 da. at $4\frac{1}{2}$ percent?
(2) \$160 for 3 yr. 7 mo. 20 da. at $6\frac{1}{2}$ percent?
(3) \$850 for 3 yr. 2 mo. 14 da. at $5\frac{1}{2}$ percent?

7. Find the interest and the amount for the following:

(1) \$700 for 1 yr. 7 mo. 15 da. at 3 percent.
(2) \$400 for 2 yr. 9 mo. 27 da. at 3 percent.
(3) \$210 for 2 yr. 5 mo. 28 da. at 6 percent.
(4) \$150 for 1 yr. 11 mo. 13 da. at 7 percent.
(5) \$280 for 1 yr. 6 mo. 19 da. at $4\frac{1}{2}$ percent.
(6) \$360 for 1 yr. 4 mo. 5 da. at 4 percent.
(7) \$260 for 2 yr. 3 mo. 11 da. at $3\frac{1}{2}$ percent.
(8) \$300 for 2 yr. 7 mo. 12 da. at 5 percent.

TO FIND THE PRINCIPAL, *P*

8. What principal at 8% will furnish \$16 interest in 2 years?

What interest will \$1 produce at 8% in 2 yr.? How many dollars will yield \$16 at 8% in 2 years?

9. What principal at 8% will produce \$30 interest in $2\frac{1}{2}$ years?

10. What principal at 6% will amount to \$112 in 1 yr. 6 months?

What is the amount of \$1 at 6% interest for 1 yr. 6 mo.?

11. Find the principal which will yield \$61.25 interest in 3 yr. 6 mo. at 7 percent.

12. Find the principal which will amount to \$972.40 in 3 yr. 2 mo. 12 da. at $4\frac{1}{2}$ percent.

13. Make a rule for finding the principal when the rate, time, and interest are given.

14. Make a rule for finding the principal when the rate, time, and amount are given.

15. Supply the value for the letter in each of the following:

	Rate	Time	Interest	Principal	Amount
(1)	6 %	1 yr. 7 mo. 15 da.	\$19.50	P	A
(2)	8 %	3 yr. 3 mo. 18 da.	\$169.02	P	A
(3)	$8\frac{1}{3}$ %	4 yr. 7 mo. 21 da.	I	P	\$998.50
(4)	7 %	6 yr. 8 mo. 16 da.	I	P	\$198.42
(5)	4 %	2 yr. 6 mo. 18 da.	\$122.50	P	A
(6)	5 %	1 yr. 10 da.	\$176.00	P	A
(7)	$4\frac{1}{2}$ %	90 da.	\$32.50	P	A

TO FIND THE RATE, r

16. At what rate percent will \$320 yield \$39.60 interest in 2 yr. 9 months?

How much interest will \$320 yield in 2 yr. 9 mo. at 1%? At what rate percent, then, will the same sum yield \$39.60 interest in 2 yr. 9 months?

17. At what rate will \$780 yield \$512.72 in 5 years 8 months?

18. Make a rule for finding the rate when the principal, time, and interest are known.

19. Find the correct value to the nearest .1 for the letter in each of the following problems:

	Principal	Time		Interest	Rate
(1)	\$580.00	1 yr.	5 mo.	\$46.50	r%
(2)	\$798.45	2 yr. 8 mo.	15 da.	\$258.65	r%
(3)	\$3698.50	1 yr. 5 mo.	19 da.	\$568.75	r%

TO FIND THE TIME,

20. How long will it take \$80 to earn \$14 interest at 4% annually?

How much interest will \$80 earn in one year at 4%? In how many years, then, will \$80 earn \$14 at the same rate?

21. At 8% how long will \$125 take to earn \$57.50?

22. At 7%, how long will it take \$648 to yield \$69.84?

When the time results in decimals of a year the decimal may be reduced to months and days by the method used in problem 51, page 178.

23. At 8% how long will \$750 take to yield \$750?

24. Make a rule for finding the time, t, required for a given principal, P, to amount to a given sum, A, at a given rate percent, r.

25. Make a rule for finding how long it will take a given principal to earn a given interest at a given rate percent.

26. Find the time, t, in years, months, and days under the conditions stated in each of the following problems:

	Principal	Rate	Interest	Time
(1)	\$66.00	7 %	\$28.60	t
(2)	\$460.00	6 %	\$31.05	t
(3)	\$750.00	$5\frac{1}{2}$%	\$147.47	t

GIVEN AMOUNT, PRINCIPAL, AND RATE; TO FIND TIME

27. The amount is \$180.80, the principal is \$160, and the rate is 4%. Find the time.

$I = \$180.80 - \$160 = \$20.60$; $.04 \times \$160 = \$6.40$ interest for 1 year.

$$\text{time} = \frac{\text{total interest}}{\text{interest for 1 yr.}} = \frac{\$20.80}{\$\ 6.40} = 3.25 \text{ yr.} = 3 \text{ yr. } 3 \text{ mo.}$$

28. A principal of \$80 loaned at 6% amounted to \$102.40. How long was it loaned?

29. How long will it take \$100 to double itself at 5% simple interest?

30. How long will it take \$10 to double itself at 6%?

31. How long will \$975 take to amount to \$1225 at 5%?

What is the total interest? What is the interest on \$975 at 5% for 1 year?

GIVEN AMOUNT, PRINCIPAL, AND TIME; TO FIND RATE

32. At what rate will \$50 amount to \$60 in 4 years?

$I = \$60 - \$50 = \$10$. Interest on \$50 at 1% for 4 yr. = \$2.00.

$\frac{\$10}{\$2} = 5$. Answer 5%.

33. At what rate will \$600 amount to \$700 in $4\frac{1}{6}$ years?

34. At what rate will \$100 double itself in 5 years?

35. A man invested \$2000 for 2 yr. 7 mo. 27 da. and received \$2638 at the end of this time. What rate percent of interest did his investment earn for him?

36. A man bought 120 A. of land at \$85 and sold it 2 yr. 8 mo. later for \$100 per A., after having received \$900 in rents and having twice paid taxes at 75 cents per acre. What was his annual rate percent of profit?

Add the difference between the rent and the taxes to the selling price to find the net receipts.

GIVEN AMOUNT, RATE, AND TIME; TO FIND PRINCIPAL

37. The amount is \$15.60, the rate 6%, and the time 5 years. Find the principal, P.

The amount of \$1 at 6% for 5 yr. is \$1.30

$$\frac{\textit{amount on } p \textit{ dollars}}{\textit{amount on 1 dollar}} = \frac{\$15.60}{\$1.30} = 12;\ \text{principal, } \$12$$

38. What principal at 7% will amount to \$595 in 10 years?

39. What principal at 3% will amount to \$625 in 8 years?

40. What principal at 3% will amount to \$625 in 8 yr. 4 mo.?

MISCELLANEOUS PROBLEMS

41. Supply the value for which each letter stands in the problems of the following table:

	Principal	Rate	Time	Interest	Amount
(1)	$60.00	6%	3 yr. 3 mo. 8 da.	I	A
(2)	$175.00	5%	4 yr. 9 mo. 15 da.	I	A
(3)	$800.00	7%	t	I	$926.00
(4)	$475.00	8%	t	$142.50	A
(5)	$1266.00	4%	t	I	$1349.85
(6)	P	6%	5 yr. 7 mo. 27 da.	$509.25	A
(7)	$1575.30	r%	1 yr. 4 mo. 18 da.	I	$1662.46
(8)	$728.25	8%	t	$209.736	A

42. Complete, to mills, the following interest table:

INTEREST TABLE: PRINCIPAL $100.

Time	Rate				
	3%	4%	5%	6%	7%
1 da.	$.008	$.011	$.014	$.017	$.019
2 da.					
3 da.					
4 da.					
5 da.					
6 da.					
1 mo.					
2 mo.					
3 mo.					
6 mo.					
1 yr.					

43. Compute by the table the interest on $758 at 7% for 4 mo. 12 da.; for 7 mo. 8 da.; for 1 yr. 3 mo. 10 days.

44. Calculate by the table the interest on $865 at 4% for 3 yr. 10 mo. 18 da.; for 8 yr. 11 mo. 24 days.

45. Calculate by the table the amount on $1280 at 6% for 4 yr. 4 mo. 17 da.; for 9 yr. 8 mo. 16 days.

§177. Promissory Notes.

A **promissory note** is a written promise, made by one person or party, called the **maker,** to pay another person or party, called the **payee,** a specified sum of money at a stated time.

The sum of money for which the note is drawn is called **the face value,** or the **face** of the note.

The date on which the note falls due is called **the date of maturity.**

$250.00 *Kankakee, Ill.,* June 1, 1903.

Ninety days *after date I promise to pay to the order of* A. M. Baker, Two Hundred Fifty and $\frac{00}{100}$ *Dollars, for value received, with interest at* six *percent per annum from date.*

Due Aug. 30, 1903. H. R. Newman.

1. Who is the maker of the above note? The payee? What is the face of the note? The date? The rate of interest? The date of maturity?

2. A promissory note, unless otherwise specified in the note, draws interest on its face value at the rate mentioned in the note from the date of the note until it is paid. Compute the interest and the amount on the foregoing note if it is paid Aug. 30, 1903.

3. Find the interest on the following note, paid Aug. 30, 1903:

$875. URBANA, ILL., May 18, 1897.

One year after date I promise to pay to James Black, or order, Eight Hundred Seventy-Five and $\frac{00}{100}$ Dollars, at Busey's Bank, for value received, with interest at the rate of seven percent per annum from date.

Due May 18, 1898. Henry Osborn.

To find the time for which interest is to be computed, proceed thus:

CONVENIENT FORM

	yr.	mo.	da.
Date of payment, . .	1903	8	30
Date of note,	1897	5	18
Time,	6	3	12

4. Find the amount of each of the following notes:

	FACE	RATE	DATE OF NOTE	DATE OF PAYMENT	INTEREST	AMOUNT
(1)	$250	6 %	Mar. 12, 1899	Jan. 1, 1902	.....	.....
(2)	$635	7 %	Mar. 12, 1896	Jan. 1, 1900	.....	.....
(3)	$2400	5½%	Dec. 12, 1899	Jan. 8, 1903	.....	.....
(4)	$3865	4½%	Oct. 13, 1901	Sept. 7, 1903	.....	.....
(5)	$3640	8⅓%	Aug. 17, 1900	Mar. 3, 1903	.....	.....

§178. Exact Interest.

Interest calculated by counting twelve 30-day months to the year is called **simple interest**. The kind of interest we have been studying is simple interest. Most banks and business firms use simple interest.

The U. S. Government calculates interest for the exact number of days, on the basis of a 365-day common year and a 366-day leap year. Exact interest is the only correct interest, and it is coming more and more into use.

The exact interest is $\frac{360}{365}$, or $\frac{72}{73}$ of the simple interest, because the simple interest year is 360 days and the exact interest year is 365 days, commonly. For leap years the fraction is $\frac{60}{61}$ of the simple interest.

Hence the method of calculating exact interest is first to calculate the simple interest and then, take $\frac{72}{73}$ of it.

Show that $\frac{72}{73} = 1 - \frac{1}{73}$; also that $\frac{60}{61} = 1 - \frac{1}{61}$.

Multiplying by $\frac{72}{73}$ is the same as multiplying by $1 - \frac{1}{73}$. Multiplying by 1 leaves the multiplicand unchanged.

Hence we may make this rule for calculating exact interest:

Calculate the simple interest and subtract from it $\frac{1}{73}$ of itself.

Or, exact interest may be calculated by:

Calculating the exact number of days and multiplying the interest for 1 yr. by the ratio of the number of days to 365, or to 366, according as a common or leap year is in question.

Cancellation is often useful in calculating exact interest.

For example, to calculate the exact interest on \$803 at 4% for 55 days, arrange work thus:

$$\frac{.04 \times \overset{11}{\cancel{803}} \times \overset{11}{\cancel{55}}}{\underset{73}{\cancel{365}}} = .04 \times 11 \times 11 = \$4.84$$

§179. Exercises.

1. Find the exact interest.

(1) \$240 for 146 days at 3%.
(2) \$438 for 62 days at 5%.
(3) \$730 for 84 days at $3\frac{1}{2}$%.
(4) \$876 for 93 days at $4\frac{1}{2}$%.
(5) \$584 for 215 days at $5\frac{1}{2}$%.
(6) \$547.50 for 315 days at 3%.
(7) \$693.50 for 126 days at 5%.
(8) \$766.50 for 99 days at 4%.
(9) \$182.50 for 75 days at $5\frac{1}{2}$%.
(10) \$328.50 for 480 days at $3\frac{1}{2}$%.

2. Find the exact interest on \$3650 at $3\frac{1}{2}$% from Feb. 14, 1911, to Oct. 7, 1911.

3. Find the exact interest on \$7300 at 4% from Feb. 10, 1908, to Aug. 13, 1908.

4. Find the exact interest on \$2044 at 4% from Jan. 15, 1910, to Oct. 15, 1910.

PERCENTAGE AND INTEREST AS PROPORTION

§180. Percentage as Proportion.

1. The percentage on a certain base at 6% is \$30. What is the percentage on the same base at 3%? At 2%? At 1%? At 12%? At 18%?

2. The percentage on a certain base at the rate of $r\%$ is \$48. What is the percentage on the same base at one-half the rate $r\%$ or at $\frac{r}{2}\%$? $\frac{1}{3}$ the rate $r\%$ or $\frac{r}{3}\%$? At $\frac{r}{6}\%$? At $\frac{r}{10}\%$? At $2r\%$? $3r\%$? $5r\%$? $10r\%$?

3. Make a statement of the law underlying questions 1 and 2.

4. Compare your statement with the following:
The percentage on a given base is proportional to the rate.

$$\frac{\text{Percentage at } 4\%}{\text{Percentage at } 12\%} = \frac{4}{12} = \frac{1}{3}$$

5. Calling p the percentage on a certain base at $r\%$, and p' (read *p-prime*), the percentage on the same base at $r'\%$, write the law just stated as a formula.

6. Compare your formula with:

$$\frac{p}{p'} = \frac{r}{r'}$$

In words: *On a fixed base, percentages and rates are proportional.*

7. The percentages on a fixed base at different rates are \$12 and \$36. and the smaller rate is 3%. Find the larger rate.

8. The percentages being as in problem 7, and the larger rate being 3%, find the smaller rate.

9. The percentage on a certain sum of money at 8% is \$73.60. Find the percentage on the same sum at 2%.

10. Two rates of percentage of a number are $2\frac{1}{2}\%$ and 10%, and the smaller percentage is \$16. Find the larger.

11. The percentage at a fixed rate on the base \$60 is \$3.90. What is the percentage at the same rate on \$120? \$180? \$300? \$480? \$30? \$20? \$15? \$6?

12. The percentage at a certain rate on a given base b, is \$96. What is the percentage at the same rate on a base twice as large ($2b$)? Three times as large ($3b$)? On $4b$? On $\frac{1}{2}b$? On $\frac{b}{3}$? On $\frac{b}{8}$? On $\frac{b}{10}$?

13. State a law connecting the percentage at a fixed rate and the base.

14. Compare your statement with the following:
The percentage at a fixed rate is proportional to the base.

$$\frac{\text{The percentage at } r\% \text{ on } \$80}{\text{The percentage at } r\% \text{ on } \$480} = \frac{80}{480} = \frac{1}{6}$$

15. Calling p the percentage at $r\%$ on the base b, and p' the percentage at the same rate, $r\%$, on the base b', write the law just stated as a formula.

16. Compare your formula with:

$$\frac{p}{p'} = \frac{b}{b'}$$

In words: *At a fixed rate percentages and bases are proportional.*

17. At the same rate, the percentages on two different bases are \$24 and \$192, and the smaller base is \$600. Find the larger base.

18. With other conditions as in problem 17, but with the larger base \$600, find the smaller.

19. Two bases are \$112 and \$784, and the percentage on \$112 is \$6.16. Find the percentage at same rate on \$784.

20. At a certain rate the percentage on \$784 is \$41.16; find the percentage at the same rate on \$112.

THE CONCLUSION

The substance of all the preceding work of this article might be stated thus:

The percentage is proportional to the rate, when the base is fixed, and to the base when the rate is fixed.

If the percentage at any rate, say 6%, is known, the percentage at 1% is $\frac{1}{6}$ of the known percentage, and at 7%, it is the sum of the percentages at 6% and at 1%; while at 5% it is the difference of the percentages at 6% and 1%, as we have seen before.

21. The percentage at 6% on a certain base is \$42. Find the percentage on the same base at 5%; at 7%; at 10%; at $3\frac{1}{2}$%; at $2\frac{1}{2}$%.

22. The percentage at 6% on a certain base is \$$p$. Express the percentage on the same base at 3%; at 5%; at 7%; at $3\frac{1}{2}$%.

§181. Interest and Proportion.

1. The interest at a certain rate for a certain time on \$150 is \$18. What principal at the same rate and for the same time will give \$36 interest? \$54 interest? \$90 interest? \$108 interest?

2. At the same rate and for the same time two principals produce \$12 and \$72, and the smaller principal is \$60. Find the larger principal.

3. Two principals, \$300 and \$1200, at the same rate and for the same time yield \$42 and \$$i$. Find the number that i stands for.

4. Two principals, P and P', at the same rate and in the same time yield \$$I$ and \$$I'$ interest. Write an expression showing how P, P', I, and I' are related.

5. State a law connecting the principals and the corresponding amounts of interest they yield at the same rate and for the same time.

6. Compare your statement with:

The principal is proportional to the interest when the rate and time are the same.

7. A certain sum at a rate of 4% produces $16 interest in a certain time. What interest would the same sum for the same time produce at 6%? At 8%? At 2%?

8. State the law employed in question 7.

9. Compare your statement with:

Interest is proportional to rate, when principal and time are fixed.

10. When interest and rate are constant or fixed, how are principal and time connected?

11. Compare you answer with:

When interest and rate are constant, the principal and time are oppositely or inversely proportional.

12. Show the meaning of the law just stated supposing the principals $35 and $350 produce the same interest at the same rate, but the first in 10 months and the second in one month.

GENERAL REVIEW PROBLEMS

§182. The Parcel Post, and Shipping Goods.

By the parcel post law as in effect January 1, 1914, parcels weighing up to 50 lb. may be shipped by post anywhere in the local zone * and zones 1 and 2. For other zones, not more than 20 lb. may be sent by parcel post. The rates are now (January, 1914) as shown in the following table:

Weight of Package	†Local zone.	†Zones 1 and 2, up to 150 mi.	Zone 3, over 150 mi. to 300 mi.	Zone 4, over 300 mi. up to 600 mi.	Zone 5, over 600 mi. up to 1000 mi.	Zone 6, over 1000 mi. up to 1400 mi.	Zone 7, over 1400 mi. up to 1800 mi.	Zone 8, over 1800 mi.
Over 4 oz. up to 1 lb.	5¢	5¢	6¢	7¢	$0.08	$0.09	$0.11	$0.12
Over 1 lb. up to 2 lb.	6	6	8	11	.14	.17	.21	.24
Over 2 lb. up to 3 lb.	6	7	10	15	.20	.25	.31	.36
Over 3 lb. up to 4 lb.	7	8	12	19	.26	.33	.41	.48
Over 4 lb. up to 5 lb.	7	9	14	23	.32	.41	.51	.60
Over 5 lb. up to 6 lb.	8	10	16	27	.38	.49	.61	.72
Over 6 lb. up to 7 lb.	8	11	18	31	.44	.57	.71	.84
Over 7 lb. up to 8 lb.	9	12	20	35	.50	.65	.81	.96
Over 8 lb. up to 9 lb.	9	13	22	39	.56	.73	.91	1.08
Over 9 lb. up to 10 lb.	10	14	24	43	.62	.81	1.01	1.20
Over 10 lb. up to 11 lb.	10	15	26	47	.68	.89	1.11	1.32
Over 11 lb. up to 12 lb.	11	16	28	51	.74	.97	1.21	1.44
Over 12 lb. up to 13 lb.	11	17	30	55	.80	1.05	1.31	1.56
Over 13 lb. up to 14 lb.	12	18	32	59	.86	1.13	1.41	1.68
Over 14 lb. up to 15 lb.	12	19	34	63	.92	1.21	1.51	1.80
Over 15 lb. up to 16 lb.	13	20	36	67	.98	1.29	1.61	1.92
Over 16 lb. up to 17 lb.	13	21	38	71	1.04	1.37	1.71	2.04
Over 17 lb. up to 18 lb.	14	22	40	75	1.10	1.45	1.81	2.16
Over 18 lb. up to 19 lb.	14	23	42	79	1.16	1.53	1.91	2.28
Over 19 lb. up to 20 lb.	15	24	44	83	1.22	1.61	2.01	2.40

*"Local zone" means within the free delivery territory of any central post office.
†For the local zone and zones 1 and 2 these rates continue up to 50 lbs.

Answer all you can orally

1. Begin with 5, count by ones to 24, and compare your counts with the numbers of the column for zones 1 and 2 of the table.

2. Begin with 6, count by twos, and compare your counts with rates for zone 3.

3. Begin with 7, count by fours, and compare with the rates for zone 4.

4. Begin with 8, count by sixes, and compare with rates for zone 5.

5. What sort of count will give the successive rates for zone 6?

6. What sort of count will give the rates for zone 7? For zone 8?

7. Find from the table the cost of sending by parcel post a package weighing $2\frac{1}{2}$ lb. to a place in the local zone of the shipping point; to a place in zones 1 and 2; in zone 5; in zone 7.

8. Find the cost by parcel post of shipping 8 lb. to a place in zones 1 and 2; in zone 4; in zone 6; in zone 8.

9. A pair of shoes weighing with wrapper $3\frac{1}{2}$ lb. was shipped by parcel post to a place 500 miles distant. How much did the stamps cost?

10. How much would it cost to ship 60 lb. (=1 bu.) of beans by parcel post in four $15\frac{1}{2}$ lb. packages (the containers weigh 2 lb.) to a point 75 miles distant? 250 miles distant?

11. A man paid \$1.10 for 1 doz. steel traps and had them sent 200 miles to his home by parcel post. When ready for shipment the package weighed 8 lb. How much did the 12 traps cost him laid down at his home? How much apiece?

12. How much more would it cost to send twenty 3 lb. packages by parcel post to a place in zone 4 than to send three 20 lb. packages? To a place in zones 1 and 2? In zone 7?

13. Butter is 40 cents a pound at a city grocery and 25 cents a pound on a farm within the local zone of that city. How much does a man living in the city save on 10 pounds of butter by buying it of the farmer and having it sent by parcel post? Add one pound for the weight of the wrapper.

The following table gives the four different freight rates per 100 lb. between Chicago and the places named in the first column, the minimum charge, and the approximate distance from Chicago.

Between Chicago and	Freight Rates by Classes in Cents per 100 Pounds				Minimum Ch'ge	App. Dist. to Chicago
	1	2	3	4		
South Bend, Ind.	22	20	17	12	25	78 mi.
Ft. Wayne, Ind.	28½	25	20	13½	29	140 mi.
Indianapolis, Ind.	31½	27	21½	14	32	180 mi.
Richmond, Ind.	36	31	23	16	36	200 mi.
Evansville, Ind.	40	34	25	17	40	300 mi.
Mobile, Ala.	110	90	75	58	110	930 mi.
Hartford, Conn.	82	71	55	39	82	960 mi.
Savannah, Ga.	130	110	97	85	75	1150 mi.
Salt Lake City, Utah.	245	207	172	139	172	1500 mi.
Phoenix, Ariz.	290	251	209	175	147	1600 mi.
Portland, Ore.	340	295	245	207	295	2200 mi.

14. Find from the table the first-class freight rate per 100 lb. from Chicago to Evansville; to Richmond; to Mobile, Alabama; to Hartford, Connecticut; to Portland, Oregon; to Savannah, Georgia.

15. Answer the questions of problem 14 for second-class freight per 100 lb.; for fourth-class freight.

16. Freight rates are the same for packages weighing less than 100 lb. as for 100 lb. Give the four freight charges on a 70 lb. package from Chicago to Indianapolis. Notice the minimum charge.

17. Give the four freight charges on a 560 lb. shipment from Chicago to Evansville; from Chicago to Richmond; to Mobile, Ala.; to Phoenix, Ariz.; to Portland, Ore.

The freight rates are the same for the cities given in the table as for small towns and places near these cities.

18. A man living near Ft. Wayne, Ind., wants to know the cost of shipping from Chicago to his home a 50 lb. parcel (a) by parcel post, (b) by first-class freight. Solve his problem. (Observe the minimum charge.)

19. Compare the cost of shipping 100 lb. of sugar by first-class freight, with the cost of shipping it in five 20 lb. packages by parcel post from Chicago to Richmond, Ind.; from Chicago to South Bend, Ind.

20. Compare the cost of shipping 24 doz. eggs in two parcels of 20 lb. each by parcel post with the cost of shipping them in a single package of 40 lb. by first-class freight from Ft. Wayne, Ind., to Chicago. From Evansville to Chicago.

21. A farmer near Ft. Wayne, Ind., can get 40¢ a dozen for eggs in Chicago or 28¢ a dozen in his home town. How much would he gain on 30 dozen eggs sent in a 48 lb. package by parcel post to Chicago, after paying the postage and a commission of $1.20, over what he would get by marketing them in his home town?

22. A farmer living near a town 120 miles from Indianapolis ships fifty 22 lb. boxes of celery by first-class freight to Indianapolis. He receives 40¢ a box for his shipment. He pays the freight and a commission of $2.00 to an agent to sell the shipment for him. How much does he receive net if the freight charge is 26¢ per 100 lb?

23. How much would the farmer of problem 22 have received net, if he had shipped by parcel post in fifty 22 lb. parcels, commission being as in problem 22?

Other problems based upon the table on page 291 may be made if desired.

24. The express charges on 5, 10, and 20 lb. parcels from Chicago to Denver are 40¢, 60¢, and $1.00 respectively. Compare these with parcel post charges on the same packages, the distance from Chicago to Denver being 1022 miles.

25. On 5, 10, and 20 lb. parcels the express charges from Chicago to San Francisco (distance 2274 miles) are 65¢, $1.11, and $2.02. Compare these with the parcel post charges for the same parcels.

26. The express charge on a 10 lb. parcel from Chicago to New York City is forty-two cents. (Feb. 1, 1914.) Compare this with the cost of sending a 10 lb. package by parcel post. (New York is in the 5th zone from Chicago.)

27. Compare the cost of sending a 5 lb. package by parcel post from Chicago to Butte, Montana (6th zone), with the express charge for the same package and distance, fifty-two cents. (Feb. 1, 1914.)

28. Other problems may be made if desired, to show the comparative costs of shipping by parcel post, by express, and by freight.

§183. Applications to Live Stock.

1. A farmer builds a convenient barn and saves 30 minutes per day in doing his chores? How many hours does he save in a year? How much is this time worth at 20¢ per hour? What will be saved in 10 years?

2. A dairy farmer hauls 278 lb. of milk to the creamery. It tests 3.5% fat. How many pounds of butter-fat does it contain? How much is the butter, made by mixing $1\frac{3}{4}$ lb. of water to the fat, worth at present prices?

3. A second farmer hauls 250 lb. of milk to the creamery. It tests 4.5% fat. How many pounds of fat does it contain?

4. How much is the fat worth at the present price? How much more money does the second farmer get than the first? Does it pay to keep a low grade of cows?

5. A cow gives an average of 21 pounds of milk per day for 300 days of each year, testing 4% fat. How many pounds of butter-fat does she give? How much is the butter-fat worth at present price?

6. A cow gives 6000 pounds of milk in a year, testing 5% fat. How many pounds of butter-fat does she give? How much is the butter-fat worth at present price?

7. The average cow in a certain state gives about 3600 pounds of milk a year, testing about 3.4% fat. How many pounds of butter does she give? What is it worth at present price?

8. During a certain year ten cows in a state agricultural college dairy herd averaged 7200 pounds each of milk, testing 5.5%. At this rate what would be the yearly income from each cow at the present price of butter? How does this compare with the average cow of the state of problem 7?

9. For bodily maintenance a cow requires at each feeding .07 lb. of protein, .7 lb. of carbohydrates, and .01 lb. of fat per 100 lb. live weight. How many pounds of these nutrients at each feeding are required to maintain a cow weighing 1000 lb.? 800 lb.? 1200 pounds?

10. If .051 lb. of protein, .25 lb. of carbohydrates and .018 lb. of fat are required to produce 1 lb. of $4\frac{1}{2}\%$ milk, how many pounds of each nutrient are required to produce 20 lb. of $4\frac{1}{2}\%$ milk?

11. How much protein, carbohydrates, and fat will a 1000 pound cow giving 20 lb. of $4\frac{1}{2}\%$ milk require? See problems 9 and 10.

12. What is the cost of a ration composed of 4 lb. corn, 1 lb. bran, and 12 lb. alfalfa hay at present cost of corn, bran, and alfalfa hay?

13. A cow fed the above ration gives 20 lb. of 4.1% milk. What is the milk worth at present price of butter?

14. A horse is fed 1.8 qt. of oats three times a day half the year and 3.5 qt. three times per day for the balance of the year. How many bushels of oats will he be fed in a year? How much is it worth at present price?

15. A horse is fed 16 pounds of hay per day. How many tons will he be fed in a year? How much is it worth at present price?

16. If it costs \$10.44 per acre to grow a crop of corn and husk it from the standing stalks, what does it cost to produce a bushel of corn when the crop is 60 bu. per acre? 40 bu. per acre? 75 bu. per acre?

17. If it costs \$8 to produce a crop of 2.5 tons of clover per acre, how much does it cost per ton to produce clover hay?

18. If corn is worth 40¢ per bu. (56 lb.) and clover hay is worth \$8 per ton, what will it cost to feed a horse 10 lb. of corn and 20 lb. of clover hay?

19. If bran is worth \$24 per ton, corn 40¢ per bu., and hay \$8 per ton, what is the cost of 8 lb. of corn, 6 lb. of bran, and 16 lb. of hay?

20. What is the cost of 16 lb. of hay and 5 lb. of oats, if hay is worth \$8 per ton and oats are worth 35¢ per bu.?

21. What is the cost of 12 lb. corn fodder and 10 lb. clover hay, if corn fodder is worth \$4 per ton and clover hay is worth \$8 per ton?

22. A farmer buys 100 sheep at \$5.50 each. When shorn the 100 sheep average $8\frac{1}{2}$ lb. of wool. How many pounds of wool do these sheep furnish? How much is it worth at 30¢ per pound?

23. From the 100 sheep the farmer raises 95 lambs worth $3 each. How much are the lambs worth? What is the average income for lambs from the 100 sheep? If it costs $2.90 per head a year to feed the 100 sheep, what percent does the farmer make on his first investment?

24. If a bushel of corn will produce 10 pounds of pork, how much will the feed for the production of a pound of pork cost at present price of corn?

25. If 200 lb. of shorts worth $25 per ton and 600 lb. of corn worth 40¢ per bu. are mixed together, what is the average cost per pound? If a 300 lb. hog be fed $\frac{3}{4}$ lb. of the mixture per 100 lb. live weight, how much will the feed cost per day?

26. It costs $1.75 to plow an acre of land, 75¢ to harrow it 3 times, 30¢ to plant it, $2.00 to cultivate it 4 times, and 60¢ to fertilize it. If 30¢ is spent for seed, and $4 for rent, how much does it cost to raise an acre of corn?

27. If it requires 8 pounds of ear corn to make one pound of pork, how many pounds of pork will 60 bushels of ear corn make? (72 lb. per bu.) How much will the pork made from 60 bushels of corn be worth at present price of pork?

§184. Miscellaneous Problems for Testing.

1. Add 13 hundred, 13 and 13 hundredths; 65 thousand, 65 and 65 thousandths; 9 thousand, 90 and 9 thousandths; and 128 thousand, 128 and 128 ten-thousandth.

2. Subtract 80 thousand, 603 and 607 ten-thousandths from 100 thousand, 100 and 1 thousandths, and check.

3. Reduce 3 pk., 6 qt., 1 pt., to bushels.

4. Reduce .640625 of a bushel to pecks, quarts, and pints.

5. What will it cost to dig a cellar 40 feet by 25 feet and 6 feet deep at 60¢ a cubic yard?

6. Express decimally $\frac{5}{8}\%$; $3\frac{1}{2}\%$; $16\frac{2}{5}\%$; 375%; $5\frac{7}{8}\%$.

7. A railroad right of way 66 feet wide runs square across a quarter section of land. For the right of way the owner receives $225 an acre. How much does the owner get for the right of way?

8. Give the face, the maker, the payee, the date, the rate of interest, and the date of maturity of the following promissory note:

$650. Boston, Mass., June 20, 1910.

Three months after date, I promise to pay Henry James, or order, six hundred fifty and $\frac{00}{100}$ dollars, at the First National Bank of Boston, value received, with simple interest at 6% per annum from date.

Due Sept. 20, 1910. Thomas Morgan.

9. Find the amount due at maturity for the above note.

10. Find the simple interest on $4375 for 2 yr. 3 mo. 15 da. at $4\frac{1}{2}\%$ per annum.

11. A firm fails with assets (property) worth $16,650, from which 30% of its indebtedness can be paid. Find the total indebtedness.

12. An article is bought for 20% less than its value and sold for 20% more than its value. What is the gain percent?

13. If 20% of the selling price of an article is profit, what is the rate percent of gain?

14. I sold two horses for $150 each. On one I gained 25% and on the other I lost 25%. How much money did I gain, or lose altogether?

15. I bought two horses for $130 each. I sold one at 10% profit and the other at 10% loss. How much money did I gain, or lose in both transactions?

16. An article is sold at a loss of 10%. If it had sold for $45 more there would have been a profit on it of $12\frac{1}{2}\%$. Find the original cost of the article.

17. A sold B some goods for $7760, losing 3%. B sold the same goods to C at a profit of 3%. Who paid more for the goods, A or C, and how much more?

18. A sold goods to B, losing 5%. B sold them to C for $798, gaining 5%. How much more did they cost A than C?

19. A man invested a sum of money and sold at 10% profit. He then invested the principal and profit and sold at 4% loss. He again invested all that remained and sold at 25% profit, and finally had $660. What was the original sum invested?

20. An agent sold a consignment of wheat for $867 and invested the money in flour. His rate of commission was the same for both buying and selling. His total commission of the two deals was $34.68. What was the rate of commission?

21. Which is better for the buyer of a bill of goods, a single discount of 45% or a trade discount of 25% and 20%? How much is the difference on a bill of $250?

22. What distance must a carriage move that the wheel 4 ft. 8 in. in diameter may make 300 revolutions? (Use $\pi = 3\frac{1}{7}$.)

23. How many revolutions must a wheel 5 feet 4 inches in diameter make to go one mile?

24. A dishonest grocer used a weight of $15\frac{1}{4}$ oz. for a pound. What percent does he gain through the dishonest measure? What percent do his customers lose?

25. What percent does a grocer gain by using a liquid quart of 57.75 cu. in. for measuring produce to be sold by the dry quart of 67.2 cu. in.? What percent do the customers lose?

26. I received \$80.56 commission at the rate of $1\frac{1}{4}\%$, for selling coffee. What were the net proceeds of the sale?

27. At what price must a merchant mark an article costing \$8.64 that he may drop 20% and still make 25% profit?

28. At what price must an article costing \$8.64 be marked that a discount of 10% may be allowed on the marked price, 20% loss may be allowed for bad debts, and still a profit of 25% be made?

29. At what percent above cost must goods be marked that the marked price may be discounted 22% and 30% may still be realized?

30. A grocer exchanges goods at 20% profit with a farmer for eggs two in every dozen of which are unusable. What profit does the grocer realize?

31. How long will it take \$840 to yield \$149.45 at 7% simple interest?

32. What is the gain on 500 yd. of satin at 48¢, discount 15, 10, and 5, if sold at 35% above cost?

33. A dealer buys 40 tons of coal, 2240 lb. each, at \$6.75. He sells it at \$7.75 a ton of 2000 lb. What is his gain percent?

34. A dealer sells coal so as to make \$1 and 240 lb. of coal on every long ton (2240 lb.). What is the gain percent on coal costing \$6.25 a long ton?

35. Draw a cash account form and balance it for the following data:

Apr. 1912. On hand Apr. 1, \$2640. Apr. 4, sold a horse for \$215. Apr. 6, bought farm tools for \$96.75; paid for labor \$28.60. Apr. 10, sold corn for \$112.50. Apr. 13, paid for labor, \$14; bought household provisions for \$60.85. Apr. 17, paid rent \$150. Apr. 20, bought lumber for \$67.50; paid for labor, \$24. Apr. 25, sold hay for \$72. Apr. 27, paid for labor, \$14.

36. Write a promissory note, yourself payer, John Doe, payee, dated July 1, 1911, due in 1 yr., rate of interest 6%. Find the amount due on the note if it was not paid until Nov. 11, 1912.

37. A commission house sent an agent \$571.20 to pay for a quantity of corn bought for the house. The sum, \$571.20, included a 2% commission for the agent. How much did the corn cost? How many bushels were bought if the price paid was $66\frac{2}{3}$¢ per bushel?

38. Simplify $\frac{12\frac{1}{3}}{25\frac{2}{3}} \times \frac{31\frac{1}{6}}{21\frac{1}{4}}$.

39. Simplify $\frac{13\frac{2}{3}}{32\frac{2}{15}} \div \frac{42\frac{1}{2}}{6\frac{4}{5}}$.

40. If a train had run at a $12\frac{1}{2}$% higher average rate, it would have run 12 mi. further than it did run in a certain time. How far did the train run?

41. If a train had run at a $12\frac{1}{2}$% higher average rate it would have run 25% less distance in 1 hr. less time than it did actually run. How long did the train actually run?

42. What number increased by $66\frac{2}{3}$% of itself becomes 40?

43. What number diminished by $33\frac{1}{3}$% of itself becomes 32?

44. $\frac{5}{6}$ is what per cent of $\frac{3}{4}$?

45. Find $\frac{7}{8}$% of 64.

46. Calculate the difference between the simple interest and the exact interest on \$900 @ 5% for 292 days.

47. Plot the following temperatures:

6 A.M.	7 A.M.	8 A.M.	9 A.M.	10 A.M.	11 A.M.	12 M.
−9	−8	−8	−6	−3	0	0

1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.	6 P.M.
+1	+2	+3	+1	−2	−5

PART IV

PRACTICAL APPLICATIONS FOR REVIEW

§185. Applications for Review (The Panama Canal).

The Panama Canal is made up of the following channels:

1. In Limon Bay, $6\frac{1}{2}$ miles;
2. In Gatun Lake, 24 miles;
3. In Culebra cut, 9 miles;
4. In a small lake, $1\frac{1}{2}$ miles;
5. In a bay at the Pacific end, $8\frac{1}{2}$ miles;
6. Through 6 locks, each 1000 feet.

1. Find the length of the canal to the nearest .01 mi.

2. Work on the canal was begun May 4, 1904. If it is completed by Jan. 1, 1915, how long will it have taken to build the canal?

3. It was announced that the canal would be ready for business July 1, 1913. If it had been, how long would it have taken to build the canal?

4. If the canal is actually completed on May 4, 1915, and it costs $375,000,000 to build it, what will be the average cost per year?

5. The new Panama canal railroad is to be 46.2 mi. long. How much longer is the canal than the railroad?

6. The canal route was first explored in 1527. In 1803 Humboldt first advocated building a canal here. How long was it from the first to the second date?

7. The French Panama Canal Co. began building a canal there in 1881. How long was this after the route was first explored? First advocated? How long will it have been from the date of the first exploration to 1915?

8. From N. Y. City to Colon is 1587 mi.; from Colon to Panama 50 mi., and from Panama to San Francisco 3768 mi. The route from N. Y. City to San Francisco, via Cape Horn, is 14,840 miles. How much is the sea route from N. Y. City to San Francisco shortened by the canal?

9. From N. Y. City to Colon is 1587; from Colon to Panama 50 mi.; from Panama to Honolulu 3950 mi., and from Honolulu to Yokohama is 3915 miles. The sea route, via Cape Horn, from New York to Yokohama is 18,755 mi. How much does the Panama Canal shorten this route?

10. From the data of problem 9, how much does the Panama Canal shorten the sea route from N. Y. to Honolulu?

§186. Industrial Problems for Review.

The table below gives the numbers of men that were employed from month to month during 1900, in factories making pins, needles, hooks, and eyes.

Month	Penn.	N. Y.	N. H.	Conn.	All other States	Total The U. S.
Jan.....	44	75	195	1180	339	1833
Feb.....	41	75	201	1223	327	
Mar.....	41	76	199	1237	331	
Apr.....	38	76	200	1213	321	
May.....	38	76	195	1209	314	
June.....	37	76	187	1207	314	
July.....	37	74	189	1196	314	
Aug.....	40	74	169	1259	328	
Sept.....	40	75	178	1279	331	
Oct.....	40	75	181	1271	328	
Nov.....	42	74	185	1265	326	
Dec.....	42	74	189	1257	327	

1. Add horizontally, check, and tell what the totals mean. Time yourself.

2. Find the average number of men thus employed per month in the several states and in "All other states."

3. What percent to the nearest 1%, of the total number of these men employed in the United States were employed in Connecticut in January? In February? In March? In September?

The corresponding table for women employed in these factories is here given:

Month	Penn.	N. Y.	N. H.	Conn.	All other States	Total The U. S.
Jan....	188	185	166	940	358	1837
Feb....	188	185	169	967	363	
Mar....	188	185	175	973	366	
Apr....	188	185	163	970	370	
May....	188	185	161	964	362	
June....	188	185	155	951	376	
July....	188	185	148	963	368	
Aug....	188	185	139	954	357	
Sept....	188	185	156	970	372	
Oct....	188	185	159	978	360	
Nov....	188	185	159	984	352	
Dec....	188	185	158	966	352	

4. Add horizontally, check the work, and tell what the totals mean. Time yourself.

5. Give a quick way of finding the sums for Pennsylvania, and for New York. Check the work in this quick way for these two columns.

6. Find the average number of women employed per month in the several states and in "All other states."

7. Compare the monthly average number of men with the monthly average number of women in the whole United States.

§187. Exercises for Practice and Review.

1. Solve the following exercises:

(1) $\frac{1}{2}+\frac{2}{3}=?$ (6) $\frac{2}{3}-\frac{1}{2}=?$ (11) $\frac{5}{6}-\frac{1}{2}+\frac{1}{8}=?$

(2) $\frac{1}{4}+\frac{3}{5}=?$ (7) $\frac{4}{5}-\frac{2}{3}=?$ (12) $\frac{7}{8}-\frac{3}{4}+\frac{5}{12}=?$

(3) $\frac{2}{3}+\frac{3}{4}=?$ (8) $\frac{4}{7}-\frac{2}{9}=?$ (13) $\frac{11}{12}-\frac{7}{8}+\frac{4}{11}=?$

(4) $\frac{4}{5}+\frac{6}{7}=?$ (9) $\frac{12}{13}-\frac{3}{7}=?$ (14) $8\frac{2}{3}+6\frac{1}{2}-10\frac{9}{11}=?$

(5) $\frac{4}{7}+\frac{3}{14}=?$ (10) $\frac{7}{8}-\frac{3}{5}=?$ (15) $16\frac{3}{4}-4\frac{1}{6}+7\frac{1}{9}=?$

2. Solve the following exercises, doing all the work mentally:

(1) $9\times\frac{5}{36}=?$ (6) $16\times\frac{7}{8}=?$ (11) $2\frac{1}{5}\times 5=?$

(2) $21\times\frac{9}{14}=?$ (7) $24\times\frac{11}{12}=?$ (12) $6\frac{1}{4}\times 4=?$

(3) $10\times\frac{11}{15}=?$ (8) $\frac{7}{9}\times 18=?$ (13) $5\times 1\frac{2}{5}=?$

(4) $\frac{5}{9}\times 9=?$ (9) $\frac{5}{18}\times 36=?$ (14) $8\times 1\frac{7}{8}=?$

(5) $2\frac{5}{64}\times 8=?$ (10) $75\times\frac{2}{25}=?$ (15) $6\frac{2}{3}\times 3=?$

3. Solve the following, using pencil only when necessary:

(1) $3\frac{1}{3}\times 4=?$ (6) $5\times\frac{3}{4}=?$ (11) $25\times 8\frac{3}{5}=?$

(2) $\frac{2}{3}\times\frac{3}{4}=?$ (7) $8\times\frac{3}{5}=?$ (12) $\frac{7}{8}\times\frac{3}{10}\times\frac{8}{9}=?$

(3) $\frac{7}{8}\times\frac{8}{14}=?$ (8) $\frac{4}{5}\div\frac{2}{5}=?$ (13) $2\frac{1}{2}\times 6\frac{2}{5}=?$

(4) $10\frac{5}{6}\times 6=?$ (9) $\frac{9}{10}\div\frac{8}{11}=?$ (14) $3\frac{1}{4}\times\frac{7}{13}\times\frac{6}{7}=?$

(5) $50\times 5\frac{9}{10}=?$ (10) $\frac{7}{4}\div\frac{14}{30}=?$ (15) $\frac{3}{4}\div 5=?$

4. Solve the following using pencil only when necessary:

(1) $4\frac{4}{5}\div 8=?$ (6) $21\div\frac{7}{11}=?$ (11) $3\frac{1}{8}\div 4\frac{5}{7}=?$

(2) $12\frac{4}{7}\div 11=?$ (7) $50\div 4\frac{3}{7}=?$ (12) $\frac{9}{49}\div\frac{2}{11}=?$

(3) $3\frac{2}{3}\div 7=?$ (8) $56\div 5\frac{4}{9}=?$ (13) $27\frac{1}{2}\div 2\frac{3}{4}=?$

(4) $6\frac{1}{3}\div 8=?$ (9) $3\frac{3}{7}\div\frac{9}{14}=?$ (14) $12\frac{2}{5}\div 4\frac{1}{6}=?$

(5) $9\frac{1}{9}\div 6=?$ (10) $3\frac{3}{4}\div 5\frac{5}{8}=?$ (15) $19\frac{7}{8}\div 3\frac{2}{3}=?$

§188. Applications.

1. A long ton is 2240 pounds and a short ton is 2000 pounds. How many short tons are there in 78 long tons of coal?

2. A gain of 93¢ a long ton is equivalent to a gain of how much per short ton?

3. A coal dealer buys 78 long tons of coal at \$6.75 and sells it at \$7.75 a short ton. How much does he gain on each long ton? How much does he gain on all 78 tons of coal?

4. Ten cases of eggs of 30 dozens each were divided among 3 merchants. The first merchant received $\frac{1}{5}$, the second, $\frac{1}{4}$ of the remainder, and the third, what was left. The eggs cost $12\frac{1}{2}$¢ a dozen. What was the cost to each merchant?

5. An estate was divided so that $\frac{1}{3}$ went to the first daughter, $\frac{1}{2}$ of the remainder to the second daughter, and what was left was divided equally among 4 grandchildren, each of whom received \$1800. What was the value of the whole estate?

6. An estate was willed partly to three sons and partly to charity in the following shares: the first son received $\frac{1}{3}$ of the estate; the second son, $\frac{3}{4}$ of the remainder; the third son, $\frac{3}{5}$ of what still remained, and the rest of the estate, which was \$2475, was willed to charity. What was the value of the entire estate?

7. A, B, and C together have 1485 sheep. A has $\frac{2}{3}$ as many as B, and B has $\frac{3}{4}$ as many as C. How many has each?

8. $\frac{2}{3}$ of the time past noon is $\frac{2}{5}$ of the time to midnight. What is the hour?

9. $\frac{1}{4}$ of the time past noon is $\frac{1}{8}$ of the time to midnight. What is the hour?

§189. Expressing Numbers Approximately.

In recording measured distances it is often sufficiently accurate to use only the nearest .1 or .01 of a foot, or yard, or mile. Sometimes even the nearest whole foot is correct enough. Sums of money are not usually expressed more accurately than to the nearest cent. Sometimes the nearest whole dollar is accurate enough. The case is similar with other measured magnitudes.

In practical work then, it is convenient to use *approximate* (nearly correct) values for long decimals. For example, instead of \$15.648246, we would write \$15.65, and say this is accurate enough for practical purposes. This shortening must be done in accordance with the plan described here.

\$15.65 is said to express \$15.648246 to the nearest .01 of a dollar (or to the nearest cent), because \$15.65 differs from \$15.648246 by *less than* $\frac{1}{2}$ *of* .01 *of a dollar*, or by *less than* $\frac{1}{2}$ *of a cent*. We know that 5 units in any decimal place is $\frac{1}{2}$ a unit of the next place to the left. More than 5 units is more, and less than 5 units is less than $\frac{1}{2}$ of a unit in the next place to the left. When the next place beyond (to the right of) the one we want to keep is more than 5 (as the 8 in \$15.648246), we *increase* the last figure by 1 (making \$15.65 and not \$15.64) and drop the rest of the decimal. When the next decimal figure is less than 5, we drop *all* figures after the one to be kept. Thus if 15.648246 were to be correct to the nearest .001, we would write it 15.648, because the 2 is less than 5. Thus 15.648 expresses 15.648246 to the nearest .001, or to within less than $\frac{1}{2}$ of .001 (or .0005).

If the next figure beyond the one to be kept is *exactly* 5 we increase the last digit to be kept by 1, if the increase makes the digit *even*, otherwise we drop the 5. Thus 15.675 would be shortened to 15.68; but 15.645 would be shortened to 15.64.

To find a quotient *to the nearest .1* means to carry the division to two places of decimals and then shorten to one; to find it *to the nearest .01* means to get 3 decimal figures, and shorten to two, etc.

1. Find the following quotients to the nearest .01:

(1) $8\overline{)17}$ (5) $33\overline{)43.33}$ (9) $43\overline{)101.835}$

(2) $7\overline{)23}$ (6) $67\overline{)82.946}$ (10) $236\overline{)739.86}$

(3) $9\overline{)25}$ (7) $56\overline{)181.776}$ (11) $421\overline{)1711.365}$

(4) $12\overline{)37}$ (8) $62\overline{)120.714}$ (12) $567\overline{)3433.185}$

2. Find the following quotients to the nearest .001:

(1) $4\overline{)11}$ (5) $1.7\overline{)18.0803}$ (9) $12.3\overline{)16.41435}$

(2) $7\overline{)15}$ (6) $2.9\overline{)10.5946}$ (10) $36.3\overline{)48.18825}$

(3) $9\overline{)41}$ (7) $3.5\overline{)12.8912}$ (11) $4.84\overline{)30.4357}$

(4) $11\overline{)73}$ (8) $3.2\overline{)503.0666}$ (12) $8.76\overline{)49.875708}$

3. Express as decimals to the nearest .001:

(1) $\frac{4}{9}$ (4) $\frac{7}{9}$ (7) $\frac{6}{7}$

(2) $\frac{5}{7}$ (5) $\frac{2}{3}$ (8) $\frac{9}{11}$

(3) $\frac{5}{9}$ (6) $\frac{5}{6}$ (9) $\frac{7}{11}$

§190. Circulates.

Some fractional numbers can not be expressed exactly in decimals.

Show by the ordinary method of reducing common to decimal fractions (§102) that:

1. $33\frac{1}{3} = 33.3333+$
2. $37\frac{4}{11} = 37.363636+$
3. $16\frac{13}{37} = 16.35135135+$
4. $24\frac{2}{7} = 24.28571+$
5. $6\frac{357}{1100} = 6.32454545+$
6. $\frac{4811}{1111} = .43294329+$

These are all *non-terminating decimals*. Those decimals in which a digit, or a set of digits, is repeated are called *circulating decimals*, or *circulates*. The part of the decimal made up of the repeated digit is the *repetend*.

In the ordinary process of attempting to express some common fractions decimally circulates are derived from their *generating fractions*. It is a little more difficult to find the generating fractions from the circulates.

Circulates are written with a dot over the repeated digit, or with a dot over the first and last digits of the repeating set, thus: $33.\dot{3}$, $37.\dot{3}\dot{6}$, $16.\dot{3}5\dot{1}$, $.\dot{4}3\dot{2}\dot{9}$, etc.

Observe carefully the treatment of the following:

$33.\dot{3} = 33\frac{3}{9} = 33\frac{1}{3}$ $\qquad$ $16.\dot{3}5\dot{1} = 16\frac{351}{999} = 16\frac{13}{37}$

$37.\dot{3}\dot{6} = 37\frac{36}{99} = 37\frac{4}{11}$ $\qquad$ $.\dot{4}3\dot{2}\dot{9} = \frac{4329}{9999} = \frac{481}{1111}$

The law is plain; viz.:

Form a common fraction by making the numerator the number denoted by the repeating set of digits, and the denominator as many 9's as there are repeating digits

Find the generating fraction of each of the following circulates.

1. $.\dot{9}$	**5.** $.\dot{1}\dot{2}$	**9.** $.\dot{1}0\dot{8}$	**13.** $.\dot{1}88\dot{1}$
2. $.\dot{6}$	**6.** $.\dot{2}\dot{7}$	**10.** $.5\dot{4}0$	**14.** $.\dot{1}0\dot{8}\dot{9}$
3. $.\dot{7}$	**7.** $.\dot{5}\dot{4}$	**11.** $.0\dot{5}\dot{4}$	**15.** $.0\dot{2}9\dot{7}$
4. $.\dot{4}$	**8.** $.\dot{7}\dot{2}$	**12.** $.6\dot{4}8$	**16.** $.\dot{1}00\dot{8}$

17. Prove the rule: *the number of rails, each 10 yd. long, passed over by a train in 20 seconds, is (nearly) the rate of running of the train in miles per hour.*

Let the rate of the train in miles per hour be r, and the number of rails run over in 20 seconds be n.

Then the distance in yards run in 1 hr. is $1760 \times r$, and the distance in yards run in $\frac{1}{3}$ minute ($=20$ seconds) is $10 \times n$, since a rail is 10 yd. long.

Then:

Distance run in 1 hr. is to distance run in $\frac{1}{3}$ min. as 60 min. is to $\frac{1}{3}$ min.

Or, $\dfrac{1760 \times r}{10 \times n} = \dfrac{60}{\frac{1}{3}}$

$176 \times r/n = 180$

$r = \dfrac{180}{176} \times n$

$= 1.022727. \times n$

$= 1.0\dot{2}2\dot{7} \times n = n$, nearly.

The number, n, is the number of clicks of the wheels passing over the joints between the rails. These may be counted for 20 seconds by the passengers.

Show that r would be still more nearly n if the clicks were counted for $20\frac{1}{2}$ seconds, and that $r = n$, for $20\frac{5}{11}$ seconds.

18. Express the exact value of $1.0\dot{2}\dot{2}\dot{7}$ by aid of the generating fraction of the repetend $.00\dot{2}\dot{7}$.

19. Find the equivalents of the following circulates, expressing the repetends as common fractions:

(1) $7.\dot{6}$	(7) $3.8\dot{4}\dot{5}$	(13) $3.1\dot{1}\dot{0}\dot{8}$
(2) $15.3\dot{6}$	(8) $5.2\dot{5}\dot{4}$	(14) $8.6\dot{1}3\dot{5}$
(3) $38.1\dot{3}$	(9) $16.\dot{6}$	(15) $.\dot{8}\dot{1}$
(4) $10.2\dot{3}\dot{6}$	(10) $7.1\dot{6}\dot{3}$	(16) $12.7\dot{2}$
(5) $3.2\dot{7}$	(11) $37.0\dot{8}\dot{1}$	(17) $23.1\dot{6}\dot{2}$
(6) $8.7\dot{2}$	(12) $5.2\dot{9}$	(18) $4.2\dot{1}6\dot{2}$

MODELING SOLIDS

§191. To Model a 3-Inch Cube.

The cube is much used by architects in buildings and parts of buildings. Some crystals found in Nature also have the cubical form. Can you suggest any uses or occurrences of the cube?

To draw the development, or pattern of the outside of a 3″ cube, Fig. 1, draw a line as *ac* nine inches long, and mark it off into 3-inch spaces as at *p* and *b*. At *p* draw a line perpendicular to *ac*; prolong it.

Exercise: *To draw a perpendicular to a line as ac, at a point, as p, on the line:*

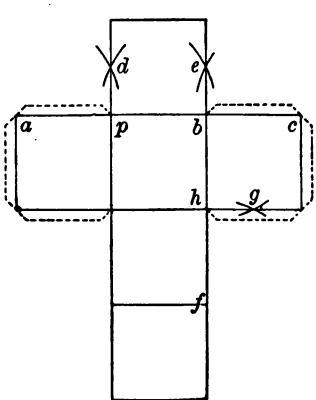


FIGURE 1
Development of 3″ cube

Measure off equal distances, as *pa* and *pb*, on opposite sides of *p*.

Put the pin-foot of the compasses, at *a*, and open the points farther than the distance *ap* and draw one of the arcs at *d*.

Using the same distance of compass points, put the pin-foot on *b* and draw the other arc at *d*.

With a ruler draw a line through *p* and the crossing point of the arcs at *d*.

The line *pd* is the perpendicular.

Another way is to place the square corner of a box, or book, at *p* and with one edge of the box or book, along *ap*, draw a line *dp*, along the other edge.

Through *b* draw another perpendicular, as *eb*, and prolong it. Divide it into 3-inch lengths. Mark the division points.

Through the point *h* draw a perpendicular, as *hg* to the line *ef*.

Set off 3-inch spaces on these lines and complete the development as shown in the figure. Leave flaps as indicated.

Crease the paper over the edge of a ruler along the lines and fold and paste a 3-inch cube. Leave the top unpasted.

1. What is the area of each surface? All surfaces?

2. How many edges has the cube? How many faces meet in each edge?

The surfaces of the cube meet each other in the edges, forming lines.

Call the corners vertices (ver'ti-ses). A single corner is a vertex.

3. How many vertices has the cube? How many edges meet at each vertex?

The edges meet each other in the corners of the cube, forming points called vertices of the cube.

4. In how many faces does each vertex lie?

5. Do cubes have length? Breadth? Thickness? Do surfaces? Do lines? Do points?

6. What are the limits of cubes? Surfaces? Lines?

Patterns of the surfaces of figures like those of Fig. 1, page 310, will hereafter be called *developments*.

7. Place an inch-cube in the corner of the 3-inch cube. How many inch-cubes will fill the row along one edge of the larger cube?

8. How many such rows form a layer on the bottom?

9. How many such layers will fill the cube?

10. How many inch cubes will fill a 3-inch cube?

11. How many cubic inches are there in a 4-inch cube? In a 10-inch cube?

Pupils should make their own models, different pupils using different lengths of lines and even devising different forms of pattern. For example, let one pupil use a 3-inch fundamental distance instead of an inch, another a 4-inch distance, and so on, each comparing his completed model with other models.

§192. To Model Prisms.

(a) *A square prism.*

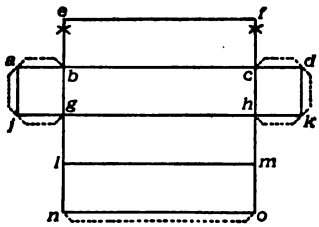


FIGURE 1
Development of the Square Prism

Draw a straight line *ad* (Fig. 1) 12 inches long, and make *ab* and *dc* each 2 inches long. Draw *en* and *fo* perpendicular to *ad* at *b* and *c*. Lay off distances *eb*, *bg*, *gl*, *ln*, *fc*, *ch*, *hm*, *mo*, each equal to 2 inches. Draw *ef*, *jk*, etc., and complete the development. Provide flaps. Cut and paste the model.

1. What is the area of each of the end surfaces?

2. Give the area of each side surface; of all surfaces.

3. How many inch-cubes will the model hold?

4. By measuring the edges how could you find the number of cubic inches the model will hold?

5. How many cubic inches would it hold if it were 1 inch longer? Twice as long?

6. Answer questions 1 to 4, assuming every line in the pattern twice as long as in the development.

(b) *A flat prism.*

Draw a line *ae* (Fig. 2) 18 in. long and mark off $ed=3$ in., $dc=6$ in., $cb=3$ in., and $ba=6$ inches.

At *c* and *d* draw perpendiculars to *ae* and extend them 11 in. below *c* and *d* to *r* and *s*, and 3″ above *c* and *d* to *k* and *l*.

Make *co* and *dp* 8 in. long and draw *mq* through *o* and *p*.

Complete the development by drawing the necessary lines.

Provide it with flaps, as shown, and paste up the model, leaving the top *ckdl* open.

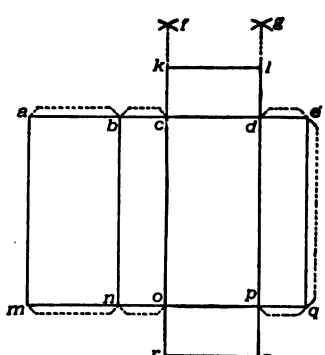


FIGURE 2
Development of Flat Prism

1. With the aid of the model of an inch-cube, find how many cubic inches would fill the flat prism.

2. To what is the product of the 3 different edges equal?

3. How could you obtain the capacity in cubic inches of such a prism by measuring the lengths of its edges?

4. How many square inches in the whole surface of the flat prism of Fig. 2, page 312?

§193. Comparison of Prisms.

(a) *A right prism.*

Using the lengths of lines as shown in Fig. 1, draw a pattern on heavy paper and construct the model of a right prism, as was done in Fig. 1. Provide flaps and paste up the model, leaving one end open, so that it may be filled with sand.

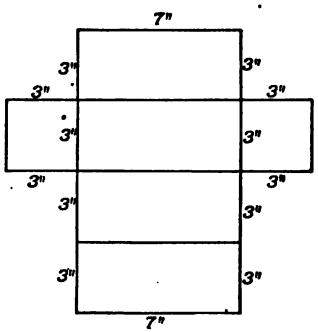


FIGURE 1
Development of Right Prism

1. How many cubic inches of sand will it take to fill the model?

2. How can you find the capacity of the model by using the lengths of the edges?

(b) *An oblique prism with parallelograms for bases.*

Similarly draw a pattern, cut, fold, and paste up a model of the oblique prism, as shown in Fig. 2, leaving an end unpasted for sand.

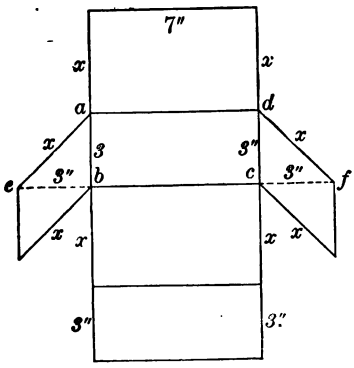


FIGURE 2
Development of Oblique Prism

3. When the model of the right prism is full of sand, the sides not being bulged out, pour the sand into the model of the oblique prism. Does it fill the second model?

4. What is the ratio of the capacities of the two models?

5. What is the ratio of the areas of the bases (ends)?

(c) *A triangular prism.*

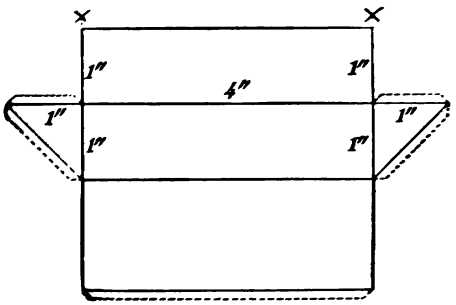


FIGURE 1
Development of Triangular Prism

Model a triangular prism like the one shown in Fig. 1.

6. Compare the capacity of this model with that of the model of a square prism 1″×1″×4″. What is their ratio?

7. Model a triangular prism such as that of Fig. 1, but use for lengths 3″ instead of 1″ and 7″ instead of 4″. How does its capacity compare with that of the prism of Fig. 1, p. 313? Of Fig. 2, p. 313? Give the ratio in each case.

8. Compare the ratios of the bases.

§194. Volume of an Oblique Prism.

The volume of a figure is the number of cubical units in the space enclosed by its bounding surfaces.

1. How many cubic inches of paper are there in a straight pile of visiting cards 2″ high, if each card is 2″×3″?

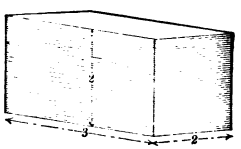


FIGURE 2
Square Pile or Right Prism

2. How many cubic inches would there be in the pile if it were 5″ high? 9″ high? *a* in. high?

3. Push the straight pile of problem 1 over as in Fig. 1, p. 315. How many cubic inches of paper are there in this oblique pile?

4. Has the height of the pile been changed in Figure 1 by the fact that the pile was pushed over? Has the area of the base been changed? The volume?

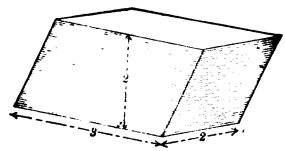


FIGURE 1
Oblique Pile, or Oblique Prism

5. How can the number of cubic units in a right prism be found from the area of its base and its height? In an oblique prism?

6. Find the volumes of square prisms having edges of the following lengths:

(1) $3'' \times 5'' \times 6''$
(2) $6'' \times 5'' \times 9''$
(3) $15'' \times 9'' \times 8''$
(4) $4\frac{2}{3}' \times 6' \times 12'$
(5) $16' \times 10\frac{3}{8}' \times 20'$
(6) $45'' \times 16\frac{2}{5}'' \times 21''$
(7) $3\frac{3}{4}' \times 16\frac{1}{2}' \times 20\frac{3}{8}'$
(8) a in. $\times b$ in. $\times c$ in.

7. Find the volume of an oblique prism having rectangles 5 feet by 6 feet for upper and lower bases, and an altitude of 3 feet.

8. Find the volume of oblique prisms having rectangular bases of dimensions indicated by the first two dimensions of the parts of problem 6 and altitudes equal to the third dimensions of the parts of problem 6.

§195. Perimeters.

The PERIMETER of any figure is the sum of the lines bounding the figure.

In (6), Fig. 1, p. 316, if p denote the perimeter, then:

(I) $$p = x + y + z.$$

1. What does $x+2x+4x$ mean? How may it be more briefly written?

2. What is the coefficient (the multiplier) of x in the answer to problem 1?

3. Write the perimeter, p, of (1), Fig. 1, in two ways.

4. Write the perimeter, p, of (13) in three ways.

5. Write the perimeter, p, of (12) in three ways.

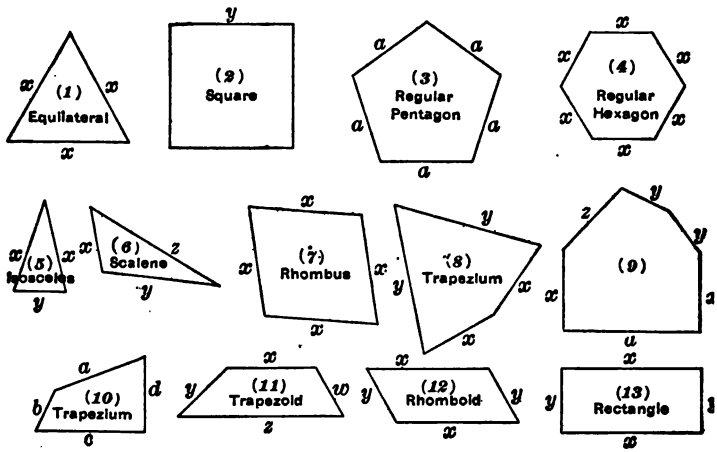


FIGURE 1

6. Write $\frac{1}{2}$ of the perimeter, p, of (13) in two ways.

7. Write an expression showing that the two answers to 6 are equal.

8. In (12), if $x=80$ rods, and $y=40$ rods, how many feet are in the perimeter?

9. Forms (1) and (13) are combined into a single figure. Write p for the new figure in two ways, supposing x the same in both.

§196. Quadrilaterals.

1. Forms (2), (7), (8), (10), (11), (12), and (13), Fig. 1, are different kinds of quadrilaterals. What is a *quadrilateral?*

2. What quadrilaterals have their opposite sides parallel? These are parallelograms. Define a *parallelogram.*

3. What parallelograms have *all* their sides *equal?* What is a *rhombus?*

4. What rhombus has *all* its angles *equal?* What is a *square?*

5. What quadrilaterals have their opposite sides equal but consecutive angles not equal? Define a *rhomboid.*

6. What parallelograms have their angles all equal? Define a *rectangle.*

7. What quadrilateral has only *one* pair of sides parallel? Define a *trapezoid.*

8. Is a trapezoid a parallelogram? Is it a quadrilateral?

9. Is a rectangle necessarily a quadrilateral? Is it a parallelogram? A square? *May* a rectangle be a square?

10. Is a square necessarily a quadrilateral? Is it a parallelogram? A rectangle? A rhombus?

§197. Perimeters of Miscellaneous Figures.

1. Denote the perimeter of each of the forms in Fig. 1, p. 316, by p and write an equation like (I) in §195, showing the value of p for each figure.

2. Omitting the lines on which the forms join, write an equation showing the value of p when (2) and (6), Fig. 1, p. 316, are joined on y, which has the same value in both.

3. In the same way join (3) and (10) on a, and write an equation showing the value of p.

4. Join (2) and (7) in which x and y are equal. If the perimeter of the figure thus formed is 240 rods, find the value of x in feet.

5. What is the length of the perimeter of a figure like (6), whose sides are 2 in., a in., and b in. long? x in., 8 in., and z in. long?

6. Like (10), with sides c ft., $2c$ ft., d ft., and x ft.? Like (12), with sides $2x$ rd., $4y$ rd., $2x$ rd., and $4y$ rd.?

In such forms as those from (5) to (10), Fig. 1, below, some line, or lines, must be found by subtracting others. In (6) for example, note that the ends are each $x-y$. The perimeter is then $x+x+(x-y)+y+y+(x-y)=4x-y-y+y+y=4x-2y+2y=4x$.

All sides not lettered must be expressed without using other letters than those given on the figure.

One-half the difference of a line m and a line n is written

$$\frac{m-n}{2} \text{ or } \tfrac{1}{2}(m-n)$$

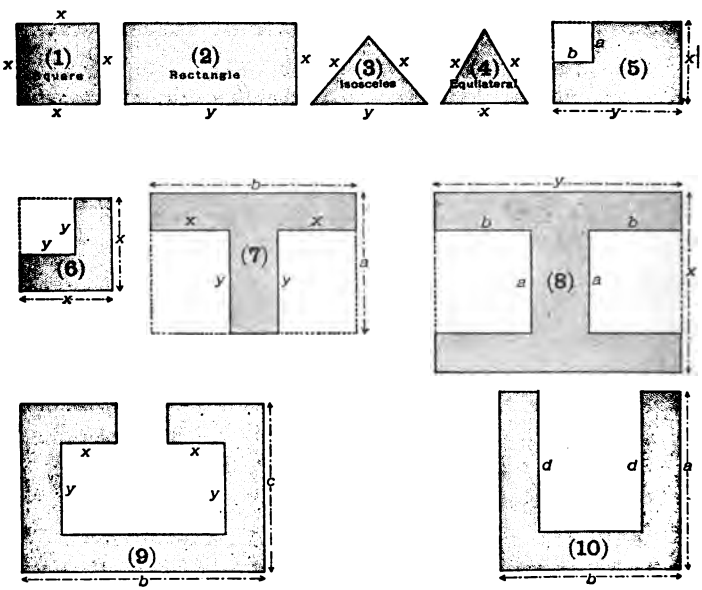


FIGURE 1

7. Write the perimeter (p) of each form of Fig. 1, above.

8. In (8), $a=15'$, $b=12'$, $x=25'$, and $y=30'$. Find p.

9. Find the area of (8) enclosed by the solid lines.

10. Make and solve other similar problems.

ANGLES AND ARCS

§198. Measuring Angles and Arcs.

Answer all you can orally

We may measure the amount of turning of each clock hand in either of two ways:

(1) By the length of the circular arc passed over by the tip of the rotating hand; or,

(2) By the wedge-shaped space having its point at the hand-post, A, over which the stem (A-III), of the rotating hand moves.

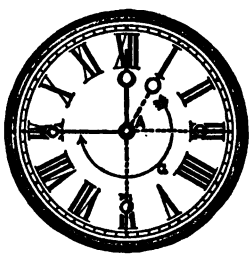


FIGURE 1

While the clock hand turns from XII to III its tip moves over 1 QUADRANT of arc and its stem moves over a right angle.

1. While the hand turns from XII to VI over how many quadrants does its tip move? Over how many right angles does its stem turn?

2. Answer the same questions for a turn of the hand from XII to IX; from XII to XII again; from XII around through XII to III.

Any wedge-shaped part of the face moved over by the stem of a clock hand as it turns around the hand-post is called an angle. The curve passed over by the tip of the hand, while the stem of the hand moves over the angle, is called the arc of the angle.

As the hand moves from A-XII to A-VI, that is, so that the two positions of the hand are in the same straight line, the hand moves over a straight angle.

3. A straight angle equals how many right angles?

4. The arc of a straight angle equals how many quadrants? How many quadrants make a complete circle?

5. How many 5-min. spaces make the circumference of a complete circle? How many 1-min. spaces make the circumference of a complete circle?

6. What part of a right angle is passed over by the stem of the hand as its tip moves from one end to the other of a 1-min. space?

7. If lines were drawn from the hand-post to the ends of all the 1-min. spaces, the whole face of the clock would be divided up into how many small angles?

The instrument in common use for measuring angles and arcs is the *protractor*. (See Fig. 1.)

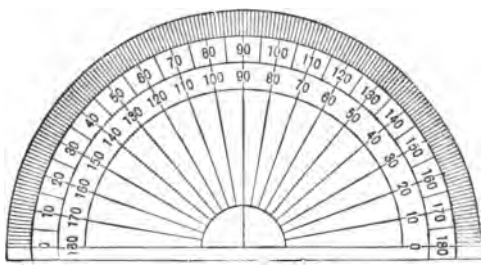


FIGURE 1

Instead of dividing the right angle up by radiating lines into 15 equal parts, the protractor divides the right angle into 90 equal parts, one of which is called the angular degree.

8. These same lines would divide the quadrant up into how many equal parts?

Each of these parts is a DEGREE OF ARC.

9. How many angular degrees are there in a straight angle? How many degrees of arc are there in the arc of a straight angle?

10. How many angular degrees are swept over by a clock hand while moving entirely around once? How many degrees of arc in the circumference of a circle?

11. Study the protractor, Fig. 1; notice how its marks are numbered. How many degrees of arc are there between the closest lines on the outer edge? How many degrees of

angle are there between the lines which converge toward the center?

For smaller angles a shorter unit is $\frac{1}{60}$ of a degree of angle and the smaller unit is called the minute of angle. $\frac{1}{60}$ of the minute, called the second, is a still smaller unit.

12. How many minutes of angle are there in a right angle? In a straight angle? In a complete revolution, called a *perigon?* How many seconds, in each case?

13. The arcs between the sides of the minute and the second of angle are the minute and the second *of arc.* How many minutes of arc in a quadrant?

A paper protractor can be purchased at any bookstore, and each pupil should supply himself with one.

To measure an angle as DAF, the center of the protractor is placed on the vertex, A, and the O-line through A is placed along one side AD (see Fig. 1). The reading of the mark under which the other side, AF, falls, is the number of degrees in the angle, or in the arc of the protractor included between AD and AF.

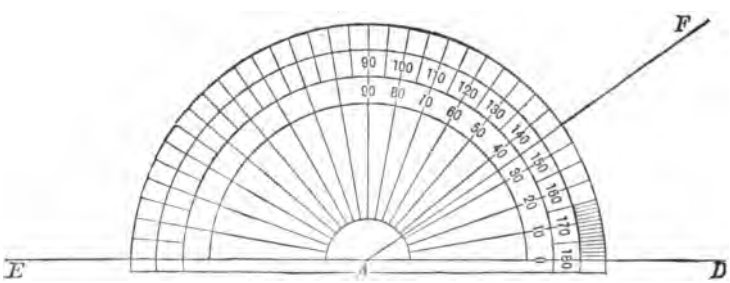


FIGURE 1

14. Draw a triangle, and, with a protractor, measure its angles. To what is the sum of all three equal?

15. With ruler draw a half dozen triangles of different shapes, carefully measure each angle, and find the sum of the three for each triangle.

16. Draw two straight lines crossing each other as in Fig. 1. With the protractor, measure and compare angles *a* and *c;* angles *b* and *d*.

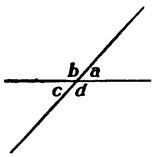


FIGURE 1

The angles a and c, or b and d, lying opposite each other, are called opposite angles, or vertical angles.

17. Draw two crossing lines in different positions, and in each case compare the measures of a pair of opposite angles. What do you find to be true?

18. Draw a pair of parallel lines, as lines *a* and *a*, Fig. 2 below. Now draw a third straight line, *C*, cutting across these two parallels. Measure with a protractor and compare the four angles, in which 1 is written in Fig. 2. Measure and compare those in which 2 is written. What do you find to be true?

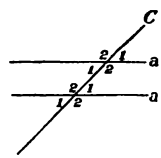


FIGURE 2

19. Draw a line perpendicular to another line and measure each of the 4 angles thus formed. What do you find?

The lines which include the angle are sides of the angle. The point where the sides meet is the vertex of the angle.

20. Draw a given angle at a point on a given line.

Let the given line be *ED*, the given angle 35 degrees, and the given point, *A*, Fig. 1, p. 321. Place the protractor with its center at *A*, and with the diameter of the protractor along the line *ED*. Make a dot opposite the 35-degree mark on the protractor. With the ruler draw a straight line through *A* and this dot to *F*. The angle, *DAF*, is the required angle.

21. Draw a straight line, mark a point on the line and draw the angle 75 degrees; 95 degrees; 100 degrees; 135 degrees; 55 degrees and 30 minutes.

TABLE OF UNITS OF ANGLE AND ARC MEASUREMENT

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
360 degrees	=	1 circumference (or perigon)
90 degrees	=	1 quadrant (or right angle)

§199. The Sum and Difference of Angles.

EXERCISE I.—*With ruler and compass, draw* AB *and* CD *perpendicular to each other.*

1. Suppose a protractor supplied with a pointer as shown in Fig. 1, the center of the protractor being placed at the point O; over how many degrees of arc would the tip of the pointer turn while the stem of the hand turns from line OB to line OC? From OB through OC to OA?

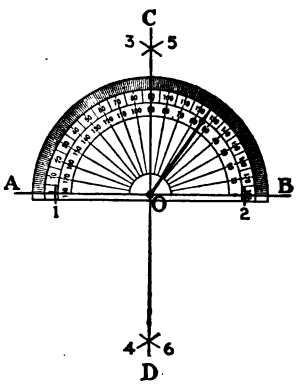
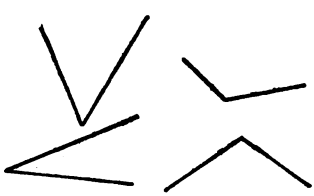


FIGURE 1

2. How many degrees are there in the angle BOC? In angle BOA? In COD? In AOD? In the angle from OA around through OD to OB? From OA around to OC? From OA entirely around to OA again?

3. How many degrees of angle fill the space around a point, as O, on one side of a straight line, as AB?


Acute Angles Obtuse Angles
FIGURE 2

4. How many degrees of angle fill the space around O on both sides of AB?

5. Draw a triangle having a right angle, using the square corner of a card.

6. Draw a triangle having an obtuse angle.

An angle that is smaller than a right angle is called an acute angle (see Fig. 2).

An angle that is larger than a right angle is called an obtuse angle.

7. Draw a triangle having three acute angles.

EXERCISE II.—*Find the sum of two angles.*

8. Draw two acute angles like those at the top of Fig. 1, carefully cut them out, and place them as in the lower part of the figure. Push their vertices and nearer sides entirely together. Draw two lines along the other sides of the angles. Remove the angles and have an angle like that on the right, which is the *sum* of the two given angles.

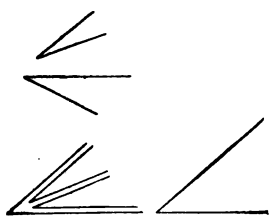


FIGURE 1

9. Calling the larger angle x, and the smaller y, what denotes the last angle, the sum?

10. Draw an angle equal to the sum of an acute and an obtuse angle.

EXERCISE III.—*Find the difference of two angles.*

11. Draw two angles like those at the top of Fig. 2; cut them out, and place them as in the lower, left-hand part of the figure. Mark along the lower side of the upper angle and cut along the mark. The lower part of the larger angle (shown at the left) equals the *difference* of the two angles shown separately at the right.

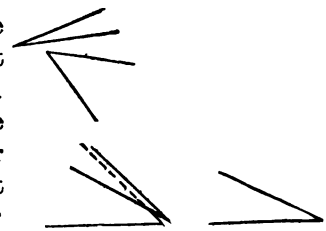


FIGURE 2

12. If the larger angle equals 52° and the smaller equals 18°, how many degrees are there in the difference?

13. If the larger angle equals 63° and the smaller equals x degrees, how many degrees are there in the difference?

14. If the larger angle equals x degrees and the smaller equals y degrees, how many degrees are there in the difference?

Two angles whose sum equals a right angle, or 90°, are called complemental angles (Fig. 1).

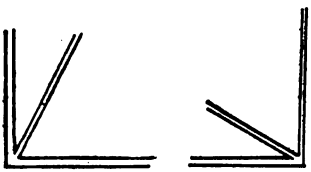


FIGURE 1

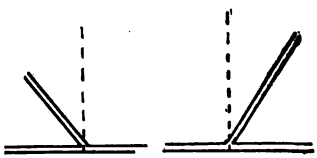


FIGURE 2

Two angles whose sum equals two right angles, or 180°, are called supplemental angles (Fig. 2).

15. The sum of all angles that just cover the plane on one side of a straight line is equal to how many right angles (See Figure 3)? To how many degrees is this sum equal?

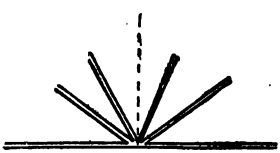


FIGURE 3

EXERCISE IV.—Draw and cut out an equilateral (equal-sided) triangle. Tear off a corner and fit it over each of the other corners in turn. How do the three angles compare in size? Tear off the other corners and place them as in Fig. 4. To what is the sum of all the three angles equal?

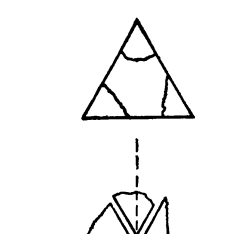


FIGURE 4

16. If one of the angles is x degrees, how many degrees are there in the sum of all three angles?

As the sum of the three angles equals both $3x$ degrees and also 180°, we may write the equation

$$3x = 180$$

If $3x = 180$, to what is x equal? How many degrees are there in one of the angles of an equilateral triangle?

EXERCISE V.—Draw any scalene triangle (no two sides equal). Tear off the corners and place them as in Fig. 1. To how many right angles is the sum of all the angles of the triangle equal? To how many degrees?

FIGURE 1

17. Letting x, y, and z denote the numbers of degrees in the respective angles, in what other way may we write the sum of all three? What equation may we then write? Tell what the equation $x+y+z=180$ means.

18. Draw other scalene triangles, tear off the corners, place them as in the figure, and find whether $x+y+z=180°$ in all cases.

19. Crease a right triangle and find whether the equation, $x+y+z=180°$, is true for it.

20. Cut along the creases and tear off the two acute angles of a carefully creased right triangle and fit over a carefully creased right angle. What seems to be true? If this is true what equation may you write for the sum of the two acute angles (x and y) of a right triangle?

§200. Longitude and Time.

The earth may be regarded as an immense sphere turning, like a top, around one of its diameters as an axis from west to east once every 24 hours. This carries the surface of the earth and all objects fixed upon it around through 360° in 24 hours.

One complete turn (rotation) of the earth is equivalent to 360°; in time units, it is equivalent to 24 hours.

This gives us the following tables of equivalent measures:

360° correspond to 24 hours.

1° corresponds to 4 min.	1 hr. corresponds to 15°.
1′ corresponds to 4 sec.	1 min. corresponds to 15′.
1″ corresponds to $\frac{1}{15}$ sec.	1 sec. corresponds to 15″.

All objects seen on the sky, as the sun, may be *regarded* as stationary, while the turning of the earth carries us past them from the west toward the east. This makes the sun *appear* to rise in the east, move over, and set in the west.

1. Will the sun pass over eastern or western places earlier? Which places have later local* times, those over which the sun passes earlier, or later? Which places have earlier local times, eastern places, or western places?

2. What time is it at a place 45° west of Washington when it is 10 o'clock at Washington? At the same instant what time is it at a place 30° east of Washington?

Longitude is the distance in degrees, minutes, and seconds (of arc) due eastward or westward from a chosen meridian, called the prime meridian.

Astronomers and navigators have agreed that the prime meridian shall be the meridian of the Royal Observatory at Greenwich, England.

3. The difference between the local times of two places is 4 hr. 3 min. What is the difference in longitude between the two places?

4. The difference of longitude between two places is 105° 45′. What is the difference of their local times?

5. It is 4: 50 (4 hr. 50 min.) P. M. at a certain place and 1: 48 P. M. at another. Which place is east of the other and what is the difference of their longitudes?

6. Two men met in Chicago with their watches keeping the correct local times of the places whence they came. On comparing their times one watch showed 10: 47 P. M., and the other 3: 18 P. M., when it was 9: 26 P. M. in Chicago. From which direction did each man come?

---

*Local sun time is obtained by setting timepieces at XII as the sun crosses the meridian.

7. Basing your work on the time of the watches (problem 6) what were the differences of longitude between the places and Chicago?

8. Explain how it happened that the announcement of King Edward's death was read in the Chicago newspapers at an earlier hour than the announced hour of death.

9. How may it happen that a cable message sent Wednesday forenoon may be received at a remote place Tuesday?

10. The local times of two ships at sea differ by 4 hr. 18 min. 15.6 sec. Find their difference of longitude.

11. One ship is in longitude 186° 40′ 12″ west and another is in longitude 20° 16′ 48″ west. What is the difference of their longitudes? Of their times?

12. The longitude of one ship is 3° 28′ 10″ west and that of another is 18′ 38″ east. What is the difference of their longitudes? Of their times?

13. The longitude of the Observatory of Madrid, Spain, is 3° 41′ 17″ west and that of the Berlin Observatory is 13° 23′ 43″ east. What is the difference of their longitudes? Of their times?

14. The longitudes of the Cambridge (Eng.) and of the Paris Observatories are 5′ 41.25″ east and 2° 21′ 14.55″ east respectively. What is the difference of their longitudes? Of their times?

15. When the sun is on your meridian, that is, when it is noon at your place, at what places on the earth is it forenoon? Afternoon? Night? Midnight?

The longitudes from Greenwich, Eng., of places are also often given (as following) in hours, minutes, and seconds *of time*. The plus (+) sign means that the place is west of the meridian of Greenwich, and the minus (−) sign, that the place is east of this meridian.

Place	Longitude from Greenwich	Place	Longitude from Greenwich
	Hr. Min. Sec.		Hr. Min. Sec.
Albany	+4 54 59.99	Denver	+6 59 47.63
Algiers	−0 12 08.55	Edinburgh	+0 12 44.2
Allegheny, Penn.	+5 20 02.93	Glasgow	+0 17 10.55
Ann Arbor, Mich.	+5 34 55.19	Madison, Wis.	+5 57 37.93
Berkeley, Cal.	+8 09 02.72	Madrid	+0 14 45.12
Berlin	−0 53 34.85	Mexico	+6 36 26.73
Bombay	−4 51 15.74	New York	+4 55 53.64
Cambridge (Eng.)	−0 00 22.75	Paris	−0 09 20.97
Cambridge (Mass.)	+4 44 31.05	Philadelphia	+5 00 38.51
Cape of Good Hope	−1 13 54.76	St. Petersburg	−2 01 13.46
Chicago	+5 50 26.84	Washington	+5 08 15.78

16. Give from the table the difference of local times of Greenwich, England, and of each of the following places and state whether their time is earlier or later than Greenwich time: Albany; Algiers; Ann Arbor; Berlin: Bombay; Chicago; New York; Paris; St. Petersburg.

17. What is the longitude in degrees (°), minutes (′), and seconds (″) of arc of Albany? Of Algiers? Of Cambridge (Mass.)? Of Washington?

18. Which of each of these pairs of places has the earlier local time and what is the difference of time?

Albany and Allegheny.
Ann Arbor and Berkeley.
Ann Arbor and Bombay.
Chicago and Cambridge, Mass.
Chicago and Paris.
Edinburgh and Madison.
Cape of Good Hope and Bombay.
Paris and Bombay.

19. Give the differences of longitude in each case of problem 18.

20. Make and solve other similar problems on the table.

§201. Standard Time.

For convenience of railway traffic a uniform system of time-keeping, known as Standard Time, was agreed upon in 1883 by the principal railroad companies of North America. It was decided that places within a belt of 15°

extending (roughly) $7\frac{1}{2}°$ on each side of the 75th meridian west of Greenwich should use the time of the 75th meridian. All places in similar belts extending about $7\frac{1}{2}°$ on each side of the 90th, of the 105th, and of the 120th meridian should take the times of those meridians respectively, and hence should have times just 1 hr., 2 hr., and 3 hr. earlier (less) than the time of the 75th meridian *time belt.* This system has now been generally adopted by most civilized countries. In practice, however, the dividing lines of the time belts are irregular lines running through railroad terminals. The following map shows the time belts and the names used to distinguish the times of the several belts which cover continental United States. In this system the times at all places in the United States differ only by whole hours.

FIGURE 1

1. When it is 8 o'clock A. M. (Standard Time) at Chicago, what is the time at each of the following places: New York? Pittsburg? St. Louis? Omaha? Denver? Spokane? San Francisco? (See map.)

2. When it is 9 hr. 10 min. 45 sec. P. M. at Omaha, what is the time at New Orleans? Philadelphia? Buffalo? Washington? Austin (Tex.)? Boise City? Los Angeles? El Paso? Salt Lake City? (Refer to a Geography.)

3. Answer the questions of problem 2 for 2: 15 P. M. at St. Paul; for 3: 25 P. M. at Dodge City, Kan.

4. Answer for other places marked on the map such questions as are asked in problems 2 and 3 for Omaha, for St. Paul, and for Dodge City.

5. When it is 10:30 P.M. Pacific time, what is the Mountain time? Central time? Eastern time?

6. When it is 10:30 P.M. Eastern time, what is the Central time? Mountain time? Pacific time?

7. When it is 11:45 A.M. on the 90th meridian, what time is it:

(1) On the 105th meridian?
(2) On the 75th meridian?
(3) On the 120th meridian?
(4) On the 60th meridian?
(5) On the 135th meridian?
(6) On the 45th meridian?
(7) On the 150th meridian?
(8) On the 0th meridian?
(9) On the 180th meridian?

MENSURATION

§202. Areas and Volumes.

We collect here what has been learned of areas and volumes, state the results as laws, and derive some laws of area and of volume of forms not yet studied.

For definition of perimeter, see §195.

§203. The Rectangle.

The area of a rectangle is the product of the base by the altitude. See page 40.

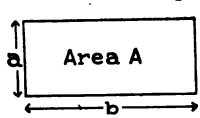


FIGURE 1

The law of area is:

$Area = base \times altitude$, or, $A = a \times b$.

The law may be expressed:

$a = A \div b$, or $b = A \div a$.

State the two latter forms in words and tell for what kind of problems they would be used.

1. How many square inches in a pane of glass $16'' \times 20''$? How many square feet?

2. A room is lighted through 3 windows and a door. Two of the windows have 8 panes each of glass, and the third has 10 panes. The panes are all 10 in. square. The door has a glass $22''$ by $34''$. Through how many square feet of glass does light come into the room?

3. Find the cost, at 4 cents a square foot, of enough wire screen for 12 windows, each $15'' \times 23''$, inside the frame, and 8 windows each $34''$ by $46''$, inside the frame, allowing the screen wire to overlap on the frames 1 inch all around.

4. Find the weight of a sheet of steel $\frac{1}{2}$ in. thick, 4 ft. wide and 30 ft. long, if it weighs 450 lb. per cubic foot.

5. Find the area of a square whose sides all touch, but do not cut, a circle of radius 14 inches.

6. How many square feet of blotting paper would it take to cover the top of your desk?

7. How many squares of roofing in a roof 16 ft. wide by 12 yd. long? (A square of roofing is a 10-foot square.)

§204. The Parallelogram.

The area of a parallelogram is the product of the base by the altitude. See problem 22, p. 131.

The law of area is:

Area = *base* × *altitude*, or, $A = b \times a$. This law may also be expressed:

$a = A \div b$, and $b = A \div a$.

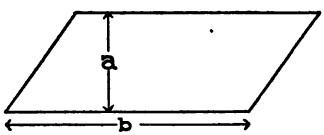


FIGURE 1

State the two latter forms in words. The altitude is the distance *square across*.

1. How many square inches of paper are there in a paper parallelogram $8\frac{1}{2}$ in. long and $3\frac{1}{4}$ inches wide?

2. A city block is a parallelogram 436 ft. by 656 ft. How many square feet does it contain?

3. A strip $49\frac{1}{2}$ ft. wide of a diagonal street 924 ft. long between two cross-streets is to be paved. How many square yards of paving will there be?

4. Find the number of square yards remaining in the area of a parallelogram l ft. long by w ft. wide after a square hole s ft. long and wide has been cut away.

5. Give the value of the ratio of the area of a parallelogram 14 (or l) ft. long and 8 (or w) ft. wide to the area of a rectangle 14 (or l) ft. long and 8 (or w) ft. wide.

6. What is the ratio of any two equal numbers?

§205. The Triangle.

The area of a triangle is $\frac{1}{2}$ the product of the altitude by the base. See problem 28, page 132.

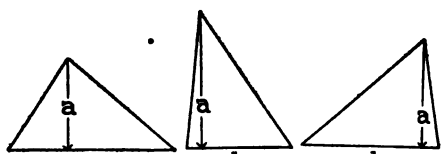


FIGURE 1

Any side of a triangle may be taken as the base; but the altitude is always the perpendicular distance from the opposite vertex to the base.

The law of area is: $Area = \frac{1}{2} \times alt. \times base$, or, $\frac{1}{2} \times a \times b$. This law may also be expressed: $a = \frac{2A}{b}$, or, $b = \frac{2A}{a}$, or $ab = 2A$. Translate these forms into words.

The triangle law might also be written $A = \frac{a}{2} \times b$, or, $A = a \times \frac{b}{2}$. Translate these forms into words.

1. Two triangular gables of a house, each 28 ft. long and 14.5 ft. high, are to be shingled. Find the number of squares to be shingled.

2. A triangular field is 20 rd. long and the altitude is 15 rd. Find the number of acres in it, and show by sketches some of the shapes it might have.

3. The inner part of a quilt is made up of 1650 triangular pieces each having a base of 2 in., and an altitude of 1.73 in. Find the area in square feet of this part of the quilt.

4. A diagonal street cuts from a city block a triangular corner worth \$150 a square foot. The triangle has a base of 212.6 ft. and an altitude of 178.3 feet. What is the triangle worth?

5. The roof of an octagonal tower is made up of 8 triangles each having a base of 6 ft. and an altitude of 14 feet. How many squares of roofing are there in the tower roof?

6. The base of a triangle and also of a parallelogram is 32 ft. (or b ft.), and the altitude of each is 23 ft. (or a ft.). Give the ratio of the area of the triangle to the area of the parallelogram.

7. What is the ratio of any number to its double? Its half?

§206. Areas, Roofing and Brickwork. (THE TRAPEZOID.)

1. Give a rule for computing from its dimensions the area of a square; a rectangle; a parallelogram; a triangle.

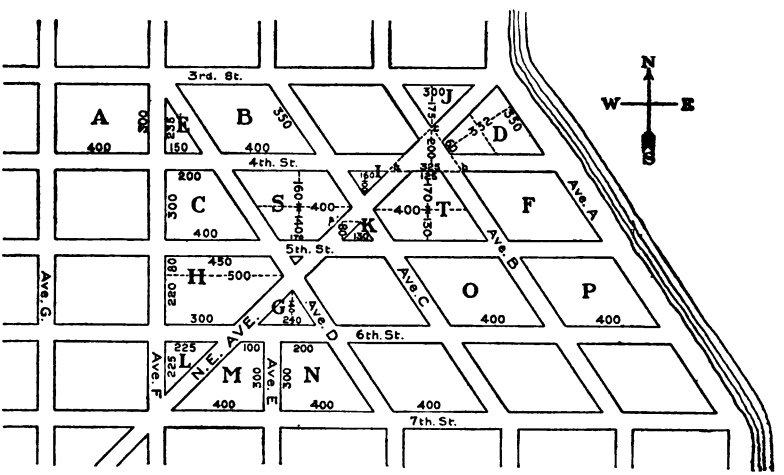


FIGURE 1

Fig. 1 represents a plot of the streets and blocks of a part of a certain city. The streets are all 75 ft. wide between sidewalks. The numbers written on the lines indicate their lengths in feet. The dotted lines show how to divide up the areas into parts for computation.

2. If $240 per foot of frontage on both 3d and 4th streets was paid for block B (Fig. 1, p. 335), how much did the block cost per square foot? Per square yard?

3. Find the area in square yards of other parallelograms, such as O, P, etc., of Fig. 1, p. 335.

4. Find the areas in square feet of the following triangles of Fig. 1, p. 335; E; G; I; J; K; L; *abc*.

Other areas of Fig. 1, p. 335, require a knowledge of trapezoids.

FIGURE 1

A four-sided figure (a quadrilateral) like Fig. 1, having one pair of parallel sides is called a trapezoid. The altitude of a trapezoid is the perpendicular distance between the two parallel sides (the bases).

5. Study the trapezoids of Fig. 2, and find how to get the length of EF, the line connecting the middle points of the two non-parallel sides, from the lengths of the two bases. After computing EF, find for each trapezoid a rectangle whose area equals the area of the trapezoid. Find the areas of the trapezoids of Fig. 2.

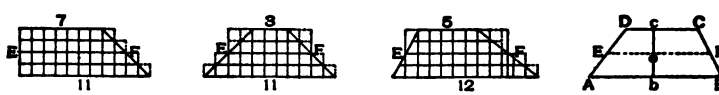


FIGURE 2

The lengths o, b, and c of the last trapezoid are called its dimensions.

$\frac{1}{2}$ the sum of x and y is written $\frac{1}{2}(x+y)$. n times the half sum is written $\frac{1}{2}n(x+y)$.

6. How does the sum of the bases of a trapezoid compare with the base of a parallelogram having an altitude and area equal to the altitude and area of the trapezoid (Fig. 3)?

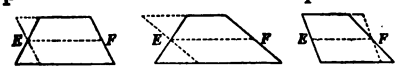


FIGURE 3

7. Supposing *o*, *b*, and *c* are the dimensions, in feet, of the trapezoids of Fig. 2, what are the areas of these trapezoids?

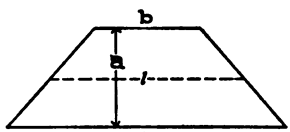
8. Find the areas in square feet of C; of D; of M: of N; of H (Fig. 1, p. 335).

9. Calling z the area of any trapezoid, whose bases are b and c and whose altitude is a, write an equation showing how to find z from a, b, and c.

The area of a trapezoid is the product of the altitude by the half-sum of the parallel sides or by the mid-line.

The parallel sides are the bases, and the altitude is the distance, square across, between them.

The mid-line, l, equals the half-sum, $B+b$. The law of area is:



The mid-line, l=the half-sum of B and b.
FIGURE 1

Area = alt. × half-sum of bases, or,

$$A = a \times \frac{B+b}{2}$$

This law may be expressed also thus:

$$2A = a \times (B + b), \text{ or, } B + b = \frac{2A}{a}, \text{ or } B = \frac{2A}{a} - b \text{ or,}$$

$$b = \frac{2A}{a} - B \text{ or } a = \frac{2A}{B+b}$$

Express some of these forms in words.

10. Find the area in square yards of A, B, N, S, T, Fig. 1, p. 335.

A square of roofing means a 10′ square of roof surface, or 100 sq. ft. A shingle is said to be laid 4″, 4½″, or 5″ to the weather when the lower end of each course of shingles on the roof extends 4″, 4½″, or 5″ below the course next above it.

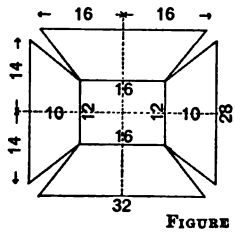
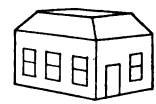


FIGURE 1



11. Draw two perpendicular center lines, and with the aid of the dimensions given in the left part of Fig. 1, complete an enlarged drawing to a convenient scale, of the development of the roof shown on the left.

12. If 1000 shingles laid 4″ to the weather cover a square of roofing, how many shingles will be needed to cover a square, if laid 5″ to the weather? $4\frac{1}{2}$″ to the weather? 3″ to the weather? $3\frac{1}{2}$″?

13. The dimensions on the development (Fig. 1) being in feet, find the cost of the shingles, laid 4″ to the weather, at \$1.10 a bunch of 250, needed to cover the four sides of the deck roof of Figure 1. Find the cost of enough tin to cover the 12′×16′ flat deck at 15¢ a 20″×28″ sheet, the long sides of the sheets being laid parallel to the long side of the deck, and allowing 10% loss for joints and overlap at edges.

14. When shingles are laid 4″ to the weather, 1000 shingles are estimated to cover a square (of roofing). Find the number of shingles, laid 4″ to the weather, needed to cover a roof which, with gables, is made up of the following parts:

SHAPES	ALTITUDES	BASES
2 rectangles	10′	40′
2 parallelograms	12′	$26\frac{1}{2}$′
2 parallelograms	12′	14′
2 triangles	7′	14′
4 triangles	$8\frac{1}{2}$′	17′
2 trapezoids	12′	12′ and $20\frac{1}{2}$′
2 trapezoids	12′	5′ and $13\frac{1}{2}$′
2 trapezoids	12′	6′ and $14\frac{3}{4}$′

15. Find the cost of the shingles of prob. 14 @ 90¢ a bunch of 250.

16. Ten years after building this house a new roof was put on it. It cost \$1.25 per M. to remove the old shingles, \$6 per M. for the new ones, and \$2.75 per M. for the labor of putting on the shingles. How much did it cost to remove the old shingles and to re-roof with new shingles?

17. If it takes 14 bricks per square foot of outside surface to lay a 2-brick wall, how many bricks will be needed to lay the side wall of the building shown in Fig. 1, deducting for 5 windows each $3\frac{1}{2}' \times 6\frac{1}{2}'$ and for 5 each $3\frac{1}{2}' \times 9\frac{1}{4}'$?

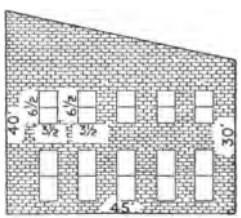


FIGURE 1

18. Make other similar problems from your own measurements or from dimensions obtained from an architect or builder.

19. The walls of a square brick house, $24' \times 24'$, start 6′ below the ground and rise 24′ above the ground to the eaves. From the bottom to 12′ above the ground the walls are 3 bricks thick and from this line to the eaves they are 2 bricks thick. Find the cost of laying the bricks of these walls @ \$4.25 per M.

No allowance for openings of any kind is made in figuring the *cost of laying* bricks in such walls, and outside dimensions are used, thus counting corners twice.

20. From the number of bricks found in problem 19 deduct for the following openings:

Downstairs	Upstairs
1 door $3'6'' \times 6'9''$	2 doors $3'6'' \times 6'6''$
1 door $3' \times 6'6''$	1 window $5' \times 6'6''$
2 windows $5' \times 6'6''$	12 windows $3' \times 6'$
9 windows $3' \times 6''$	

and 1400 bricks for double counting of corners, and find the cost of the bricks @ $\$12\frac{1}{2}$ per M.

§207. The Circle.

The circumference of a circle is π (called pi) *times the diameter.* See §155, p. 245.

For fairly accurate work take $\pi = 3\frac{1}{7}$; for greater accuracy use $\pi = 3.1416$.

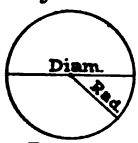


FIGURE 1

The law of circumference is:

Circumf. $= \pi \times$ *diam.*, or, $C = \pi d$, or, $C = 2\pi r$, r being the radius, or $\frac{1}{2}$ the diameter.

The area of a circle is π *times the square of the radius,* or, what is the same thing, π times $\frac{1}{4}$ the square of the diameter. See §156.

The law of area is: $A = \pi r^2$, or, $A = \frac{\pi d^2}{4}$.

§208. The Circular Ring.

The area of a circular ring is the difference of the areas of two circles.

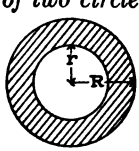


FIGURE 2

The law of area of the ring is:

$$A = \pi \times R^2 - \pi \times r^2 = \pi \times (R^2 - r^2).$$

1. How long is the wire of a wire ring of 1 inch radius? Of 9 inches diameter?

2. How many square inches are there in the area of a circle of 1 inch radius?

3. How many square feet are there in a circle 1 yard in diameter?

4. How many square inches of iron are there in the cross-section of a $1\frac{1}{2}$-inch iron rod?

5. How many square inches of metal in the cross-section of a hollow iron tube of 1 inch outer radius and $\frac{3}{4}$ of an inch inner radius?

6. In a tin circle of 4 inches radius it is desired to make a central circular hole by removing $\frac{1}{4}$ of the tin. Find the radius of the circular hole.

7. How many square inches of tin in the end-surfaces of a tin can 3 inches in diameter?

§209. The Cube.

The surface of the cube is 6 times the square of the edge, and the volume is the cube of the edge.

The law of surface is:

$Surf. = 6 \times e \times e = 6e^2$.

This law may be stated: $e = \sqrt{\frac{A}{6}}$, or, the *square root* of $\frac{1}{6}$ of A.

The law of volume is:

$Vol. = edge \times edge \times edge$, or, $V = e^3$.

This law may be expressed:

$e = \sqrt[3]{A}$, or, the cube root of A.

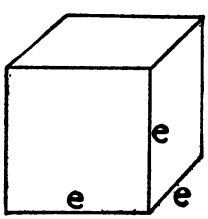


FIGURE 1

§210. The Prism.

The volume of a prism is the product of the area of the base by the altitude. (See §194.)

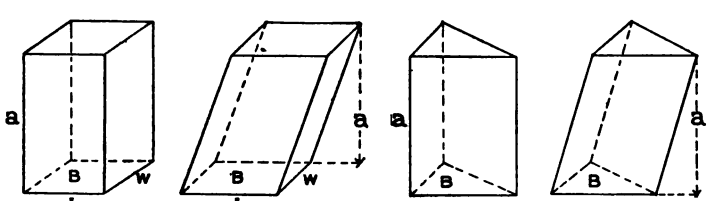


FIGURE 2

The law of volume is:

$Vol. = base \times alt.$, or $V = B \times a$. (See Fig. 2.) This law may also be stated:

$a = \frac{V}{B}$, or, $B = \frac{V}{a}$. If the prism is square cornered: $B = l \times w$, and $V = a \times l \times w$.

For a square cornered prism the law of surface is: $Surf. = 2hl + 2hw + 2lw$. (h is height, or altitude.)

1. Find the surface and volume of a cubical block of 4 ft. edge.

2. How many cubic feet of grain will a cubical bin of $8\frac{1}{2}$ ft. edge (inside measurement) contain?

3. A room is in the form of a 4-yard cube. How many cubic feet of air does it contain?

4. How many square yards of surface are there in the walls, ceiling, and floor of the room of problem 3?

5. How many cubic feet are there in the volume of a tank 3 ft. by $4\frac{1}{2}$ ft. by 10 ft.?

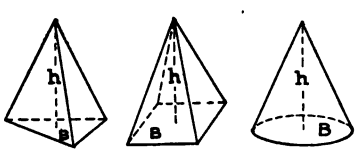
6. How many bushels of wheat, allowing $1\frac{1}{4}$ cu. ft. to the bushel, will a box-car 8 ft. by 30 ft. (inside measurement) contain, when filled to a depth of $4\frac{1}{2}$ ft? 6 feet?

7. Find the volume of a triangular prism of base 18 sq. ft., and height $5\frac{1}{2}$ feet.

8. Find volumes of the forms of hollow beams, each 12 ft. long, having cross-sections like (6)—(10) Fig. 1, p. 318.

9. Give the volumes of the beams of problem 8, if the length of the beam in each case is l feet.

§211. The Pyramid.



Pyramids Cone

FIGURE 1

The contents of a hollow pyramid filled with sand or sawdust, poured 3 times into a hollow prism of the same base and height, will just fill the prism.

Geometry proves that: *The volume of a pyramid is $\frac{1}{3}$ of the volume of a prism of the same base and altitude as the pyramid.* The law of volume of a pyramid is:

Vol. $= \frac{1}{3} \times$ *base* $\times$ *alt.* or $V = \frac{1}{3}B \times a$. This law may also be written: $B = \frac{3V}{a}$, or, $a = \frac{3V}{B}$.

1. Find the volume of a triangular pyramid whose altitude is 18″ and whose base is a triangle having a base of 8″ and an altitude of 5 inches.

2. Find the volume of a pyramid whose base contains 16 sq. in. and whose altitude is 12 inches.

3. The Great Pyramid of Egypt is 481 ft. high and its base is 756′ square. If it were solid and had smooth faces, how many cubic feet of masonry would it contain?

4. At the close of the nineteenth century the United States had 195,887 mi. of railroad track. How many times would these railroad tracks, if placed end to end, encircle the earth? (Use $\pi = 3\frac{1}{7}$. The radius of the earth $= 3959$ miles.)

5. The ties used for these roads would contain wood enough to make a pyramid 1395′ high with a 2192′ square for its base. How many cubic feet of wood were used for the railroad ties?

6. It has been computed that the materials used for the road beds for these railroad tracks would make a solid pyramid 2470 ft. high and having a 3870 ft. square for its base. How many cubic feet would this pyramid contain?

§212. The Cylinder.

If a cylinder is blackened with charcoal, laid on its side, and rolled over once, it will blacken a rectangle as wide as the height of the cylinder, and as long as the distance around the cylinder. The distance around the cylinder is $2\pi \times r$, r being the radius of the base. See §155, page 245.

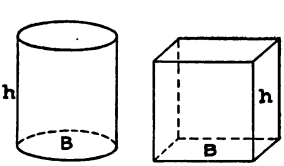


FIGURE 1

The law of area of the curved surface of a cylinder is then: $Surf. = 2\pi \times radius \times height$, or, $S = 2\pi \times r \times h$.

In words, the curved surface is 2π times the radius times the altitude of the cylinder.

This law might also be written: $r = \frac{S}{2\pi h}$, or, $h = \frac{S}{2\pi r}$.

Express the two latter forms of the law in words.

Suppose the number of square inches in the bases of the cylinder and the prism of Fig. 1, p. 343, is the same, say 12.57 sq. in. How many cubic inches of sand are there in a layer 1 in. deep in either the cylinder or the prism? In a layer 4 in. deep? h in. deep?

Geometry proves that:

The volume of a cylinder is the same as the volume of a prism of equal base-area and height.

The law of volume of the cylinder is, then:

$Vol. = height \times base\text{-}area$, or, $V = h \times B$.

The base-area of a circular cylinder is $B = \pi r^2$, r being the base-radius. See §156, p. 246.

The law of volume of a circular cylinder is, then:

$Vol. = \pi \times r^2 \times h$, or, $V = \pi r^2 h$.

1. Find the volumes of these circular cylinders, using $\pi = 3.1416$.

(1)	Diam.	4″	alt.	6″	(4)	Diam.	$18\frac{1}{2}$″	alt.	8″
(2)	"	7″	"	7″	(5)	"	6′	"	10.5′
(3)	"	10″	"	$8\frac{1}{2}$″	(6)	"	$2r'$	"	a'

2. Find the volume of a prism having a base r inches square and an altitude h inches.

3. Compare the result of problem 2 with the formula for the volume of a cylinder of base-radius r inches and altitude h inches.

4. Find the volume of a prism having a base $2r$ ft. square and an altitude h ft.

5. Show that the volume of a cylinder of base-radius r ft. and altitude h ft. equals .7854 times the volume of the prism of problem 4.

6. The inside diameter of the cylindrical water tank of a street sprinkler is 3.6 ft., and its length is 10 ft.; how many liquid gallons will the tank hold?

7. The tank of the sprinkler is filled through a hose of $2\frac{1}{2}''$ inside diameter from a hydrant from which the water flows at the rate of 300 linear feet per minute. How long will it take to fill the tank?

In 1 min. a cylinder of water as large as the inside of the hose and 300′ long flows into the tank.

8. Find the weight of an iron rod $\frac{1}{2}''$ in diameter and 24′ long, if iron weighs 450 lb. per cubic foot.

9. How many cubic inches of air are there in the hollow tire of a bicycle wheel 28″ in diameter (from center line to center line of tube), the hollow having a diameter of $1\frac{3}{4}''$?

10. How many cubic inches of rubber are there in the hollow tire of a 32″ automobile wheel if the inside diameter of the cylindrical tube is 3″ and the outside diameter is 4 inches? (Mean diam. of hollow tire = 32″.)

§213. The Cone.

In § 211 on the pyramid it does not matter what is the *shape* of the base. Only the area is of consequence. The base may even be a *circle*. Instead of a pyramid, we then have a cone, and the law of volume of a cone is:

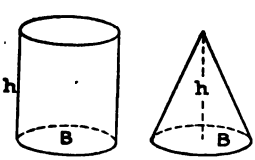
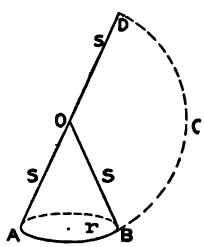


FIGURE 1

Vol. $= \frac{1}{3} \times$ *height* $\times$ *base-area*, or, $V = \frac{1}{3}h \times B$. If the base is a circle, $B = \pi \times r^2$. See § 156, p. 246.

The law of volume of a circular cone is:

$V = \frac{1}{3} \times \pi r^2 \times h$, or, $V = \frac{1}{3} \pi r^2 h$. Or, if a hollow cone, made of stiff paper, be filled with sand or sawdust and poured 3 times into a hollow cylinder of the same base and altitude, it will just fill the cylinder. Since the volume of the cylinder is $\pi r^2 h$, the volume of the cone is: $V = \frac{1}{3} \pi r^2 h$.



Conical Surface
FIGURE 1

To find the area of the curved surface of a cone, suppose a blackened cone to be rolled just once over on a sheet of paper. It will blacken a circular area, Fig. 1, as O B C D, of radius s, the *slant height* of the cone, and such a part of a whole circle as the arc, BCD (which is the length of the circumference of the base) is of a whole circumference of radius s, or, Area of curved surface of cone:

$$= \frac{arc\ BCD}{circumf.\ of\ base} \times area\ of\ a\ circle\ of\ radius\ s$$

$$= \frac{2\pi r}{2\pi s} \times \pi s^2, \text{ (as } arc\ BCD = circumf.\ of\ base\ 2\pi r)$$

$$= \frac{r}{s} \times \pi s^2$$

$$= \pi \times r \times s$$

The law of area of curved surface of a circular cone is:

Surf. $= \pi \times$ *radius* $\times$ *slant height*, or,

$S = \pi \times r \times s$

This law may be expressed: $r = \frac{S}{\pi s}$, or, $s = \frac{S}{\pi r}$

State the two latter forms of the law in words.

Geometry proves that for a right circular cone:

$$S = \pi r s$$

and $V = \frac{1}{3} \pi r^2 h$

1. Find the volumes of these circular cones:

(1) Diameter of base 3″, altitude, 7″
(2) " " 6″ " 20″
(3) " " 6.8″ " 2.25′
(4) " " 3.95″ " 10.25″

2. How many cubic inches of water are there in a funnel-shaped vessel, if the water is 4″ deep and the diameter of the surface of the water is 4.5″? (See Fig. 1.)

3. How many cubic inches of water will it take to fill the same vessel to twice the depth, or 8″? (See Fig. 1.)

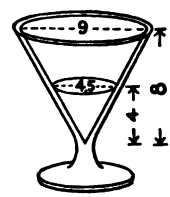


FIGURE 1

4. How many cubic inches had to be poured into the vessel of problem 3 to fill it to 8″ depth if the depth of the water was 4″ at the beginning?

5. A mountain peak has the shape of a circular cone whose altitude is 1.25 mi. and the diameter of whose base is 2.35 miles. Find its volume in cubic miles.

6. How many cubic inches of water will it take to fill a conical vase 10″ deep and 3.225″ across at the top?

7. The conical tower of a building is 24.75′ across at the base and 36.8′ high. How many cubic feet of space does it occupy?

8. A canvas tent has the shape of a circular cone. The base is a circle of 8.7 ft. radius and the altitude is 11.6 ft. Using $\pi = 3\frac{1}{7}$, find the number of cubic feet of space within the tent.

9. If the length of the sloping side (slant height) of the tent of problem 8 is 14.5 ft., how many square feet of canvas are there in the covering of the tent?

10. How many cubic inches are there in an ice cream cone of circular base 3 in. in diameter and of altitude $3\frac{1}{2}$ inches? Use $\pi = 3\frac{1}{7}$.

SURFACE OF THE SPHERE

§214. The Sphere.

If a cord be wrapped around the curved outside surface of a *half* croquet ball (see Fig. 1), as the string is wound

FIGURE 1

around a top, until the surface is covered, and then if the same cord be wrapped around on the flat circular base beginning at the center until the circle is covered, the former length of the cord will be found to be just twice the latter. The area of the surface of the *whole* ball is then how many times the area of the circular section of the ball? What is the length of the radius of this circular section?

It is proved in geometry that:

The area of the surface of a sphere equals 4 times the area of a circular section going through the center of the sphere.

The law of surface of the sphere is:

$S = 4\pi r^2$, r denoting the radius of the sphere.

1. The radius of a ball is $1\frac{1}{2}''$. What is the area of the greatest circular section of the ball? What is the area of the surface of the ball?

2. Measure the circumference of a baseball and from these measurements find how many square inches of leather there are in the cover of the ball.

3. How many square inches of paint will it take to cover the surface of a globe of 10″ radius? Of 10″ diameter?

4. The average diameter of the earth is 7918 miles. How many square miles are there in its surface?

VOLUME OF THE SPHERE

If the entire surface of a globe, or sphere, were divided up into small triangles and rectangles, and the sphere were cut up by planes cutting along the curved sides of these figures and passing through the center, O, the volume of the sphere would be divided up *approximately* into small pyramids, having their vertices at the center (see Fig. 1). The volume of each pyramid would be the product of its base by $\frac{1}{3}$ of the radius of the sphere. The sum of the areas of the bases of all these pyramids would equal the surface of the whole sphere and the sum of the volumes of all the pyramids would equal the surface of the whole sphere multiplied by $\frac{1}{3}$ of the radius of the sphere.

FIGURE 1

1. Calling V the volume and r the radius of a sphere, give the meaning of the formula:

$$V=\frac{r}{3}\times 4\pi r^2=\frac{4}{3}\pi r^3$$

2. Find the number of cubic inches in a sphere of 2″ radius.

3. How many cubic inches are there in a croquet ball 4″ in diameter? In a tennis ball 1.75″ in diameter? In a baseball 2.9″ in diameter? In a globe 10.15″ in diameter?

4. Calling the sun, the moon, and the planets all spheres with diameters in miles as in the following table, compute for two or three of them, circumferences in miles,

surfaces in square miles, and volumes in cubic miles:

Moon.....2160	Mars.....4230	Uranus..... 31,900
Mercury.....3030	Jupiter.....86,500	Neptune.... 34,800
Venus.....7700	Saturn.....73,000	Sun.....866,400
Earth.....7918		

5. What is the ratio of the surface (including the ends) of a cylinder having an altitude 12 inches and a radius of 6 inches, to the surface of a sphere 12 inches in diameter? What is the ratio excluding the end surfaces?

6. What is the ratio of the areas of the curved surfaces of a right cone and a right cylinder, each having an altitude 12 inches and radius of base of 6 inches?

7. Find the ratio of the volume of a sphere 12 inches in diameter to the volume of a right cylinder 12 inches high and 12 inches in diameter.

8. Find the ratio of volumes of a right cone 12 inches high and having a circular base 12 inches in diameter, and a sphere 12 inches in diameter.

9. Find the ratio of volumes of a cone and a cylinder each 12 in. high and having a circular base 12 in. in diameter.

§215. The Right Triangle.

A right triangle is a triangle that has a right angle.

EXERCISE I.—*To find the relation of the squares of the sides of an isosceles right triangle.*

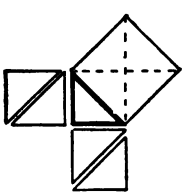


FIGURE 1

Construct an isosceles right triangle (a right triangle having two equal sides) and on each of its three sides draw a square. Draw the dotted lines and cut the side squares as shown in Fig. 1. Fit the pieces over the large square on the hypothenuse (the side opposite the right angle). If the area of each side square were 9 sq. in. what would be the area of the large square?

EXERCISE II.—*To find the relation of the squares of the sides of a right triangle, having one of the shorter sides twice as long as the other.*

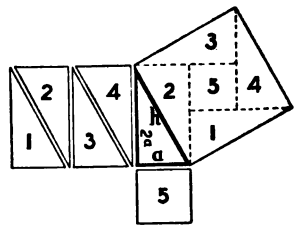


FIGURE 1

Draw a right angle and mark off sides a and $2a$, one twice as long as the other. Draw the hypothenuse, h.

With a square-cornered card draw squares on each of the three sides. Cut off the side-squares. Cut the larger side-square first into two equal rectangles, and then cut each rectangle along diagonals, as shown. Place the several pieces as the numbers in Fig. 1, page 351, indicate. It is plain that the area of the square on the hypothenuse equals the sum of the areas of the side-squares.

EXERCISE III.—*To find the relation of the squares of the sides of any right triangle.*

Draw any right triangle and then draw squares on each of the sides.

Then find the center point of the larger side-square, by marking a point where the diagonals intersect. Through this center point draw a line parallel to the hypothenuse of the given triangle. Through the same point draw also a perpendicular to the parallel. Cut as shown, and place the pieces on the largest square as shown by the numbers in Fig. 2.

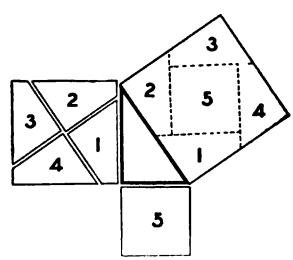


FIGURE 2

Thus, it is clear that *with any right triangle the sum of the squares on the sides equals the square on the hypothenuse.*

1. Denoting the length of either short side of Fig. 1, p. 350, by a, what denotes the area of the square drawn upon this side?

2. Denoting the length of the hypothenuse of Fig. 1, p. 350, by h, what denotes the area of the square drawn on the hypothenuse?

3. Write an equation from Fig. 1, p. 350, showing the relation between a^2 and h^2.

4. Denote the lengths of the three sides of any right triangle (Fig. 2, p. 351) by a, b, and h (h being the hypothenuse). What will denote the areas of each of the squares on the three sides?

5. Write an equation showing the relation of the squares of the sides of any right triangle.

6. The sides of a right triangle are 3″ and 4″. What is the length of the hypothenuse?

$h^2 = 3^2 + 4^2 = 9 + 16 = 25$. What is the value of h?

7. The hypothenuse of a right triangle is 10″, and one of the sides is 6″. What is the other side?

$10^2 = a^2 + 6^2$. Find the value of a.

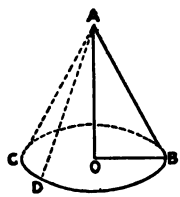


FIGURE 1

If a right triangle, AOB, be turned around one of its sides, AO, as a door turns on its hinges, one side, OB, of the triangle makes the circular base, and the hypothenuse, AB, makes the curved surface of a cone, A- BCD. It is called a *cone of revolution*.

The triangle is called the *generating* triangle.

The side, OA, that the triangle turns on is the altitude of the cone. The side, OB, is the radius of the base, and the hypothenuse is the slant height of the cone.

8. The altitude of a cone of revolution is 4″ and the base radius is 3 inches. Find the slant height of the cone.

9. The slant height of a cone of revolution is 10″, and the altitude is 8″. Find the radius of the base.

10. The slant height of a cone of revolution is 20 ft., the radius of the base is 12 ft. Find the altitude.

SQUARE AND CUBE ROOTS

§216. Squares and Square Roots.

The product obtained by using any number twice as a factor is called the square of that number. Thus, 36 is the square of 6, because 6, used twice as a factor gives 36 ($6 \times 6 = 36$).

The square of a number, as 6, is often written thus, 6^2. What does the small 2 show?

Table of squares of numbers to be learned:

Squares of Units	Squares of Tens	Squares of Hundreds
$1^2 = 1$	$10^2 = 100$	$100^2 = 10{,}000$
$2^2 = 4$	$20^2 = 400$	$200^2 = 40{,}000$
$3^2 = 9$	$30^2 = 900$	$300^2 = 90{,}000$
$4^2 = 16$	$40^2 = 1600$	$400^2 = 160{,}000$
$5^2 = 25$	$50^2 = 2500$	$500^2 = 250{,}000$
$6^2 = 36$	$60^2 = 3600$	$600^2 = 360{,}000$
$7^2 = 49$	$70^2 = 4900$	$700^2 = 490{,}000$
$8^2 = 64$	$80^2 = 6400$	$800^2 = 640{,}000$
$9^2 = 81$	$90^2 = 8100$	$900^2 = 810{,}000$

1. Write all the pairs of numbers which, multiplied together, give the product 36; the product 16; the product 64; the product 49.

Write the pairs of factors of 36 thus: 1 and 36; 2 and 18; 3 and 12; 4 and 9; 6 and 6. Proceed similarly with the rest.

The SQUARE ROOT of a number is one of the two equal factors of it. The sign of square root is $\sqrt{}$, called a radical sign. Thus, $\sqrt{25}$ means the square root of 25, which is 5.

2. Give the square roots of the following numbers:

49; 64; 400; 3600; 490,000;
16; 81; 2500; 160,000; 810,000.

To find the square root of a number not in the table given, it is necessary to learn how the square of a number is formed from the number.

From the table answer the following questions:

3. How many digits are there in the square of any number of units? Of tens? Of hundreds?

4. Find the square of each of the following decimals:

.1	.5	.7	.01	.04	.09	.001	.005	.007
	.3	.9	.03	.06	.07	.003	.006	.009

5. How many decimal places are there in the square of any number of tenths? Of hundredths? Of thousandths?

6. The square of any number of units, then, (as 5) contains only units and tens. What places of *any* number that is a square (as 576) must contain the square of the units (4) of its square root (24)?

7. What places must contain the square of the tens? Of the hundreds? Of the tenths? Of the hundredths? Of the thousandths?

If, then, we separate a number, whose square root is desired, into two-digit groups, beginning at the decimal point and proceeding both toward the left and toward the right, the first group must contain the square of the units of the square root; the second to the left, the square of the tens, etc.

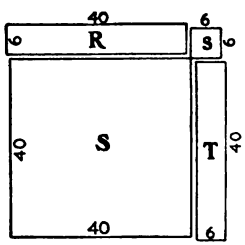


FIGURE 1

8. Find the square of 46.

COMMON METHOD	MEANING
46	$46 = 40 + 6.$
46	$46^2 = (40 + 6)^2 =$
276	$(40 + 6)\ (40 + 6) =$
184	$40 \times 40 + 40 \times 6 + 6 \times 40 + 6 \times 6 =$
2116	$40^2 + 2\ (40 \times 6) + 6^2 =$
$46^2 = 2116$	$1600 + 480 + 36 = 2116$

Show the meaning of the parts of this sum in Fig. 1.

Thus it is seen that the square of a two-figure number is the sum of (1) the square of the tens, (2) twice the product of the tens by the units, and (3) the square of the units.

This shows that if any two-figure number be denoted by $t+u$, where t denotes the tens and u the units, the square is formed thus:

$40+6$	$t+u$
$40+6$	$t+u$
$240+36$	$tu+u^2$
$1600+240$	t^2+tu
$1600+480+36$	$t^2+2tu+u^2=(t+u)^2$

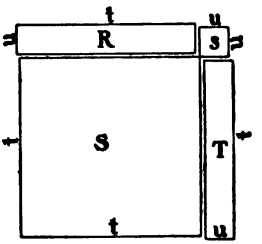


FIGURE 1

Show the meaning of the equation by Fig. 1.

9. Find the square root of 2116.

CONVENIENT FORM

46 = required root.
2116 denoted by $t^2+2tu+u^2$.
1600 = t^2, greatest square (see the table, page 353) in 2116.

$2t=80$	516	contains $2tu+u^2$, where $t=40$.
$2t+u=86$	516	denoted by $2tu+u^2$, where $u=6$.

Check: $46\times46=2116$.

SHORTENED FORM

21'16	46 = required root.
1600	
80 516	
6	
86 516	

10. Find the square roots of the following numbers:

(1) 484	(4) 1156	(7) 5625	(10) 9409
(2) 729	(5) 1225	(8) 7569	(11) 9604
(3) 1024	(6) 1296	(9) 8561	(12) 110,224

11. Find the square root of 2079.36.

CONVENIENT FORM

```
  20'79.36'  |45.6
   16'00.
  ---------
80 | 4'79.
 5 |
---|
85 | 4'25.
  ---------
90. | 54.36
 .6 |
---|
90.6 | 54.36
```

Ans. $\sqrt{2079.36} = 45.6$

Check: $45.6 \times 45.6 = 2079.36$

Separate the number into two-digit groups. Beginning on the extreme left, subtract the greatest square of tens in 20 hundreds, viz., 16 hundreds ($=40^2$), and write 4 tens on the right as the first root digit. Double the 4 tens and use the result 80 as a *trial* divisor. 80 is contained in the remainder, 479, 5 times. Write 5 as the 2d root digit and also add it to the 80, giving 85 as the *complete* divisor. Double the part of the root found, 45, giving 90 for the next *trial* divisor and complete the steps as before.

12. Find the square root of 1578 to 3 decimal places.

CONVENIENT FORM

```
   15'78.00'00'00  |39.724+
    9
   ---
60 | 678.
 9 |
---|
69 | 621.
  ---------
78.0 | 57.00
 .7  |
-----|
78.7 | 55.09
  ---------
79.40 | 1.9100
 .02  |
------|
79.42 | 1.5884
  ---------
79.440 | .321600
 .004   |
--------|
79.444  | .317776
```

.003824, Remainder

Annex zeros and proceed as before.

In practice the decimal point is used only in the root.

Check:

$39.724 \times 39.724 = 1577.996176$

rem. $= \quad .003824$

$1578.$

13. Find the square roots of the following numbers:

(1) 1900.96	(4) 61.1524	(7) .5476
(2) 4719.69	(5) 75.8641	(8) .458329
(3) 5055.21	(6) 79.9236	(9) 1.216609

14. Find to the nearest .001 the square roots of the following numbers:

(1) 5	(4) .85	(7) 1683	(10) 1.85
(2) 7	(5) .25	(8) 6875	(11) 26.79
(3) 15	(6) 125	(9) 7328	(12) 64.893

15. Find, by multiplication, the values of the following expressions:

(1) $(\frac{1}{2})^2$	(3) $(\frac{4}{5})^2$	(5) $(\frac{8}{15})^2$	(7) $(\frac{a}{b})^2$
(2) $(\frac{2}{3})^2$	(4) $(\frac{12}{13})^2$	(6) $(\frac{21}{32})^2$	(8) $(\frac{x}{y})^2$

16. Make a rule for finding the square root of a common fraction.

17. Compare your rule with the following:

Extract the square root of both numerator and denominator, then divide the square root of the numerator by the square root of the denominator.

18. Find, without reducing the common fractions to decimals, the values of the following expressions and prove them by multiplication:

(1) $\sqrt{\frac{1}{4}}$	(4) $\sqrt{\frac{144}{169}}$	(7) $\sqrt{\frac{81}{289}}$
(2) $\sqrt{\frac{4}{9}}$	(5) $\sqrt{\frac{64}{121}}$	(8) $\sqrt{\frac{400}{729}}$
(3) $\sqrt{\frac{16}{25}}$	(6) $\sqrt{\frac{49}{144}}$	(9) $\sqrt{\frac{a^2}{b^2}}$

19. Find to the nearest .001 the square root of:

(1) 25.	(5) 6400	(9) 79.21	(13) 70
(2) 2.5	(6) 640	(10) 7.921	(14) 7
(3) .25	(7) 64	(11) .7921	(15) .7
(4) .025	(8) 6.4	(12) .07921	(16) .07

20. From the results of 19 show that a number made of a given set of figures leads to only two different sets of figures in its square root.

21. The sides of a right triangle are respectively 34″ and 26″ long. How long is the hypothenuse?

22. The center pole of a circus tent is 35′ high, and a guy rope is stretched from the top of the pole to a stake 56′ from the bottom. How long is the rope, if the ground is level and the rope straight, allowing 4′ for tying?

23. 50′ of the top of a tree standing on level ground is broken by the wind and remains fastened to the stump. If the top strikes the ground 30′ from the stump, how high was the tree?

24. A horse is staked out by a rope 40′ long tied to the top of a stake 5′ high. Over what area can the horse graze?

25. The vertical mast of a hoisting derrick 35′ high is held in position by four guy ropes staked to the ground at the four corner points of a square. The stakes are 75′ from the bottom of the mast. How much will the rope for the 4 guys cost at 2.5¢ a foot, 20′ being allowed for knots?

26. What is the area of the square whose corners are at the stakes?

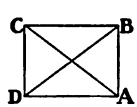


FIGURE 1

27. Find the length of the diagonals of a rectangle 48′×64′.

28. A single-track railroad runs diagonally across a rectangular farm 120 rd.×160 rd. If each line of rails is as long as a diagonal of the farm, how many yards are there in the rails?

29. The center pole of a circus tent is held by 6 ropes tied to stakes at the corners of a regular hexagon (see Fig. 2, p. 359). The ground is level, the pole is 48′ high and a side of the hexagon is 64′. How many feet of rope are needed, if the knots take 180′?

Draw a circle; with compass tips a radius distance apart, step around the circumference, marking where the tips touch the circumference. Join each mark with the next one making a regular hexagon (six-sided figure). Join the marks with the center. The joining lines are radii of the hexagon. Notice that any side of a regular hexagon equals the radius of the hexagon.

30. The sides of an equilateral triangle are 20′ long. A straight line drawn from any vertex to the middle of the opposite side is perpendicular to this side and bisects it. What is the area? See Figure 1.

FIGURE 1

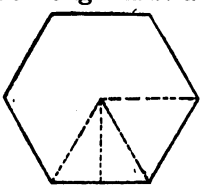


FIGURE 2

31. One side of a regular hexagon is 36′. Find the area of the hexagon.

32. A straight line drawn from the vertex of an isosceles triangle to the middle of the base is perpendicular to the base. Find the area of a regular pentagon whose sides are 24″, if the radius of the circle drawn around the pentagon is 18″. (See Fig. 3.)

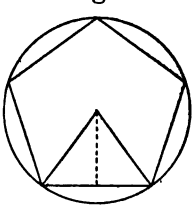


FIGURE 3

§217. Practical Problems in Division and Square Root.

1. The areas in square miles of some of the counties of Indiana are given below. If these counties were square what would the length of a side be? (Ans. to nearest .01 mi.):

Allen	661	Marshall	441
Fayette	216	Ohio	85
Floyd	148	Scott	190
Greene	643	Tippecanoe	503
Marion	397	Vermilion	254

2. The total number of acres of berries in Indiana in 1910 was 3984, and in 1911, 4709. If each of these crops were together in a square field, how long to the nearest .01 mi. would a side of the field be? (1 sq. mi. = 640 acres.)

3. The acreages of alfalfa hay in Indiana in 1910 and in 1911 were 21,742 and 23,911 respectively. If each of the alfalfa crops were in a square field, how long to the nearest .01 mi. would a side of each of the two squares be?

4. The acreages in Irish potatoes in Indiana in 1910 and in 1911 were 55,200 and 43,325 respectively. If each of these acreages made a square field, how long would a side be? (Nearest .01 mile.)

5. The acreages of several other Indiana crops for the years 1910 and 1911 were as given below. Answer questions similar to the questions asked in problems 3 and 4 for each of these crops:

Crop	1910	1911
Rye	73,283	72,815
Barley	7058	7476
Oats	1,558,472	1,623,717
Buckwheat	4670	3782

6. The Indiana acreages in oats and in hay for the five years of column 1, were as given below. If the acreages for each crop were in the form of a square field, how long in miles (to the nearest .01 mi.) would a side of the field be?

Year	Oats	Hay
1879	623,531	1,274,364
1889	1,102,479	2,330,501
1899	1,017,385	2,442,414
1909	1,667,818	2,300,579
1911	1,623,717	1,886,049

§218. Cubes and Cube Roots.

1. What is the volume of a cube whose edge is 5″? 6″? 7″? 11″? 18″? 21″? 24″?

The cube of a number is the product obtained by using the number 3 times as a factor.

Thus the cube of 4 is $4\times4\times4=4^3=64$.

Table of cubes of numbers to be learned:

$1^3=1$	$10^3=1000$
$2^3=8$	$20^3=8000$
$3^3=27$	$30^3=27{,}000$
$4^3=64$	$40^3=64{,}000$
$5^3=125$	$50^3=125{,}000$
$6^3=216$	$60^3=216{,}000$
$7^3=343$	$70^3=343{,}000$
$8^3=512$	$80^3=512{,}000$
$9^3=729$	$90^3=729{,}000$

The cube root of a number is one of its 3 equal factors. Cube root is indicated by the sign $\sqrt[3]{\ }$, called a radical sign.

Thus, $\sqrt[3]{729}$ means one of the three equal factors of 729, which is 9.

2. From the above table tell between what two whole numbers the following cube roots lie:

$\sqrt[3]{16}$ $\sqrt[3]{35}$ $\sqrt[3]{78}$ $\sqrt[3]{450}$ $\sqrt[3]{675}$

$\sqrt[3]{895}$ $\sqrt[3]{58{,}000}$ $\sqrt[3]{480{,}000}$

3. Find by multiplication the values of the following cubes:

$(2.5)^3$ $(6.7)^3$ $(12.5)^3$ $(\frac{1}{2})^3$ $(\frac{2}{3})^3$ $(33\frac{1}{3})^3$ $(18\frac{3}{4})^3$

4. Make a rule for finding the cube of any common fraction.

5. Prove the following relations by multiplication:

$\sqrt[3]{2197}=13$ $\sqrt[3]{4096}=16$ $\sqrt[3]{\frac{125}{512}}=\frac{5}{8}$

$\sqrt[3]{\frac{729}{1231}}=19$ $\sqrt[3]{\frac{33753}{49131}}=\frac{15}{17}$ $\sqrt[3]{27a^3}=3a$

6. Make a rule for finding the cube root of any common fraction.

7. Find the cube roots of the following:

$\frac{1}{8}$ $\frac{27}{64}$ $\frac{125}{343}$ $\frac{729}{8000}$ $\frac{343}{1000}$ $\frac{729}{125000}$

8. Find the cube root of 57,512.456.

57512.456 | 38.6 cube root.

27

$3\times30^2=$ 2700	30512	2700 goes in 30512 about 8 times.
$3\times30\times\ 8=$ 720		
$8^2=$ 64		
3484	27872	$=8\times3484$
$3\times380^2=433200$	2640.456	
$3\times380\times6=$ 6840		
$6^2=$ 36		
440076	2640.456	$=6\times440.076$

Hence, $38.6=\sqrt[3]{57512.456}$.

9. Find the cube roots of the following:

(1) 1,906,624 (2) 12,487,168 (3) 80,621.568 (4) $\frac{1728}{4913}$

10. Find the length of the edge of a cube whose volume is 1953.125 cubic feet.

11. Find the length of the edge of a cube whose volume is $\frac{15625}{32768}$ cubic inches.

RATIO AND PROPORTION

§219. Triangles Having the Same Shape (Similar Triangles).

1. Write the numerical values of the following ratios from Fig. 1:

(1) $\frac{ac}{AC}$ (2) $\frac{bc}{BC}$ (3) $\frac{ab}{AB}$ (4) $\frac{bc}{ac}$ (5) $\frac{BC}{AC}$

(6) $\frac{ab^*}{ac}$ (7) $\frac{AB}{AC}$ (8) $\frac{bc}{ab}$ (9) $\frac{BC}{AB}$

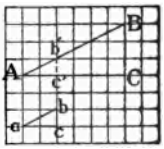


FIGURE 1

2. In Fig. 2 the triangles have the same shape. Write the values of the ratios:

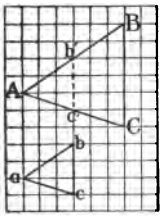


FIGURE 2

(1) $\frac{bc}{BC}$ (2) $\frac{ab}{AB}$ (3) $\frac{ac}{AC}$

3. In Fig. 3 the triangles have the same shape, and $AC = 7 \times ac$. Find the sides of triangle ABC, if $ac = \frac{7}{40}''$, $ab = \frac{1}{4}''$, and $bc = \frac{11}{40}''$.

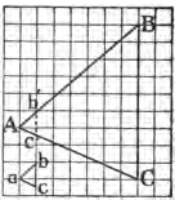


FIGURE 3

4. If ac $(= \frac{1}{7}\ AC)$ represents 7′ (Fig. 3), ab, 10′, and bc, 11′, what lengths do AC, AB, and BC represent?

5. In Figs. 4 and 5, $AB = 4\ ab$. If AB represents 1 mi., BC, 3 mi., and AC, $3\frac{1}{2}$ mi., what distances do ab, bc, and ac represent?

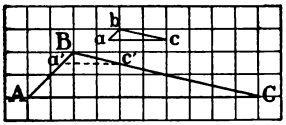


FIGURE 4

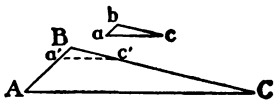


FIGURE 5

* The length of ab is $\sqrt{4+1} = \sqrt{5}$. Why? See Ex. III, page 351.

6. Draw a triangle having sides of $1''$, $\frac{3}{4}''$, and $\frac{1}{2}''$, and another having sides of $3''$, $2\frac{1}{4}''$, and $1\frac{1}{2}''$. Call the angles, opposite the sides $1''$, $\frac{3}{4}''$, and $\frac{1}{2}''$, a, b, and c, respectively, and the angles opposite the sides $3''$, $2\frac{1}{4}''$, and $1\frac{1}{2}''$, A, B, and C, respectively. Do these triangles have the same shape? Carefully cut out the triangles and place angle a over angle A; then place angle b over angle B; and, last, angle c over angle C. What do you find to be true in each case?

In triangles having the same shape, angles lying opposite proportional sides in different triangles are called corresponding angles.

7. Read the pairs of corresponding angles in Figs. 1, 2, 3, and 4, p. 363.

8. In triangles having the same shape, how do corresponding angles compare in size (see problem 6)?

9. In Fig. 4, p. 363, find the ratio of the side lying opposite the angle B in triangle ABC to the side lying opposite the corresponding angle in triangle abc. Find a similar ratio between any other pair of such sides. How do the ratios compare?

10. Answer similar questions for the triangles of Figs. 2, 3, and 4, p. 363.

In triangles having the same shape, sides, as AB and ab (Fig. 4, p. 363) or BC and bc, that lie opposite the equal angles, are called corresponding sides.

11. In triangles having the same shape state a law of relation of corresponding sides.

12. Compare your law with the following:

Corresponding sides of triangles having the same shape have equal ratios.

13. Triangles ABC and ade (Fig. 1) have the same shape. Compare their corresponding angles. What relation holds for their corresponding sides, i. e., how do these ratios compare: $AB:ad$? $BC:de$? $AC:ae$? With a protractor measure each pair of corresponding angles. How do they compare in size?

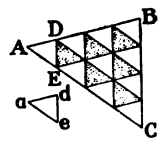


FIGURE 1

14. What is the ratio of the areas of triangles ABC and ade (Fig. 1)? How may this ratio be found from the ratio of a pair of corresponding sides, as AB and ad?

15. Draw a pair of triangles, one having sides of $1''$, $1\frac{1}{2}''$, and $1\frac{3}{4}''$, and the other having sides of $4''$, $6''$, and $7''$. Give the ratio of each pair of corresponding sides; the ratio of the areas of the triangles. How may the ratio of the areas be computed from the ratio of a pair of corresponding sides?

16. State a law connecting the ratio of the areas of triangles having the same shape with the ratio of any pair of corresponding sides.

17. Compare your law with the following:

The ratio of the areas of two triangles having the same shape is equal to the ratio of the squares of any pair of corresponding sides.

18. A pair of corresponding sides of two triangles of the same shape are $3'$ and $15'$. What is the ratio of their areas?

19. The area of the smaller of the triangles is $8\frac{2}{5}$ sq. ft. What is the area of the larger triangle?

20. Compare the ratio of the areas of two squares, one of 3 ft. side and the other of 12 ft. side, with the ratio of the squares of the sides. Does the law of prob. 17 hold also for squares?

§220. Applications of Proportion.

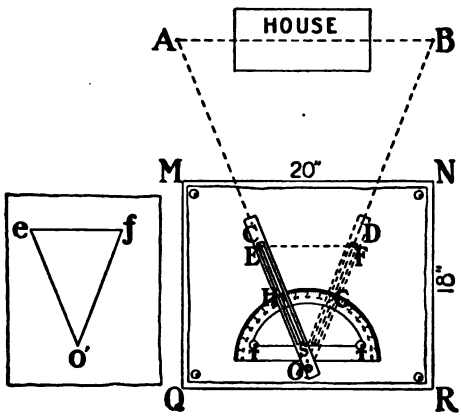


FIGURE 1

In Fig. 1 MNRQ represents an 18″×20″ board, provided with a moving arm, OC, and a protractor. The arm is made of light wood. A slot, ES is cut along the arm and a thread is stretched on its under side and fastened at the ends. The ends of the thread are sunk into the wood so as to permit the thread to move smoothly over the protractor as the arm CO is turned round O. Common pins stuck vertically at C and at O complete the apparatus. The pin at O should be driven through far enough to fasten the arm to the board at O as a pivot. The protractor is held by thumb tacks at T, T. The board may be placed on a window sill, or on a post in a fixed position, while measurements are being made. It is desired to measure the distance from A to B, points on opposite sides of a large building.

Use the proportion form of statement in all problems, calling the length of the unknown line x. Then find the value of x. Solve some problems like this from your own measures. Buildings, steeples, hills, trees, furnish good material for problems.

1. The board was placed on a window sill and the arm set in such position, FO, that when the eye sighted along the pins, O and D, these pins were in line with B. The mark on the protractor at G was 52°. The board being held firmly in place, the arm was swung around O until the pins at O and at C were sighted in line with A. The reading on the protractor at H was 114°. How many degrees were there in the angle EOF?

2. The distances from A and from B to the window at O were measured and found to be 484′ and 520′. A triangle

O′ef (shown in Fig. 1, p. 366) was drawn having angle eO′f = 62°, and O′e = 4.84″, O′f = 5.20″. The side ef was then drawn, measured, and found to be 5.18″. How long is AB?

3. A woodman desires to fell only such trees as will furnish two 10-ft. cuts between the stump and the first limb. He wishes to allow 4 ft. for height of stump and waste in cutting at the bottom and top ends. To test a standing tree, he places a 4-ft. stick QR (See Fig. 1) vertically in the ground 33 ft. from the tree, lies down on his back with his feet against the stake, and sights over the top of the stake to the first limb B. The distance from his eye, P, to the soles of his feet at Q is $5\frac{1}{2}$ feet. Should he fell the tree?

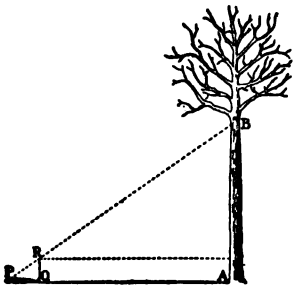


FIGURE 1

4. In Fig. 1, if PQ = 6 ft., RQ = $4\frac{1}{2}$ ft., and QA = 60 ft., how long is AB?

5. How long is AB if PQ and RQ are the length, as in problem 4, and QA = 90 feet?

6. A woodman steps off a distance of 9 steps from a tree, faces the tree, and holds his axe-handle at arm's length in front of his eye, as the ruler is held in Fig. 2, p,

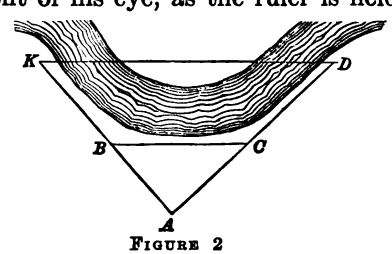


FIGURE 2

242. His arm is 28 in. long, and he finds that 2 ft. of the axe-handle just cover the distance from the ground to the first limb of the tree. If his step is 3 ft. long, how high is the first limb?

7. If AB = 35 rd., AK = 80 rd., and BC = 56 rd., how long is KD (Fig. 2, page 367)?

8. Make measures on objects in your vicinity and solve such problems as are suggested by those given.

9. Ten pounds of potatoes contain 7.83 lb. of water and 2.17 lb. of solid matter. How much water and solid matter would a 60-lb. bushel of potatoes contain? A peck?

10. Of the solid part of 10 lb. of potatoes, 1.8 lb. is starch, .22 lb. is proteid (muscle-food); .1 lb. ash; .04 lb. crude fiber, and .01 lb. fat. How much of each food-substance does 1 bu. of potatoes contain? A peck?

11. What percent of the whole weight of potato is the starch? The proteid? The ash? The crude fiber? The fat?

12. What percent of the solid portion is the starch? The proteid? The ash? The crude fiber? The fat?

13. In 12 oz. of wheat flour there is 1.37 oz. of water; 1.66 oz. of proteid; 8.63 oz. of carbohydrate, and 0.23 oz. of fat. How much water is there in 25 lb. of wheat flour? How much proteid? How much carbohydrate? How much fat?

14. 8 oz. of corn meal contain 1 oz. of water; .736 oz. of proteid; 6.032 oz. carbohydrate, .152 oz. of fat, and .08 oz. of waste matter. How much water is there in a 50-lb. sack of corn meal? How much proteid? How much carbohydrate? Fat? Waste matter?

15. One pound of milk furnishes .033 lb. of proteid, .04 lb. of fat, and .05 lb. carbohydrates. What would it cost to buy milk enough at 2¢ a lb. to furnish 1 lb. of proteid? At 3.5¢ a pound? How much fat would be bought at the same time? How much carbohydrates?

APPLICATIONS OF PERCENTAGE AND INTEREST

INSURANCE

§221. Definitions.

To insure one's person or property, against loss, or injury is to make a written agreement to pay a company a stated sum at stated times for the company's promise to pay a specified sum of money in case of loss of life, or a loss or injury to one's person or property within a stated time.

The person who insures is called the insured.

The written agreement is the policy.

The sum of money to be paid by the company in case of death, or injury, or loss, is called the face of the policy. The face of the policy may also be said to be the amount for which the property or life is insured.

The sum paid for the insurance by the insured is the premium.

The most common kinds of insurance are the following:

Life insurance, or insurance against loss of life;

Fire insurance, or insurance against loss of property by fire;

Marine insurance, or insurance against loss at sea;

Accident insurance, or insurance against loss of life, or injury to the person, or loss or injury of property, from accidents.

Insurance rates are usually stated at a certain price per $100, or $1000, payments of the premium to be made annually, semi-annually, quarterly, or every three or five years, etc. Insurance problems are, however, often treated in the language of rate percent.

§222 Problems.

1. What will it cost to insure $680 worth of household furniture, at $1.75 per $100?

2. An art gallery, valued at $500,000, is insured at the rate of $12.50 per $1000. What is the premium?

3. A vessel, valued at \$16,000, was insured for $\frac{7}{8}$ of its value. Find the face of the policy.

4. A growing crop was insured at \$5 per \$100. The premium was \$140. What was the face of the policy?

5. If it costs \$420 to insure a house for $\frac{3}{4}$ of its value, at $3\frac{1}{2}\%$, or \$3.50 per \$100, what is the house worth?

6. A stock of hardware was insured for \$7000, insurance costing \$110.75. What was the rate of insurance?

7. If a shipment of grain is worth \$840, and the premium amounts to \$17.60, what is the rate of insurance?

8. A machinist insured his tools, valued at \$360, for $\frac{7}{8}$ of their value, paying \$8.19. What rate did he pay?

9. A farmer paid a premium of \$10.50 for insuring his stock at $1\frac{1}{2}\%$, or at \$1.50 per \$100. For what amount was the stock insured?

10. A man paid \$67.50 for the insurance of a steam launch at $1\frac{1}{2}\%$. For what amount was the launch insured?

11. What is the rate paid for insuring a bridge, valued at \$15,000, for $\frac{2}{3}$ of its value, the premium being \$240?

12. A library, worth \$28,000, was insured for $\frac{6}{7}$ of its value, the premium being \$720. What was the rate of insurance?

13. The contents of a grain elevator were insured at a rate of $\frac{3}{8}\%$. What was the amount of the policy, the premium paid being \$272.40?

14. The premium paid for insuring a quantity of lumber, for two-thirds of its entire value, at 3%, was \$36. What was the value of the lumber?

15. A piece of property, valued at \$27,200, was insured for $\frac{5}{8}$ of its value. The premium was \$212.50. What was the rate?

16. A tank of oil, holding 2592 gal., worth 18¢ per gallon, was insured at 4%. Find the premium.

17. A man insured a cargo worth \$2400 at $3\frac{1}{2}\%$. $\frac{5}{8}$ of the cargo was lost at sea. What amount of insurance should the man obtain? What premium had he paid?

18. What is the rate of insurance paid for insuring 26,000 bu. of grain worth 77¢, for $\frac{5}{8}$ of its value, if the premium is \$500.50?

19. What sums must be paid for policies to insure a factory, valued at \$21,000, at $1\frac{2}{3}\%$; a house, worth \$28,000, at $\frac{1}{4}\%$; and a barn, valued at \$5600, at $1\frac{3}{8}\%$?

20. A steamboat had a cargo, valued at \$2000, which was destroyed by fire. The insurance was \$1500. What percent of the value of the cargo was covered by insurance? What was the premium, at $4\frac{1}{2}\%$?

TAXES

§223. Definitions.

The national government needs money for the support of the army and navy, for pensions, salaries of officials, etc.

The state government must care for the blind, the deaf and dumb, the insane, and for criminals; must pay salaries, provide for education, etc.

The county must build bridges, court-houses, etc., must pay salaries of county officers, support education, etc.

Cities and villages need police and fire protection, schools, street-paving, lighting, etc.

Most of these expenses are met by taxes.

A tax is a sum of money levied by the proper officers to defray the expenses of national, state, county, and city governments, and for public schools and public improvements.

There are two kinds of property tax: tax on real estate (as houses, lands, etc.), and tax on personal (or movable) property. In addition to these there is also a poll tax. This is a sum of money sometimes levied on male citizens from 21 to 50 years of age.

The tax rate is usually stated as a certain number of mills on $1, or as a certain number of cents on $100 of assessed valuation.

Assessed valuation means the estimated value of property, fixed by a public officer.

§224. Problems.

1. If the tax rate is 12 mills on the $1, what is the tax on a piece of property assessed at a valuation of $4500?

2. If the tax rate is 75 cents on $100, what is the tax on a factory valued at $480,000, and assessed at $\frac{4}{5}$ of its valuation?

3. The assessed valuation of a town is $2,000,000. A tax levy of $49,000 is made on personal property and real estate, and a collector's commission of 2% is paid. What is the tax rate?

4. What tax should be paid on property and real estate assessed at $6000 as in problem 3?

5. A town wishes to raise $23,500 to build a public hall. If the collector's commission is $2\frac{1}{2}\%$, what is the total amount to be collected?

6. The taxable property of a town is valued at $923,846 and the rate of taxation is $3\frac{2}{3}\%$. What is the whole amount of the tax?

7. The taxable property of a town is $869,472, and the rate is $8\frac{1}{2}$ mills on the dollar. What is the amount of the tax?

8. What amount of money must be collected to raise a net tax of $643,000, allowing 6% for collection?

9. The tax in a city is $49,682; the rate is 15 mills on the dollar. What is the assessed valuation?

10. What is the rate of taxation if property assessed at \$8940 pays a tax of \$160.92?

11. The net amount collected as a tax was \$2896.42. The collector's commission was $3\frac{1}{2}\%$. What was the whole amount collected?

12. The assessed valuation of the taxable property of a town was \$1,218,694. The tax to be raised was \$18,280.41. How many mills on the dollar was the rate of taxation?

13. A tax collector received \$175.18 as his 2% commission on a certain sum collected. The assessed valuation of the taxable property was \$922,000. What was the tax of a man, whose property was valued at \$18,000?

14. The real property of a certain town is valued at \$4,560,800, and the personal property at \$945,900. If \$12,000 is to be raised by taxation, how much must a man pay, whose property is valued at \$9840?

15. A net tax of \$8965 is to be raised in a certain city. The assessed valuation of the taxable property is \$5,689,243. The collector's commission is $1\frac{1}{2}\%$. What will be the tax of a man, whose property is valued at \$56,000?

16. Property on a city street is assessed 2% on its valuation. How much more will a man pay who owns 100 front feet, valued at \$125 per foot, than one who owns property valued at \$7500?

17. If the assessed valuation of the taxable property in a certain town is \$1,069,210, and the whole amount of tax collected is \$13,899.73, what is the rate of taxation and the net amount after deducting the collector's commission of $1\frac{7}{8}\%$?

18. The taxable property in a town is \$872,990. The rate of taxation is .015. What is the net amount collected after deducting the collector's commission of $2\frac{1}{2}\%$?

19. The assessed valuation of the taxable property in a certain city is \$4,968,390. The tax collected amounts to \$94,399.41. How many mills on the dollar was the rate of taxation?

20. A certain town wishes to raise by taxes \$20,500 to build a schoolhouse. What tax must be levied to cover this and the cost of collection at 4%?

BANK DISCOUNT

§225. Discounting Notes.

Discount is a deduction from the amount due on a note at the date of maturity.

In some cases promissory notes do not draw interest. The following is an example:

John C. Cannon purchased a self-binding harvester from A. R. Crow for \$120 and gave him the following note in payment:

> \$120. Peoria, Ill., June 20, 1906.
> Eighteen months after date, for value received, I promise to pay to A. R. Crow, or order, One Hundred Twenty and $\frac{00}{100}$ Dollars, without interest until due.
> Due Dec. 20, 1907. John C. Cannon.

1. On September 20, 1906, Crow sold the note to Adams at such a price that Adams received his purchase money and 8% interest on it until the date of maturity (Dec. 20, 1907). How much did Adams pay for the note?

Any principal at 8% will amount to 110% of itself in 1 yr. 3 mo. Why? Hence, \$120 = 110% of what amount? Or, better, $1.10x = \$120$. Find the value of x.

The sum of money which, at the specified rate and in the time the note is to run before falling due, will amount to the value of the note, when due, is called the present worth of the note.

The difference between the value of the note, when due, and the present worth is called the true discount.

The bank discount of a note is the interest upon the value of the note when due, from the date of discount until the date of maturity.

2. If Crow had sold the above note to a banker at a discount of 8%, the banker would have computed the interest at 8% on $120 from the date of sale (Sept. 20, 1906) until the date of maturity. How much would he have received for the note? Find the difference between the true and the bank discount of the note.

3. C. A. Thomas bought a road wagon of J. K. Duncan giving the following note in payment:

$65. Austin, Texas, Sept. 10, 1900.
Two years after date I promise to pay J. K. Duncan, or order, Sixty-five Dollars, value received, with interest at 7% per annum.
Due Sept. 10, 1902. C. A. Thomas.

On March 10, 1901, Duncan sold this note to a bank at $7\frac{1}{2}\%$ discount. How much did he receive for it?

Remember, bank discount is computed on the *amount* of the note *when due*, for the time to run from date of sale.

4. Counting money worth 7%, how much did Duncan receive for the wagon?

First, calculate the amount due at maturity on Sept. 10, 1902. Next, calculate the discount at $7\frac{1}{2}\%$ for $1\frac{1}{2}$ years on this amount, and deduct it from the amount.

Then find what sum of money at 7% will amount to this difference in $\frac{1}{2}$ year. $1.035x = \$65.76$; find x.

5. A man bought a horse, giving in payment his note for 1 yr. for $85, dated Feb. 26, 1903, and drawing interest at 7% from date. Two months later the holder of the note discounted it at a bank at 6%. What was the discount? What did the bank pay for the note?

6. Find the bank discount and proceeds (cash value at the bank) on the following notes:

	Face	Rate	Date of Note	Date of Maturity	Date of Sale	Rate of Disc.
(1)	$60	6 %	Apr. 6, 1897	Oct. 6, 1898	June 15, 1897	7 %
(2)	$275	6½%	Aug. 7, 1899	Nov. 7, 1901	Mar. 13, 1900	6 %
(3)	$350	7½%	Sep. 10, 1900	Dec. 10, 1902	Jan. 15, 1901	6 %
(4)	$700	8 %	Feb. 15, 1901	Nov. 15, 1903	June 20, 1901	6 %
(5)	$858	6 %	Jan. 1, 1900	July 15, 1903	May 19, 1900	7 %
(6)	$1260	5 %	Feb. 28, 1897	Aug. 31, 1900	Aug. 8, 1897	6 %
(7)	$1800	5½%	June 19, 1897	Aug. 19, 1901	Dec. 29, 1898	6 %
(8)	$2450	7 %	Nov. 18, 1899	Feb. 18, 1902	Dec. 1, 1900	5½%
(9)	$3865	6 %	Dec. 20, 1901	Nov. 20, 1902	Feb. 12, 1902	5 %
(10)	$8600	4½%	Oct. 18, 1900	Jan. 18, 1904	Apr. 2, 1901	4 %

STOCKS AND BONDS

§226. Definitions.

A stock or a stock certificate, is a formal statement issued by a company setting forth the number of shares owned by the holder of the certificate.

The stockholder is generally paid a certain number of dollars per *share* annually, reckoned at a certain rate per-cent of the face value of a share. A share usually has a face value of $100, $500, $50, $1000, etc. A share of mining stock and of some other kinds of stock often has a much smaller face value.

When a stock company pays to the holders of its stock $2 on every $100 of its stock, or 2% on its stock. the company is said to be paying a $2 *dividend*, or a 2% *dividend*.

When stock is paying a high rate of dividend it may sell in the market *above par*, or for more than its face value. If the rate of dividend is low, the stock may sell for less than its face value, or *below par*. When it sells for its face value it is sold *at par*.

A bond is a written agreement made by a national, state, or city government, or by a company, to pay the holder a stated sum of money with interest on it at a stated rate and time. The sum of money to be repaid is called the face of the bond.

Stocks and bonds are usually purchased through an agent, called a *broker*, who makes a business of buying and selling stocks and bonds. He charges a certain percent (usually $\frac{1}{8}\%$) of the face value for buying, and an equal rate for selling. This charge is called *brokerage*.

In the following problems, regard the face value of the stock or bond as \$100 and the brokerage as $\frac{1}{8}\%$, unless otherwise stated.

Following are newspaper quotations on stocks and bonds:

STOCKS

	Open	High	Low	Noon
Amal. Copper	65⅛	66⅛	65⅛	65⅛
Amer. Sugar	129¼	130	129¼	129¼
B. & O. com.	101¼	101¼	100¼	101
B. & O. pfd.	95½	95½	95½	95½
Chi. & Alton com.	36¼	37	36¼	36¼
Chi. & Alton pfd.	71½	72¼	71½	72
Ill. Central R. R.	148	148⅛	148	148⅛
Peoples Gas	105⅛	105⅛	105⅛	105⅛
Pressed Steel com.	65⅛	65⅛	65⅛	65⅛
Pressed Steel pfd.	94⅛	94⅛	94⅛	94⅛
Rubber Goods com.	24⅛	24⅛	24⅛	24⅛
U. S. Steel com.	37⅛	37⅛	37⅛	37⅛
U. S. Steel pfd.	87⅛	87⅛	87⅛	87⅛
St. L. & S. F. com.	78⅛	80⅛	78	80⅛
St. L. & S. F. pfd.	79⅛	80⅛	79⅛	80⅛

BONDS

Closing bid and asked prices for government bonds were as follows:

	Bid	Asked
New 2s	108⅛	109⅛
Coupons	108⅛	109⅛
New 3s	108⅛	109
New 3s coupon	108⅜	109
New 3s small	108	109
Registered 4s	112½	113
Coupon 4s	112½	113
Registered 4s new	139½	139⅛
Coupons 4s new	139½	139⅛
Registered 5s	107⅛	107⅛
Coupon 5s	107⅛	107⅛

BOND TRANSACTIONS

No. Sales		
27,000	Atchison Gen. 4s	100⅛@100⅛
12,000	B. and O. gold 4s	101 @101⅛
1000	B. and O. 3½s	94⅛
1000	C. B. and Q. 4s	106⅛
9000	C. and A. 3½s	76⅛@ 76⅛
25,000	U. P. 4s	102⅛

No. Sales		
5000	U. S. Leather 4s	85⅛
3000	B.&O. coupon 4s	106⅛@106⅛
82,000	Cen.Pac.R.R.3½s	88⅛@ 88⅛
10,000	Wabash R. R. 5s	115⅛@115⅛
10,000	Manhattan 4s	105⅛

§227. Problems.

1. A certain express company has a capital stock of \$500,000, divided among its stockholders in shares of the par value of \$100 each. In 6 mo. the net profits amount to \$10,500, which the company distributes among its stockholders as a dividend. What is the rate of dividend? How much does a holder of 200 shares receive as dividend?

2. What must a man have paid for 400 shares Amal. Copper stock on the date of the table, including brokerage, if he bought at the opening price? At the highest price? At the lowest? At the noon price?

3. Answer same question for U. S. Steel pfd. (preferred).

Preferred stocks are stocks which pay a fixed dividend (say of 7%) before any dividends are paid on common stocks.

Common stocks pay dividends dependent on the net earnings of the company above expenses and dividends on preferred stock.

4. How much did a man gain or lose by buying 2000 Am. Sugar at the opening price and selling at the highest, paying $\frac{1}{8}\%$ brokerage for buying and $\frac{1}{8}\%$ for selling?

5. Answer similar questions for B. & O. com.; for B. & O. pfd.

6. How much does a man make or lose who buys 2500 shares Ill. Cen. R. R. stock at the lowest quotation of the table and sells at the highest, paying brokerage of $\frac{1}{8}\%$ for buying and $\frac{1}{8}\%$ for selling?

7. If a man invests in St. L. and S. F. pfd. stock, paying 7% annual dividend, at the lowest quotation of the table, what interest does he receive on his investment?

The investor must pay $\$79\frac{1}{8}+\$\frac{1}{2}$ per share (of $100), and he receives $7 per year as interest.

8. Answer a similar question on C. & A. pfd.

9. What interest does a man receive on an investment in Government New 2s if he invests at the price "bid," brokerage $\frac{1}{8}$? At the price "asked"?

Government 2s, 3s, etc., are government bonds paying 2%, 3%, etc., annually.

10. Answer similar question for (1) New 3s; (2) New 3s coupon; (3) Registered 4s; (4) Registered 4s new.

11. What rate of interest did a man make who bought 20 B. & O. $3\frac{1}{2}$s at $94\frac{1}{2}$, which were paid off at par with interest, 1 year after purchase?

COMPOUND INTEREST

§228. Definitions.

With certain classes of notes, if the interest is not paid when due, it is added to the principal and this amount becomes a new principal, which draws interest. Savings banks add the interest on savings deposits at each interest-paying period and pay interest on the entire amount.

Interest on interest is called compound interest.

Bankers also collect their interest on loans and then reloan the interest, and in this way virtually receive compound interest on their money.

Compound interest is usually payable annually, or semi-annually.

§229. Problems.

1. Find the compound interest on a note of **$200** at 6% for 5 yr., interest payable annually.

Solution:

Interest on $200	for 1 yr. at 6%	=	200	×	$ .06	=	$ 12.00
Amount of $200	for 1 yr. at 6%	=	200	×	$ 1.06	=	$212.00
Amount of $212	for 1 yr. at 6%	=	200	×	$(1.06)^2	=	$224.72
Amount of $224.72	for 1 yr. at 6%	=	200	×	$(1.06)^3	=	$238.23
Amount of $238.23	for 1 yr. at 6%	=	200	×	$(1.06)^4	=	$252.52
Amount of $252.52	for 1 yr. at 6%	=	200	×	$(1.06)^5	=	$267.68

The compound interest = $267.68 − $200 = $67.68.

The $267.68 is called the *compound amount*. The expressions $(1.06)^3$, $(1.06)^4$, and $(1.06)^5$ mean $1.06\times1.06\times1.06$, $1.06\times1.06\times1.06\times1.06$, and $1.06\times1.06\times1.06\times1.06\times1.06$, and are read "1.06 cube," "1.06 to the 4th power," and "1.06 to the 5th power."

2. A man paid compound interest at 6%, interest compounded annually, on a note of $150 for 7 yr. How much was required to pay the note?

3. A note for $500, bearing 6% interest, has 3 interest coupons attached, the coupons, when due, bearing 7% interest, payable annually. If the note runs 4 yr., no coupons being paid in the meantime, how much money will be required to pay the entire debt at the end of this time?

Many notes have coupon notes attached for the amount of interest due at each interest-paying period. These coupon notes usually bear a higher rate of interest than does the note itself.

4. A note of $450, bearing interest at 7%, interest payable semi-annually, coupons bearing 8% interest, amounts to what sum at the end of $4\frac{1}{2}$ yr., no coupons being paid until the end of the time?

SAVINGS DEPOSITS

§230. Definitions.

Savings banks receive, use, and pay interest on the savings of persons who deposit their money and leave it in the bank to the end of an interest term.

The *interest term* is usually 6 months, and the rate of interest paid by the bank for the privilege of using the deposit is usually 3%, or 4%. *Interest days* are the days on which an interest term ends and the next term begins.

If, at the end of the interest term, the interest is not withdrawn, it is added to the deposit. The interest for the next period is then paid on the whole amount of the former deposit and the accrued interest. In this way savings banks pay *compound interest.*

Interest is usually calculated only on the whole number of dollars on deposits. Thus, if $255.45 is the deposit during the interest term, the interest is calculated on $255, say at 3% for $\frac{1}{2}$ year, giving $3.83. This is added to the $255.45, however, giving $259.28, and $259 draws interest for the next term.

When the account is drawn upon during the interest term the sum of the withdrawal is subtracted from the first deposit, or from the sum of the first and second deposits, etc., and interest is paid on this difference and on all later deposits for the whole number of months to the end of the term.

Interest terms are usually 6 months, beginning Jan. 1 and July 1; or 3 months, beginning on the first of Jan., Apr., July, and October.

In the following problems interest terms are taken as 6 months.

§231. Problems.

1. A man puts \$450 into a savings bank which pays 3%, and leaves it through 2 interest terms of 6 months. He then draws out both the deposit and the accrued interest. How much money does he draw out?

2. Find the withdrawal value of the following savings deposits, the rate and the number of interest terms being as shown:

	Deposit	Number Interest Terms	Rate Paid
(1)	\$75	2	3 %
(2)	\$150	3	3 %
(3)	\$365	3	4 %
(4)	\$1275	2	4 %
(5)	\$2560	3	$3\frac{1}{2}$%
(6)	\$4060	3	3 %
(7)	\$6575	2	4 %
(8)	\$5672.90	2	$3\frac{1}{2}$%

3. A man puts \$30 a month in a savings bank which pays 3%, for 12 months, and leaves it all until the last deposit has been in a full interest term. What is then the withdrawal value of his account?

4. A man opens a savings account with a bank paying 3% on deposits, and reserves the privilege of withdrawing money from the account at any time. The deposits and withdrawals, with dates, are shown here:

Date	Deposits	Withdrawals	Balance
Jan. 1.....	$280		
Feb. 26.....		$135	
Mar. 20.....	75		
May 2.....		90	
May 20.....	150		
June 15.....		80	
June 20.....	110		

Find for each date of withdrawal, the total deposits, the total withdrawals, and the balance. The interest term running from Jan. 1 to July 1, find the withdrawal value of this account on July 1.

5. A savings deposit of $1200 remains in a bank paying 3%, for 5 interest terms. What amount is due on the account at the end of the 5th term? Carry calculations to the 3d decimal place and shorten each amount to the nearest .01.

PARTIAL PAYMENTS

§232. United States Rule.

For calculating the balance due on a note or bond on which partial payments have been made, nearly all the states have adopted the following rule, known as "The United States Rule of Partial Payments," which has been made the legal rule by a decision of the Supreme Court of the United States:

Rule.—Find the amount of the principal to the time when the payment or the sum of the payments equals or exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal and proceed as before.

§233. Problems.

1. Find the balance due on the following note at maturity:

$1250. Chicago, Ill., May 21, 1900.

On or before two years after date I promise to pay to the order of P. A. Hopper, Twelve Hundred Fifty and $\frac{00}{100}$ Dollars at the Corn Exchange National Bank, for value received, with interest at 6 per cent per annum. JOHN P. MILLER.

Due May 21, 1902.

The indorsements on the back of this note were as follows:

Nov. 21, 1900..... $80.00
P. A. Hopper
Feb. 21, 1901..... $10.00
P. A. Hopper
May 21, 1901.....$150.00
P. A. Hopper
Feb. 21, 1902.....$500.00
P. A. Hopper

SOLUTION:

Principal on May 21, 1900		$1250.00
Interest for 6 mo. on $1		.03
Interest due Nov. 21, 1900	$ 37.50	
Amount Nov. 21, 1900 (date of first payment on $80)		$1287.50
First payment		80.00
New principal Nov. 21, 1900		$1207.50
Interest for 3 mo. on $1		.015
Interest to Feb. 21, 1901 (date of second payment of $10)	$ 18.11	
Payment being less than interest no settlement is made,		$1207.50
Interest for 6 mo. on $1		.03
Interest to May 21, 1901 (date of third payment of $150)	$ 36.23	
Amount May 21, 1901		$1243.73
Sum of second and third payments ($10+$150)		160.00
New principal May 21, 1901		$1083.73
Interest for 9 mo. on $1		.045
Interest to Feb. 21, 1902 (date of fourth payment of $500)	$ 48.77	
Amount Feb. 21, 1902		$1132.50
Fourth payment		500.00
New principal Feb. 21, 1902		$ 632.50
Interest for 3 mo. on $1		.015
Interest to date of maturity, May 21, 1902	$ 9.49	
Balance due at maturity, May 21, 1902		$ 641.99
Answer		$ 641.99

2. The following payments were made on a $650 note, bearing 7% interest and dated April 20, 1901:

July 30, 1901	$75.00
Jan. 15, 1902	$15.00
Aug. 12, 1902	$175.00
Jan. 1, 1903	$50.00

Find the amount due April 20, 1903.

3. On a note of $2800, dated Feb. 23, 1900, and bearing 7% interest, the following payments were made:

Feb. 23, 1901	$100.00
July 16, 1901	$50.00
Jan. 1, 1902	$800.00
July 15, 1902	$85.00
Nov. 28, 1902	$380.00

Find the amount due Feb. 23, 1903.

PROBLEMS FOR GENERAL REVIEW

§234. Practical Application for Review.

1. Counting 9 hills of corn to 2 rods, how many hills of corn are there in a row 80 rd. long?

2. Counting 9 rows 80 rd. long to the acre, how many hills of corn are there in an acre?

3. Counting $1\frac{1}{2}$ ears on an average per hill and 100 ears to the bushel, how many bushels would the acre of problem 2 yield?

4. How many bushels under the same conditions would a 40-acre field yield?

5. What would be the value of the corn-yield per acre (prob. 3) if corn sells for 40¢ a bushel?

6. Supposing it costs \$4.44 to raise 1 acre of the crop (prob. 5), what would be the net profit per acre?

7. Supposing money worth 6%, what was the value per acre this year of the field of problem 5?

8. The county that had the largest acreage of corn in 1909 of any county in a certain state, had 203,289 acres, yielding 3,862,491 bu. worth \$2,124,370.05. Find the average yield per acre and price per bushel.

9. The acreage of corn in the same county in 1910 was 238,018, yielding 3,332,252 bu. worth \$1,699,448.52. Find the average yield per acre and price per bu. of this county's corn in 1910.

10. In 1909, 183,573 A. of corn in a certain county yielded 3,671,460 bu. worth $2,129,446.80, and in 1910 the figures were 219,680 A. yielding 4,832,960 bu. worth $2,416,480.00. Find the yields per acre, and prices per bu. for this county's corn for each year.

11. For another county the corresponding figures were:

	Acreage	Yield	Value
1909	183,889 A	4,045,558 bu.	$2,144,145.74
1910	237,781 A.	1,902,248 bu.	$951,124.00

Find the average yield per acre and price per bu. for each year.

12. N. Y. did $\frac{1}{6}$, Penn. $\frac{1}{7}$, Ill. $\frac{1}{10}$, and Mass. $\frac{1}{13}$ of the manufacturing of the United States in 1900. What part was done by N. Y. and Penn. together? By N. Y. and Ill.? Penn. and Ill.? Penn. and Ill. and Mass.? By all four of these states?

13. What was the difference of the parts done by:

N. Y. and Penn.	Penn. and Ill.
N. Y. " Ill.	Penn. " Mass.
N. Y. " Mass.	Ill. " Mass.

14. What was the difference between the part done by all these 4 states and by all the other states together?

15. Represent the steel production of the United States by a line $3\frac{1}{4}$ in. long. If Germany produces $\frac{5}{9}$ as much steel as the United States, how long a line will represent the steel production of Germany?

16. Great Britain produces $\frac{1}{3}$ as much steel as the United States. Using the scale of problem 15, how long a line will represent the steel production of Great Britain?

17. France produces $\frac{1}{8}$ and Russia $\frac{1}{10}$ as much steel as the United States. Using the scale of problem 15, how long lines will represent the steel production of these countries?

18. The relative figures for pig iron production for 1903 for the leading countries are: The United States 1; Germany $\frac{5}{9}$; Great Britain $\frac{1}{2}$; France $\frac{1}{6}$; Russia $\frac{1}{8}$; Austria-Hungary $\frac{1}{12}$, and Belgium $\frac{1}{16}$. Representing the production for the United States by a strip 3.06 in. long, how long strips will represent the productions of the other countries?

19. The relative figures for coal production in 1903 are: The world 1; the United States $\frac{2}{5}$; Penn. $\frac{2}{15}$; Ill. $\frac{2}{35}$; W. Va. $\frac{4}{45}$; Ohio $\frac{1}{13}$. Representing the coal production of the U. S. by a strip $2\frac{1}{8}$ inches long, how long must strips be to represent the production of the world and each of the four states mentioned?

20. The relative figures for cotton production for 1904 are: The world $1\frac{2}{15}$; the United States 1; Texas $\frac{1}{5}$; Georgia $\frac{1}{8}$; Miss. $\frac{1}{9}$; Ala. $\frac{1}{12}$. Representing the world's cotton production by a strip $2\frac{1}{8}$ in. long, find the lengths of strips to represent the production of the United States and of each of the 4 states named.

21. The relative figures for the 1900 corn crop are: The United States 1; Ill. $\frac{1}{6}$; Iowa $\frac{1}{7}$; Kan. $\frac{1}{11}$; Neb. $\frac{1}{12}$; Mo. $\frac{1}{13}$; Ind. $\frac{1}{16}$. Representing the crop of Mo. by a strip $1\frac{3}{16}$ in. long, find the lengths of strips to represent the crops of the United States and of the other five states mentioned.

22. For the 1893 wheat crop the relative figures are: The world 1; Russia $\frac{2}{9}$; U. S. $\frac{1}{5}$; France $\frac{1}{8}$; India $\frac{1}{10}$; Austria-Hungary $\frac{1}{15}$; Italy $\frac{2}{45}$. Representing the wheat crop of the world by a strip $5\frac{5}{8}$ in. long, find lengths of strips to represent the crop of each of the other countries mentioned.

Urban population means people living in cities and towns, and **rural** population means people living on farms, or as we sometimes say, people living *in the country*. The *total* population of a state is made up of its *urban* and its *rural* populations. In a school geography the urban and

the total populations of the 7 most populous states are presented by strips of the following lengths, in inches:

State	Total	Urban	Rural
N. Y.	$2\frac{5}{8}$	$1\frac{3}{4}$	.....
Penn.	$1\frac{7}{8}$	$1\frac{1}{8}$	.....
Ill.	$1\frac{13}{16}$	$\frac{7}{8}$	.....
Ohio	$1\frac{9}{16}$	$\frac{5}{8}$	.....
Mo.	$1\frac{3}{16}$	$\frac{3}{8}$	.....
Texas	$1\frac{3}{16}$	$\frac{3}{16}$	.....
Mass.	$1\frac{1}{16}$	$\frac{7}{8}$	.....

23. Write out the length of strips to represent, to the scale used here, the rural populations of these states.

24. How long a line would represent, to the same scale, the entire urban population of these 7 states? The total population of the 7 states? The entire rural population of the 7 states?

Keep the results of the next two problems for use in problem 27.

25. The total population of N. Y. state is about $\frac{1}{10}$ of the entire population of the United States. What part of the entire population of the United States is that of Penn.?

Call the fraction that is to be found x. Then write:

$x : \frac{1}{10} = 1\frac{7}{8} : 2\frac{5}{8}$, or $\frac{x}{\frac{1}{10}} = \frac{1\frac{7}{8}}{2\frac{5}{8}}$ and solve for x.

Thus, $x = \frac{1}{10} \times \frac{15}{8} \times \frac{8}{21} = \frac{1}{14}$ Ans.

26. What part of the entire population of the United States is the total population of Ill.? Of Ohio? Of Mo? Of Texas? Of Mass.? Of all the 7 states? The entire urban population of the 7 states?

27. Express the results of problems 25 and 26 in decimal fractions to the nearest .01.

§235. Miscellaneous Review Problems.

1. Find the sum of seven hundred and 15 thousandths; six thousand four and twenty-five hundred thousandths; nine hundred sixty-five and fifty-six ten thousandths, and fifteen thousand fifteen and fifteen hundred thousandths.

2. Find the difference between one hundred thousand one hundred and one thousandth, and eight thousand ninety-seven and six hundred three ten thousandths. Check the result.

3. Find the product of six hundred thirty-two and forty seven and three-fifths hundredths by eight and seventy-six and four-ninths hundredths. Write the result correct to the nearest .1; to the nearest .01; to the nearest .001.

4. A rate of 30 mi. an hour is equivalent to how many feet a second?

5. How many quarter-inch cubes can be fitted into a box 8 in. by 10 in. by 12 inches?

6. Find the value of $22.071\frac{9}{28} \div 68.36\frac{1}{4}$. Write the result correct to the nearest .01.

7. Telegraph poles are set 99 ft. apart. A train passes 8 poles every 10 seconds. Find the rate of the train in miles per hour.

8. How many square yards are there in the surface of a 21-inch cube?

9. A dealer buys a 50-ton car load of coal at \$6.75 a long ton (2240 lb.), and sells it at \$7.75 a short ton (2000 lb.), and suffers a shrinking in the car of 3 short tons. Find the dealer's rate percent of profit on the car load.

10. How long is a string that just reaches from a corner of a room $10 \times 12 \times 16$ feet to the diagonally opposite corner? Answer to the nearest .01 foot.

11. A man takes 2 steps, each 28 in. long, every second. Find his rate of walking in miles per hour.

12. Calculate the surface in square yards of the greatest cube that can be cut from a wooden globe 6 feet in diameter.

13. A chimney, standing on level ground, is stayed by 4 guy wires, each 120 ft. long and attached 96 ft. from the ground. The ground ends are at the corners of a square. Find the area of the square in square yards.

14. If a loaf of bread weighs 1 lb. when wheat is worth 90 cents a bu., how much should a loaf weigh when wheat is worth $1.20 a bushel?

15. Find the area of the part of a circle of 5 inches radius that lies outside of the greatest square that can be cut from that circle. Use $\pi = 3\frac{1}{7}$.

16. On a lot of ground 160 ft. square, apple trees are set 20 ft. apart in rows so spaced that the trees of any row stand opposite the middle of the spaces of the rows beside it, and so that the trees of the second row are 20 ft. from those of the first row. This plan is followed over the whole lot. What is the largest number of trees that can be placed on the lot, no tree being nearer the sides of the lot than 10 ft.?

17. One man can plow a field in 5 days and another can plow it in 6 days. How long will it take both plowing together?

18. Five men plow 9 days in a field and then 3 men plowing at the same rate finish the field in 4 more days. At the same rate, how long would it have taken 3 men, working continuously, to plow the field?

19. The corners of a square are so cut away as to leave a regular octagon with 4-inch sides. A circle is then inscribed in the octagon. Find the area between the circle and the octagon to the nearest .001 square inch.

20. A hardware dealer bought a bill of goods listed at \$1875, discounts 10% and 5%, and 5% cash. What amount would pay the bill in cash?

21. Which is the better investment, and by how much, $3\frac{1}{2}\%$ bonds at 95, or 4% bonds at 115?

22. A note for \$860 dated Mar. 10, 1911, due Mar. 10, 1912, with interest at 5% from date was discounted at a bank Dec. 5, 1911, at 6%. Find the proceeds.

23. A troy ounce (480 gr.) of pure gold is worth \$248.04. How much is an avoirdupois pound of pure gold worth?

24. A speed of 60 miles an hour is equivalent to how many feet per minute? Per second?

25. Find the cost of plastering the walls and ceiling of a room 27 ft. long by 24 ft. wide by 13 ft. high at 27 cents per square yard, making no allowance for openings.

26. In 1901 the total imports of tea into the United States were 83,395,924 lb. and of coffee, 511,041,459 lb. If the average cost of tea was 37¢ per lb., and of coffee, 18¢ per lb., what was the total cost of both?

27. What will it cost to plaster the four walls and the ceiling of a hall 60 ft. long, 36 ft. wide, and 18 ft. high, at $22\frac{1}{2}$¢ per square yard, after deducting 486 sq. ft. for door and window openings?

28. A man husks two rows of corn, each row 80 rd. long, and the rows 44 inches apart. Find the yield per acre at the same rate if the two rows yield 12 bushels.

29. Two vertical chimneys stand on level ground 76 ft. apart. One chimney is 48 ft. tall and the other is 105 ft. tall. Find the distance between their tops.

30. From a field which is 80 rods square there was sold a piece of 80 square rods. Find the value of the remainder at \$120 per acre.

31. A. is 308 yd. north of a gate. B. is 277 yd. south of it. Both start at the same time to walk toward the gate, A. at the rate of $2\frac{1}{3}$ yd. per second and B. at 2 yd. per second. In how many seconds will they be equally distant from the gate?

32. Reduce $\frac{3}{4}\frac{5}{9}\frac{2}{6}$ to its lowest terms.

33. Find the L. C. M. of 87, 970, and 7035.

34. Add $\frac{7}{13}$, $\frac{15}{23}$, and $\frac{18}{43}$.

35. Subtract $\frac{15}{31}$ from $\frac{21}{32}$.

36. Add 3 to both the numerator and the denominator of $\frac{7}{8}$ and subtract $\frac{7}{8}$ from the result.

37. Subtract 1 from both the numerator and the denominator of $\frac{5}{8}$ and subtract the result from $\frac{5}{8}$.

38. Add 1 to both the numerator and the denominator of $\frac{12}{13}$ and subtract $\frac{12}{13}$ from the result.

39. Subtract 5 from both terms of the fraction $\frac{20}{21}$ and subtract the result from $\frac{20}{21}$.

40. Prove by an example of your own choosing that the value of a fraction is changed by adding the same number to both of its terms.

41. Prove by an example of your own choosing that the value of a fraction is changed by subtracting the same number from both of its terms.

42. Prove by an example of your own that the value of a fraction is not changed by multiplying both of its terms by the same number.

43. Prove that the value of a fraction is not changed by dividing both terms of a fraction by the same number.

44. How many yards of cloth @ $\$2\frac{3}{4}$ can be bought for $\$30\frac{1}{4}$? At \$d can be bought for \$D?

45. A piece of wire $72\frac{3}{4}$ in. long was cut into pieces each of which was $9\frac{3}{32}$ in. long. How many pieces were there?

46. A rectangular piece of copper plate contained $\frac{1}{6}$ sq. in. and was $\frac{5}{9}$ in. long. How wide was it?

47. A rectangular sheet of paper contains $\frac{737}{1152}$ sq. ft. and is $8\frac{3}{8}$ in. wide. How long is it?

48. Change to decimals the following:

$$\frac{3}{4} \quad \frac{17}{25} \quad \frac{49}{75} \quad \frac{135}{525} \quad \frac{1}{1200}$$

49. Change to common fractions:

$$.33\tfrac{1}{3} \quad .37\tfrac{1}{2} \quad .62\tfrac{1}{2} \quad .44\tfrac{4}{9} \quad .55\tfrac{5}{9} \quad .11\tfrac{1}{9} \quad .42\tfrac{8}{7}$$

50. Find the cube of $\frac{22}{35}$.

51. Find the *cube root* of 34.328125.

52. Simplify $\frac{2.16}{32.5} \times \frac{6.5}{1.18} \div \frac{.708}{6.25}$.

53. Find the sum of 328, 24.85, .73, 4161, and 700.603.

54. The minuend is 600.0031 and the subtrahend 78.7869. Find the difference.

55. Find the sum and the product of 87.5, $38\frac{5}{16}$, and 3.89.

56. The sum of two numbers is 1 and one of the numbers is .18379. Find the other number.

57. The product of two numbers is .0963976 and one number is 37.076. Find the other number.

58. Find to the nearest .001 rod the length of the diagonal of a square whose perimeter is 80 rods.

59. Find, in feet, to the nearest .001, the side of a square whose area is 1 acre.

60. How many sq. rd. does a rectangle $24\frac{3}{8}' \times 121\frac{1}{2}'$ contain? Find the result to nearest .001.

61. How many grains does 1 cu. in. of water weigh if 1 cu. ft. of water weighs $62\frac{1}{2}$ pounds? (Give result to the nearest .01.)

62. $\frac{3}{7}$ is what percent of $\frac{8}{9}$? $54\frac{11}{16}$ is what percent of $62\frac{1}{2}$?

63. What percent profit does a grocer make who buys berries by dry measure and sells them by liquid measure, buying and selling at the same price per quart?

64. At what price must the following articles bought by the dealer under the specified conditions, be marked to gain 35%, after allowing a discount of 25%.

12 Tables	@ $16.00	$ 192.00
10 Bedsteads	@ 40.00	400.00
6 Dressers	@ 20.00	120.00
6 Davenports	@ 48.00	288.00
		$1000.00
Less 10%		100.00
		$ 900.00

65. At what price should the articles bought as specified in problem 64 be marked to make a profit of 30%, after allowing discounts of 25% and 20%?

66. Mark the same articles, bought as in problem 64, to make a profit of 25%, after allowing for a loss from bad debts of 20%, and after allowing a discount of 20% and 10%.

67. A wholesale dry goods merchant buys in France a stock consisting of two qualities of silk, paying for one $7.38\frac{1}{3}$ francs per meter and for the other, 22 francs per meter. Calling a yard .914 meter and a franc $.193, find to three decimal places the prices per yard of the two grades of silk in United States money.

68. If the dealer of problem 67 must pay an exchange rate of $20\frac{1}{4}$ cents per franc, how much do the two grades of silk of the last problem cost in United States money?

69. What must the dealer of problem 67 mark the two qualities of silk that he may drop 20% from the marked price and still make $33\frac{1}{3}\%$ on his investment, the goods being sold in America?

70. What must the dealer mark the two qualities of silk, if he must pay an import duty of 40% on bringing the silks into this country, that he may drop 20% from the marked price and still make $33\frac{1}{3}\%$ on his investment?

71. A man contracts to do a piece of work for \$1000, to be paid \$200 at the end of a year; \$200 at the end of 2 years; and so on for 5 payments. Counting money worth 6%, find the value, at the time of contracting, of the first payment; of the second payment.

72. An author is paid a $4\frac{1}{2}\%$ royalty on the wholesale price of a book which sells, retail, at 45 cents. The wholesale price is 90% of the retail price. What is the amount of the author's royalty on 16,400 books sold?

73. Find to the nearest .01 ft. the perimeter and the diagonal of 1 square acre.

74. Find the length of the diagonal, to the nearest .01, from one lower corner of a room 12.6 ft. $\times$ 16.4 ft. $\times$ 9.8 ft. to the opposite upper corner.

75. How many square feet of area are there in the surface of a spherical ball 2 ft. in diameter?

76. Find the radius, to the nearest .01 in., of a circle whose area is 1 square yard.

77. Find the radius in inches, to the nearest .01, of a sphere whose surface is 1 square yard.

78. Find the radius to the nearest .01 in. of a sphere whose volume is 1 cubic yard.

79. A room, containing 6000 cubic feet, is twice as wide, and 3 times as long, as high. Find to the nearest .01 the length of the diagonal from a lower corner to the opposite upper corner.

80. From a square piece of tin is cut the largest possible circle. What percent of the square is cut away?

81. A wooden cylinder has an 8-in. radius. What is the length of the side of the largest square prism that can be sawed out of it?

82. Calculate the area of a rectangle whose dimensions are the diameter and circumference of a sphere of 10 inches radius, and compare it with the area of the surface of the sphere.

83. A right circular cone whose base is 2 inches in diameter is cut from the top of a similar cone, whose base is 4 inches in diameter. The altitude of the lower part is 6 inches. Find the volume of the top cone.

84. The diameter of a circle is 14 inches. The width of a rectangle of equal area is 7 inches. What is its length?

85. The outside diameter of a circular ring cut from a cast-iron plate 1 in. thick, is $10\frac{1}{2}$ inches. The hole in the center of the ring is $3\frac{1}{2}$ inches in diameter. Cast iron is 7.2 times as heavy as an equal volume of water. Find the weight of the cast-iron ring.

86. The sum of the two acute angles of a right triangle is 90°. The acute angles are to each other as 1:2. Find the angles.

Call one angle, x, and the other $2x$. Thus, $x+2x=90°$.

87. The sum of the 3 angles of any triangle is 180°. The angles are to each other as 1:3:5. Find the angles.

88. A regular octagonal flower-bed (having 8 equal sides) is made by cutting off the corners of a 12-ft. square. The distance from the center to a corner of the octagon is 6.494 feet. Find the area of the octagon to the nearest .01 square foot.

89. Find the area of a circle whose circumference is 80 rods.

90. Find the area of a rectangle whose length is 3 times its width, and whose perimeter is 80 rods.

91. The radius of a circle is $12\frac{542}{1309}$ rods long. Calling $\pi = 3.1416$, find the number of posts, set 1 rod apart, needed to put a fence on its circumference.

92. If wrought iron is 7.68 times as heavy as water, how heavy is a solid wrought iron ball having a radius of 7.4432 inches? (Call $\pi = \frac{22}{7}$)

93. A solid block is composed of a circular cylinder surmounted by an inverted hemisphere of the dimensions shown in Fig. 1. How many cu. in. are there in the solid?

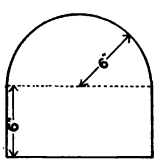


FIGURE 1

94. Suppose the moon's path to be a circle of radius 60 times the radius of the earth (=4000 mi.) and that the moon makes a complete circuit once in $27\frac{1}{3}$ days. Find the speed of the moon's motion in miles per hour. (Take $\pi = \frac{22}{7}$)

95. Find the moon's speed under the same conditions as in problem 94 in feet per second and compare it with the speed of a rifle ball moving 1100 ft. per second.

96. Find the simple interest at $5\frac{1}{2}\%$ on \$248.60 from Feb. 16, 1904, to Apr. 28, 1907.

97. How long will it take \$120.00 to amount to \$150.00 at 4% simple interest?

98. Calculate the simple interest on this note:

$865.00 Emporia, Kan., Oct. 12, 1899.

Five years after date, I promise to pay W. H. Upton, for value received, at the First National Bank of Emporia, ($865.00) eight hundred sixty-five and $\frac{00}{100}$ dollars with interest at 6% from date.

Signed: Robert O. Wallace.

Due Oct. 12, 1904.

99. What sum of money loaned Sept. 10, 1904, at 6% simple interest will amount to $324.32 July 18, 1910?

GENERAL INDEX

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